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THEORETICAL AND EXPERIMENTAL HEAT TRANSFER IN ROCKET ENGINE WITH SOLID PROPELLANT

Izabel Cecilia Ferreira de Souza Vicentin
Carlos Henrique Marchi
Department of Mechanical Engineeering, Federal University of Paraná, Centro Politécnico, Curitiba, PR, Brasil
izabeldesouza@gmail.com, chmcfd@gmail.com

Antônio Carlos Foltran
Diego Fernando Moro
Department of Mechanical Engineering, Positivo University, 5300 Pedro Viriato Parigot de Souza, Curitiba, PR, Brazil
antoniocarlos.foltran@gmail.com, difmoro@gmail.com

Nicholas Dicati Pereira da Silva
Marcos Carvalho Campos
Luciano Kiyoshi Araki
Department of Mechanical Engineeering, Federal University of Paraná, Centro Politécnico, Curitiba, PR, Brasil
ndicati@gmail.com, mccampos@ufpr.br, lucianoaraki@gmail.com

Abstract. *Accurate determination of heat flux is an important task not only in designing but also in calculation of the performance of rocket engines. In this work, the heat flux in the combustion chamber is calculated using inverse method. In this approach, the transient heat flux is determined from experimental data measured at the outer wall of the rocket engine. The physical phenomenon was modeled by the transient one-dimensional heat equation in cylindrical coordinates. The chamber material properties of the were treated as constant. The inverse problem is solved by least squares modified by the addition of Tikhonov regularization term of zero order. The sensitivity coefficients were obtained by Duhamel's theorem, so the methodology is applicable to linear problems. By using the regularization parameter, it was possible to generate good results despite the considerable experimental errors.*

Keywords: *combustion chamber, Tikhonov regularization, heat flux, conduction problem.*

1. INTRODUCTION

In a trust chamber (nozzle and combustion chamber), the amount of energy transferred as heat to the chamber walls is between 0.5 to 5 percent of total energy generated (Sutton, 1992). Nevertheless, this amount could be enough to cause structural failure. Thus, to prevent the chamber and nozzle walls from fail, it is necessary the knowledge of heat flux accurately. Furthermore, accurate determination of heat flux is also important in calculation of rocket engine performance.

For heat transfer determination, usually the combustion properties must be calculated. Due to the difficulties in predicting the combustion properties, these quantities have large uncertainties.

In order to minimize these uncertainties, Inverse Heat Conduction Problem (IHCP), or Inverse Problem (IP) for short, can be used. The standard heat conduction problem (Direct Problems or DP) is concerned with the determination of temperature distribution in the interior of the solid material. In contrast, IHCP is concerned with the determination of boundary condition by utilizing the measurement temperature history at one or more locations in the solid (Beck *et al*., 1985). The main advantage in using inverse problem is to avoid the calculus of physical properties of gas combustion in order to obtain the heat flux from combustion (Kimura, 1987).

The main difficulty in solving inverse problems is associated from the fact that they are ill-posed (Ozisik and Orlande, 2000). According to Hadamard, DP are well-posed because the solution exists, is unique and is stable under small changes in input data. The problems that not satisfy one or more of these conditions are called ill-posed (Ozisik and Orlande, 2000). In order to overcome the difficulty in solving ill-posed problems, a variety of analytic and numerical approaches have been proposed. Analytical solutions were proposed by Stoltz (1960) and Burgraff (1964). In his work, Stoltz used the Duhamel theorem to solve the heat equation for slab and spheres and Burgraff (1964) presented a solution based in series. Analytical solutions are restricted to linear problems so the technique of IP was extended to nonlinear problems by the use numeric methods. The first researcher to use a nonlinear problem in IP was Beck (1970). In his work, the Fourier equation was solved using temperature dependent properties (specific heat and conductivity). Williams and Curry (1977) calculated the heat flux for the nonlinear case in materials of high and low thermal conductivity using the least squares method. Metha (1981) used the Finite Difference Method (FDM) to determine the heat flux in a rocket engine.

The least squares method modified for a regularization term was first used by Tikhonov and Arsenin (1977). The authors showed that this method is efficient in solving systems of ill conditioned equations, resulting from data with considerable experimental errors. The authors' emphasis is performing an analysis of the stability of existing methods (quite limited in relation to stability) and the proposed method. The authors also discussed methods for determining optimal values of the regularization parameter. Alifanov and Rumyantsev (1987) presented the conjugate gradient method. In this minimization method, the regularization is implicit, made during the iterative procedure.

