LEWIS FRY RICHARDSON. D. Sc., FRS

Reproduced with permission of Bassano & Vandyk Studios

Annu. Rev. Fluid Mech. 1998. 30:xiii–xxxvi

LEWIS FRY RICHARDSON AND HIS CONTRIBUTIONS TO MATHEMATICS, METEOROLOGY, AND MODELS OF CONFLICT

J.C.R. Hunt
University of Cambridge, Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge CB3 9EW, United Kingdom

KEY WORDS: meteorology, numerical analysis, fractals, conflict analysis, statistics of wars

ABSTRACT

The life and major scientific contributions of Lewis Fry Richardson (1881–1953) are reviewed, with particular emphasis on his pioneering work in numerical analysis, meteorology, and numerical weather prediction. His later work on mathematical modeling of psychology, causes of conflict, and the statistics of wars is outlined in terms understandable to fluid dynamicists. It is included because it led to Richardson's discovery of one aspect of fractals, an analytical technique now recognized as valuable in the study of complex fluid motions.

Introduction

The discoveries of many great scientists and mathematicians, including those of Lewis Fry Richardson (LFR), have been directly related to the political, economic, and other human concerns of their period. LFR expressed himself clearly at the outset of his career (Richardson 1908): "The root of the matter is that the greatest stimulus of scientific discovery are its practical applications."

During the period of his life from 1881–1953, science and its applications developed rapidly and in quite new directions. It is well known how the lives of those involved in the development of modern physics are woven into the history of this century: Einstein and Bohr to name but two. But pioneers of other branches of mathematical sciences have also contributed with their discoveries to the great changes in the world. As with the physicists, the research and lives

0066-4189/97/0115-xiii$08.00

of many of these scientists and mathematicians were also strongly influenced by the history of this period, especially the two world wars and their peacetime repercussions. Richardson was one of these pioneers. He made outstanding contributions to several different fields. His name can be added to the list of inventors of computational mathematics (with von Neumann, Courant, and Turing), of modern meteorology and fluid mechanics (with Bjerknes, Taylor, and Prandtl), of quantitative techniques in psychology and social sciences (with James), and of analysis and modeling of complex systems (with Norbert Wiener). In each of these fields his work is still being cited. Between 1980 and 1984, in one citation index, there were over 200 references to his work.

Knowing about the lives and beliefs of creative people helps in the understanding and appreciation of their work. This is especially true in the case of Lewis Fry Richardson, much of whose