Simpler techniques require less computational cost, however, they may not have the desired stability. Otherwise, more sophisticated techniques can generate more accurate results, although it usually requires more computational cost. Currently, several methods are combined to take advantage of two or more approaches, sophisticated or not, in order to obtain a resulting algorithm (hybrid) more competitive.

This work is concerned in predicting the heat flux in combustion chamber using Least Square method modified by the Thikonov regularization parameter (Tikhonov and Arsenin, 1997). Much work has been done to obtain heat flux in nozzle, especially in throat (Mehta, 1981; Kimura, 1987; Patire Jr., 2010). The objective of this work is, by utilizing IP, to estimate the data of heat flux values in rocket combustion chamber.

2. PROBLEM DEFINITION

The combustion chamber used in this work has cylindrical geometry. Thus, the physical model could be described as a hollow cylinder of internal and external radius R_i and R_o respectively. In Fig. 1, the heat flux *q"(t)* from combustion is applied at the boundary surface at the internal radius R_i. At the external surface, at radius R_o, it is reasonable to consider insulated surface. The experimentally measured temperature *Y* is also showed in Fig. 1.

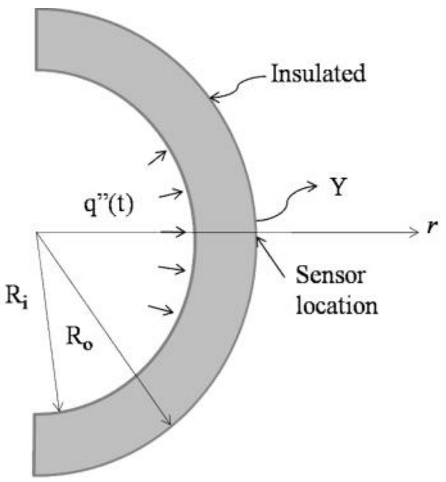


Figure 1. Geometry of problem.

The hypotheses considered for this problem are: one-dimensional in transient heat transfer (in radial direction) and the physical properties of combustion chamber are temperature and time independent.

Mathematical model, Eq. (1), is written (Ozisik, 1993):

$$^\circ\, c_p \frac{\partial T}{\partial t} = \frac{k}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right), \tag{1}$$

where k is the thermal conductivity in W/(mK), ρ is specific mass, in kg/m^3, c_p is specific heat in J/(KgK), T is the dependent variable and r and t are the independent variables.

The boundary condition at the outer surface is given by Eq. (2) (Ozisik, 1993):

$$\frac{\partial T}{\partial r} = 0 \tag{2}$$

The boundary condition at the inner surface is given by Eq. (3) (Ozisik, 1993):

$$-k\frac{\partial T}{\partial r} = q''(t) \tag{3}$$

The initial condition is Eq. (4):

$$T = T_0, \tag{4}$$

where T_0 is the initial temperature.

2.1 Direct problem (DP)

As previously commented, DP is the standard procedure which is concerned with determination of temperature distribution inside of solid. All properties, boundary conditions and initial condition are prescribed, including the boundary condition on the inner surface *q"(t).* Thus, the DP can be readily solved by classical solution techniques.

The DP was solved numerically using Finite Volume Method (FVM) (Versteeg and Malalasekera, 1995). The physical domain was discretized using a uniform mesh (all elements of size Δr) as indicate in Fig.2:

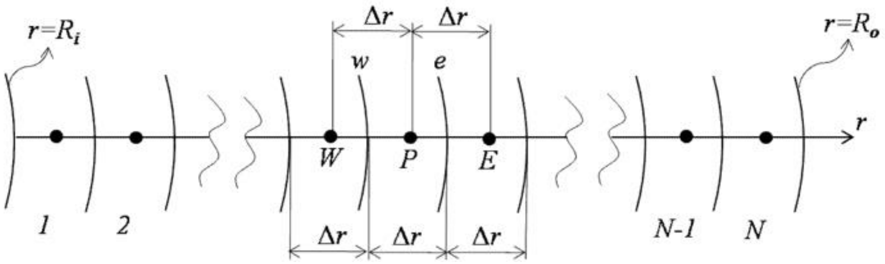


Figure 2. Uniform one-dimensional mesh.

Figure 2 illustrates internal and boundary volumes for *N* elements. Each internal volume *P* has a neighbor to the left, *W*, and to the right, *E*. Similarly each volume has faces to the left, *w*, and to the right, *e*.

The mathematical model, Eq. 1, was discretized using FVM, which provides the general equation:

$$a_P T_P = a_w T_W + a_e T_E + b_P. \tag{5}$$

The approximation used in Eq. (1) to obtain the coefficients and source term to the internal volumes was CDS-2 scheme (Central Differencing Scheme) (Maliska, 2004; Versteeg and Malalasekera, 1995). The coefficients a_P, a_w and a_e and the source term b_P, are:

$$a_P = \frac{\Delta r^2 r_P}{\alpha\,\Delta t} + r_e\theta + r_w\theta, \tag{6}$$

$$a_e = r_e\theta, \tag{7}$$

$$a_w = r_w\theta, \tag{8}$$

$$b_P = T_E^0 r_e(1-\theta) + T_W^0 r_w(1-\theta) + T_P^0(\theta-1)[r_e + r_w] + \frac{\Delta r^2 r_P}{\alpha\,\Delta t}T_P^0, \tag{9}$$

where θ is the parameter which defines the formulation used in time. In this work Crank Nicholson was used.

The approximation used to obtain the coefficients and source term of the first volume, *N=1*, was the DDS (Downstream differencing scheme) (Maliska, 2004; Versteeg and Malalasekera, 1995). The coefficients and source term are:

$$a_P = \frac{\Delta r^2 r_P}{r_e \sim \Delta t} + {}^{-}, \tag{10}$$

$$a_e = {}^{-}, \tag{11}$$

$$a_w = 0, \tag{12}$$

$$b_P = T_E^0 (1 - {}^{-}) + T_P^0 ({}^{-} - 1) + T_P^0 \frac{\Delta r^2 r_P}{r_e \sim \Delta t} + \frac{q''(t) r_w \Delta r}{r_e k}. \tag{13}$$

Finally, the approximation used to obtain the coefficients and source term of the last volume, *N*, was the UDS (Upstream differencing scheme) (Maliska, 2004; Versteeg and Malalasekera, 1995). The coefficients and source term of the last volume are:

$$a_P = \frac{\Delta r^2 r_P}{r_w \sim \Delta t} + {}^{-}, \tag{14}$$

$$a_e = 0, \tag{15}$$

$$a_w = {}^{-}, \tag{16}$$

$$b_P = T_W^0 (1 - {}^{-}) + T_P^0 ({}^{-} - 1) + T_P^0 \frac{\Delta r^2 r_P}{r_w \sim \Delta t}. \tag{17}$$

The system of linear equation of Eq.(5) was solved using the Tomas' algorithm, TDMA (Tridiagonal matrix Algorithm) (Maliska, 2004). The DP was implemented using the FORTRAN/2008 language with double precision. It was used ten volumes to obtain the solution.

2.2 Inverse problem

The mathematical formulation of IP is identical to the DP except that the boundary condition in Eq. (3), that is q"(t), is unknown. The IP consists in to determine the heat flux *q"(t)* taking account that physical properties and geometry data are known.

Ill posed problems do not satisfy one or more Hadamard's conditions of existence, uniqueness and stability. The existence and uniqueness of an IP are guaranteed by requiring that inverse solution minimize the least squares norm rather than make it necessarily zero.

The measured temperature on the outer surface is symbolized by:

$$Y_j, \qquad j = 1, 2, \ldots M, \tag{18}$$

where *j* is the instant which the temperature is measured and *M* is the total amount of measurements.

The estimated temperature on the outer surface is symbolized by:

$$\hat{T}_j(\hat{q}''_j), \qquad j = 1, 2, \ldots M, \tag{19}$$

where the superscript ^ denotes the estimated values.

In order to solve the IP is required that the estimated temperature on the outer wall, (computed from the solution of the direct problem by Fourier Equation) should match the measured temperatures as closely as possible over a specified tolerance. In order to do that, the least square norm is minimized. Here, the least square norm is modified by addition of a zero-order regularization term (Ozisik, 1993; Arsenin and Tikhonov, 1977; Beck *et al.*, 1985):

$$S(\hat{\mathbf{q}}'') = \sum_{j=1}^{M} [Y_j - \hat{T}_j(\hat{\mathbf{q}}'')]^2 + \sim \sum_{j=1}^{M} \hat{q}''^2_j, \tag{20}$$

where S is the sum of squares and α is the regularization parameter.

The traditional least squares on the right-hand side of Eq. (20) is added to a zero-order regularization term in order to reduce instability or oscillations that are inherent to the solution of ill-posed problems. The instability problem is solved by using the regularization term. The next step in the analysis is the minimization of the least square Eq.(20) by differencing it with respect to each of the unknown heat flux components (Ozisik 1993). After some algebra, it is possible to write the solution for the heat flux in the matrix form:

$$\mathbf{q}'' = (\mathbf{X}^T \mathbf{X} + \sim \mathbf{I})^{-1} \mathbf{X}^T (\mathbf{Y} - \mathbf{T}_0), \tag{21}$$

where **X** is the matrix of sensitivity.

Equation (21) is the formal solution of the inverse heat conduction problem, considered for the unknown surface heat flux. Once the regularization term α, the matrix of sensitivity **X** and the measured temperature data Y are available, the surface heat flux ***q"*** is computed from Eq. (21).

The matrix of sensitivity is, by definition the first derivatives of the dependent variable (i.e., temperature) with respect to the unknown parameter (i.e., heat flux), Eq. (22):

$$\mathbf{X} \equiv \frac{\partial \mathbf{T}}{\partial \mathbf{q}''^T} = \begin{bmatrix} \frac{\partial T_1}{\partial q''_1} & \frac{\partial T_1}{\partial q''_2} & \cdots & \frac{\partial T_1}{\partial q''_M} \\ \frac{\partial T_2}{\partial q''_1} & \frac{\partial T_2}{\partial q''_2} & & \frac{\partial T_2}{\partial q''_M} \\ \vdots & & & \\ \frac{\partial T_M}{\partial q''_1} & \frac{\partial T_M}{\partial q''_2} & \cdots & \frac{\partial T_M}{\partial q''_M} \end{bmatrix}. \tag{22}$$

The matrix of sensitivity represents the changes of dependent variable with respect to the changes of the unknown parameter. Because the direct problem can be readily solved by classical solution techniques, the sensitivity are easily determined. For this study, the direct problem was solved numerically, using Duhamel's theorem (Ozisik, 1993).

2.3 Stability analisis

To examine the accuracy of the predictions by the inverse analysis, consider that a known heat flux having a triangular shape, Eq. (23), is applied on the inner surface of hollow cylinder of Fig. 1

$$q''(t) = \begin{cases} 5t & 0 \le \mathrm{t} \le 1 \\ -5t + 10 & 1 < \mathrm{t} \le 2 \end{cases}. \tag{23}$$

The measured data is simulated by solving the direct problem using the heat flux known. Thus, solving the direct problem the temperature over the whole domain can be determined, including the temperature on the outer surface. Then the measure data is generated by introducing a random error ω, as follows:

$$Y_j^* = T_{exact} + \mu, \tag{24}$$

where ω has values between -5.0 and 5.0 The exact value of the surface heat flux is given by Eq. (23).

Figure 3 presents the heat flux **q"** estimated from Eq. (21) using different values of regularization parameters. For small value of α (i.e. α=1.10^{-12}) the solution exhibits oscillatory behavior, whereas for α=1.10^{-10} the solution is damped and deviates from the exact result. The regularization parameter whose satisfactorily approximates the solution to exact result satisfactorily is 1.10^{-11}.

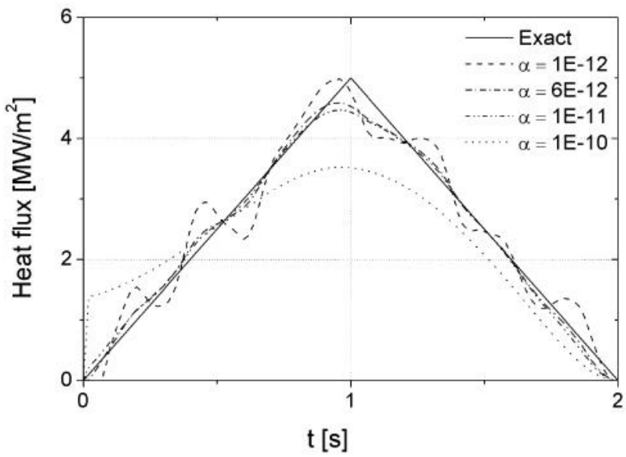


Figure 3. Effect of regularization parameter α on the stability of inverse solution.

2.4 Literature comparison

In this subsection, data of two works from the literature are used to compare to methodology presented. In order to accomplish the first comparison, an analytical correlation from Kacynski*et al.* (1987) is used. In this case the experimental temperature data was took from Kimura (1987), as well as the physical properties of rocket nozzle (Tab. 1):

Table 1. Physical properties of nozzle, from Kimura (1987).

Property	Value
Thermal conductivity (W/(mK))	45.6
Specific heat (J/(kgK))	1256.0
Specific mass (kg/m)	1830.0

The correlation developed by Kacynski et al. (1987) is an analytical solution significantly simplified by the hypothesis that the temperature derivative over time is independent of the spatial derivative over time. Figure 4 presents the results, where is possible to note good agreement between the solutions.

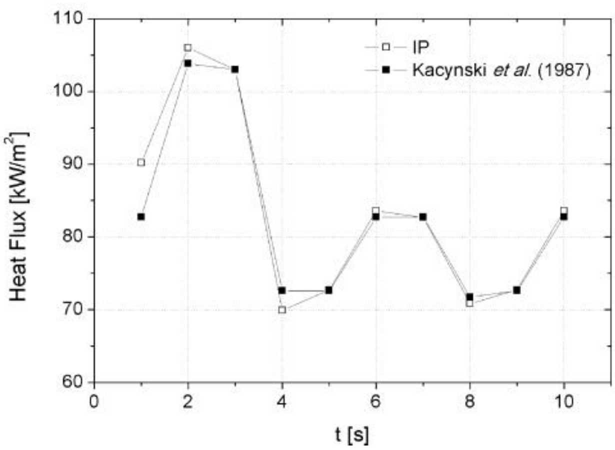
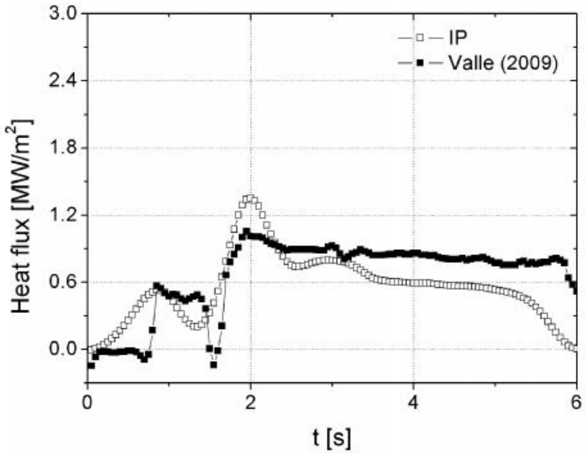
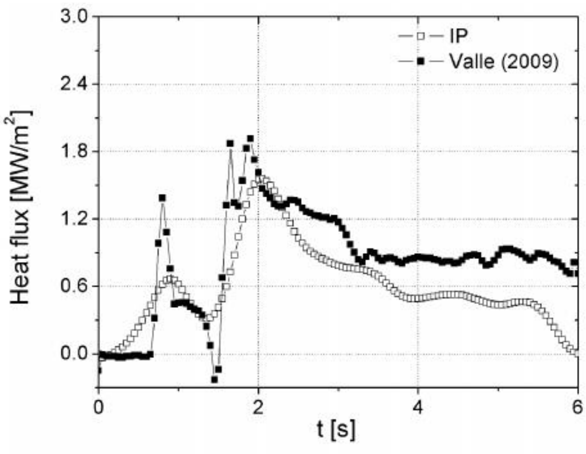


Figure 4. Comparison between Kacynski*et al.* (1987) analytical correlation and the solution for IP of this work.

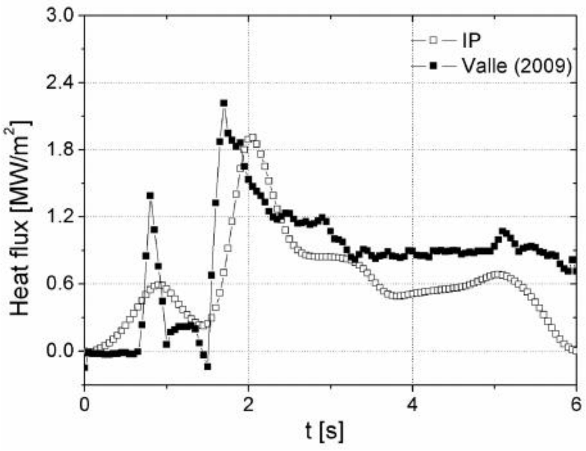
For the second problem, data from Valle (2009) is used to perform the comparison. Valle (2009) used the same boundary conditions presented previously to solve the nonlinear IP based in least square method. The geometry was a slab and the author used rectangular coordinates. Figure 5 presents the results of the heat flux in different regions of rocket nozzle (from (a) to (e)):



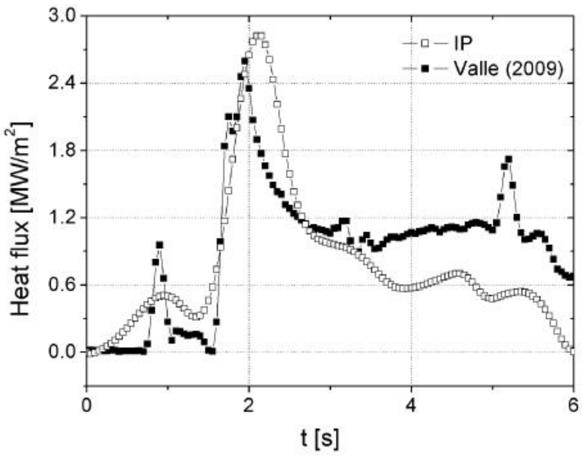
(a)



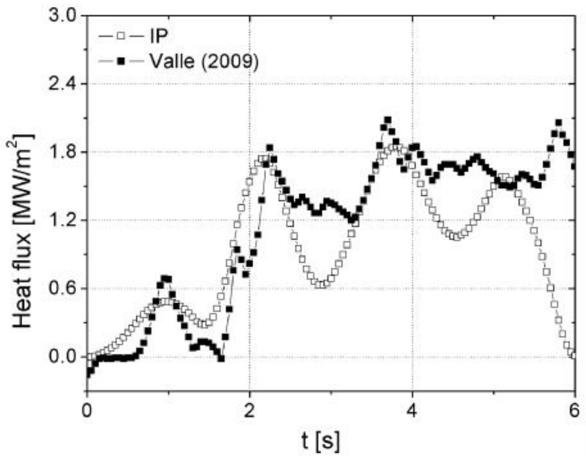
(b)



(c)



(d)



(e)

Figure 5. Comparison between Valle (2009) and the solution for IP of this work.

Valle (2009) used the least square method with regularization parameter and a covariance matrix in a iterative process, being valid for nonlinear problems. The differences observed between the results of the present study and the results obtained by Valle (2009) are possibly related to the differences between the methods. The method presented in this work is more realistic for problems which presents errors with normal distribution and zero covariance. Some error is also expected due to the fact that the data has been taken out of figures. Considering what was mentioned, the results of this work are satisfactory.

3. EXPERIMENTAL APARATUS

The tests were performed at Hydraulic Machines Laboratory (LMH), in Federal University of Paraná (UFPR). For the tests, the engine was fixed horizontally on a test bench, which allows triggering the transducer strength. Outer wall temperatures were measured by K thermocouples fixed on the outer wall of engine. According to the manufacturer, the thermocouple response time is under 0.3 seconds. Calibration report of thermocouples presented an error almost linear of 274.4 K at 393 K and 287 K at 1473 K. Data acquisition system used was HBM Spider 8 and Catman 4.5 software. The acquisition frequency was 200 Hz resulting in a 0.005 second gap of time.

Engine propellant test (MTP) was developed by Space Activities Laboratory (LAE) of UFPR with the purpose of studying propellant performance parameters. MTP is an ABNT 4340 steel engine with cylindrical geometry and external and internal diameter of 80 mm and 60 mm respectively. The total length without nozzle and cap is 208 mm. Figure 6 illustrates MTP rocket engine and its main parts. Smectite is a non reactive substance used to seal the propellant grain between cap and casing in order to avoid gas losses during burning. Powder of smectite (approximately 0.1 kg) were placed in MTP casing and conformed by mechanical press. After, powder of propellant (approximately 0.214 kg) was placed in MTP engine and similarly conformed by mechanical press. The propellant used was potassium nitrate (KNO_3)/sucrose ($C_{12}H_{22}O_{11}$), known as KNSu. The composition used was 65 wt% of KNO_3 and 35 wt% of $C_{12}H_{22}O_{11}$.

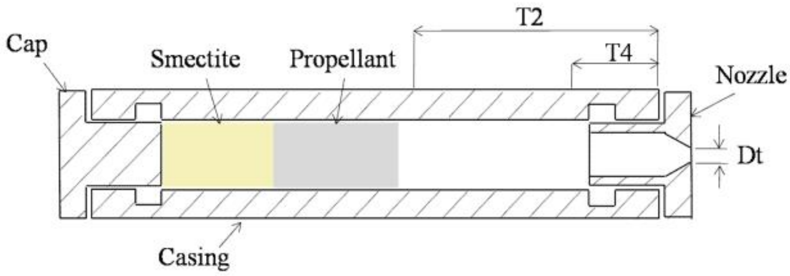


Figure 6. Main parts of MTP engine and thermocouple location.

MTP nozzle was designed with convergent only. Because of it the velocity of the exhaust gases is subsonic and may achieve sonic velocity at nozzle throat Dt. Different diameters of nozzle throat were used to accomplish the tests. Table 2 shows the nozzle throat diameter of MTP engines:

Table 2. Nozzle throat diameter of MTP engines.

MTP	Dt: Nozzle throat diameter [mm]
8	5.94
9	7.00
10	8.00
11	9.98

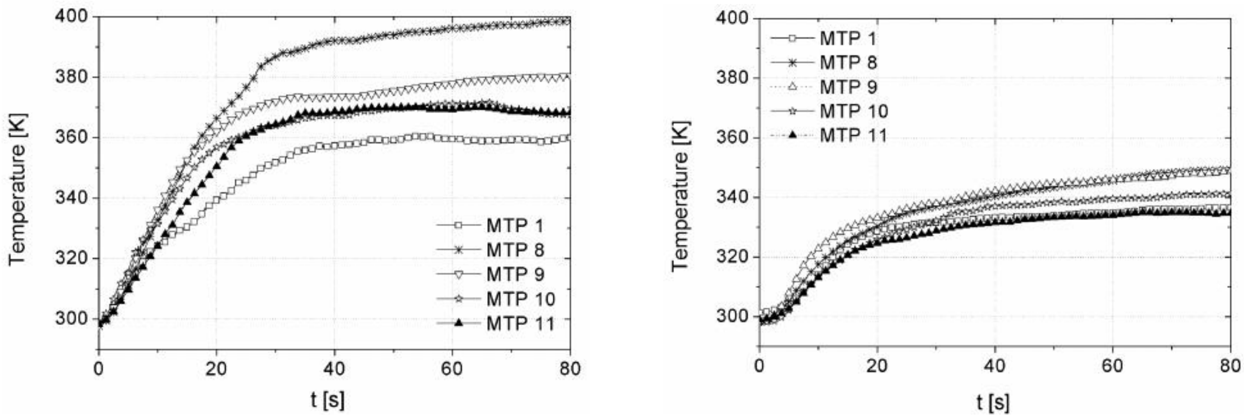
Figure 6 illustrates the locations of two thermocouples (T2 and T4). Thermocouple placed in T2 is positioned on the outer surface next to propellant grain (approximately 100 mm) and thermocouple T4 is positioned on the outer surface next to nozzle entrance (approximately 50 mm).

As mentioned, MTP engine was manufactured of ABNT 4340 steel. Table. 3 indicates the physical properties, took from Aerospace Specification Metals (ASM, 2016).

Table 3. Physical properties of MTP engine.

MTP: ABNT 4340	Values
Density[kg/m^3]	7850.0
Specific heat[J/kg.K]	475.0
Thermal conductivity [W/m.K]	44.5

Lots of tests were performed. However, because they are very similar, only the results of 06/04/2015 test are shown in this work. The temperature history of thermocouples T2 and T4 are presented in the Fig. 7:

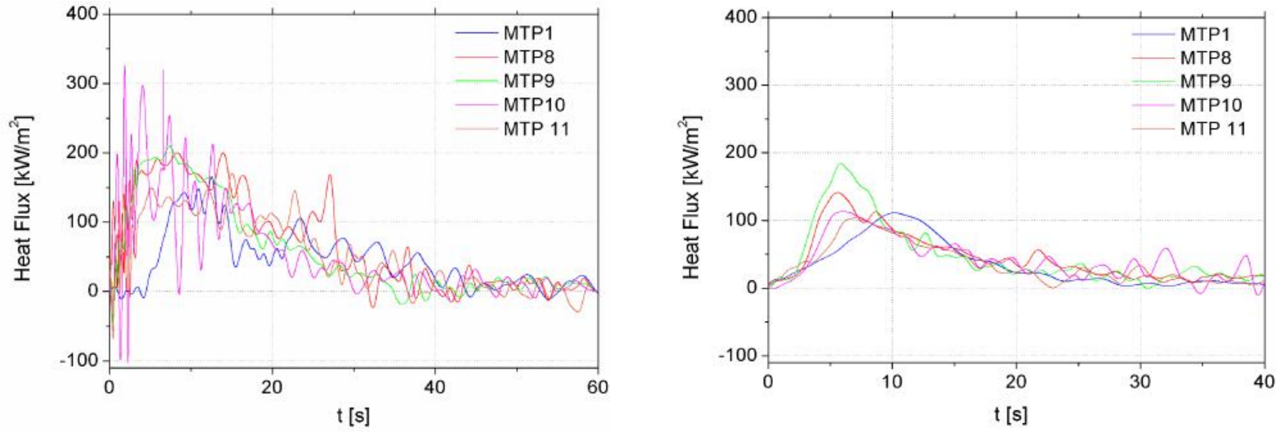


(a) (b)

Figure 7. Experimental temperature of (a) T2 and (b) T4.

4. RESULTS

Applying the methodology discussed in the previous sections, the following heat fluxes, as present in Fig. 8 were obtained.



(a) (b)

Figure 8. Heat flux from temperature history of T2 (a) and T4 (b).

We can note that MTP 8 and 9 have the larger heat flux peak, as expected. This occurs because the pressure is larger in combustion chambers with small throat diameter nozzles. The theoretical chamber pressure varies from 8.10^5 Pa in MTP 8 to 9.10^4 Pa in MTP 12.

The heat flux obtained from temperature history of thermocouple T2 has more instability. It is known that there is an optimal regularization parameter for each thermocouple (Beck *et al*., 1985), which must be determined by an analysis of sensitivity matrix. In the present study both heat fluxes were obtained with the same regularization parameter

5. CONCLUSIONS

A methodology for heat flux determination using experimental temperature history was tested. The solution for the inverse problem is computed from a linear equation, that is, the solution is given directly, it is not an iterative process; resulting in simple program code. The main advantage of the method is to avoid the calculus of the physical properties of the gases from combustion. Another point to emphasize is that the methodology presented is not very sensitive to experimental errors.

The programming code for solving IP was used to solve similar problems in literature. The results agreed satisfactorily. The methodology was applied in rocket engines during static tests at UFPR. It was used two thermocouples in the chamber pressure region. Results demonstrate instabilities for heat flux obtained from thermocouple next to burning surface area and the flame.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

ASM, 2016. “Aerospace Specification Metals”. 1 Aug 2016 < http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=m434ae>.

Alifanov, O.M. and Rumyantsev, S.V. 1987. “Application of iterative regularization for the Solution of Incorrect Inverse Problems”. *Journal of Engineering Physics*, Vol 53, p. 843-851.

Beck, J.V., 1970. “Nonlinear estimation applied to the non linear inverse heat conduction problem”. Journal Heat Mass Transfer, Vol. 13, p 13:16.

Beck, J.V., Blackwell, B. and St. Clair Jr, C.R., 1985. *Inverse heat conduction-Ill Posed Problems*. John Wiley & Sons, New York.

Burggraf, O.R., 1964. “An exact solution of the inverse problem in heat conduction theory and applications”. *Journal of heat Transfer*, Vol 86, p. 373-380.

Kacynski, J.K., Pavli, A., Smith, T., 1987. “Experimental Evaluation of Heat Transfer on a 1030:1 Area Ratio Rocket Nozzle”. Technical paper 2726. NASA.

Kimura, A.L., 1987. *Transferência de calor em motor-foguete*. PhD thesis, Instituto Tecnológico de Aeronáutica, São José dos Campos.

Maliska, C.R., 2004. *Transferência de Calor e Mecânica dos Fluidos Computacional*. Livros Técnicos e Científicos, Rio de Janeiro, 2nd edition.

Mehta, R.C., 1981. “Estimation of heat transfer coefficient in a rocket nozzle”. *AIAA Journal*, Vol 19, p. 1085-1086

Ozisik, M.N., 1993. *Heat Conduction*. John Wiley & Sons, North Carolina, 2nd edition.

Ozisik, M. N., Orlande, H. R. B., 2000. *Inverse Heat Transfer: fundamentals and applications*. Taylor & Francis, New York.

Patire Jr., H., 2010. *Determinação da carga térmica na parede da câmara de empuxo de propulsores bipropelente usando método inverso*. PhD Tesis, Instituto Tecnológico de Aeronáutica, São José dos Campos.

Stolz, G., 1960. “Numerical solutions to an inverse problem of heat conduction for simple shapes”. Journal Heat Transfer Vol. 82, p. 20-25.

Sutton, G.P., 1992. *Rocket Propulsion Elements*. John Wiley & Sons, New York, 6th edition.

Tikhonov, A.N. and Arsenin, Y.V., 1977. *Solutions of ill-posed problems*. Winston & Sons, Washington DC.

Versteeg, H. and Malalasekera, W., 1995. *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*. Prentice Hall, London.

Valle, D., 2009. *Aplicação do Método Inverso para a Determinação do Fluxo de Calor em Câmaras de Combustão Não-Regenerativas de MFP*. Instituto Tecnológico de Aeronáutica, São José dos Campos.

Williams, S.D. and Curry, D.M., 1977. “An Analytical and Experimental Study for Surface Heat Flux Determination”. *Journal of. Spacecraft and Rockets*, Vol. 14, p. 632-637.

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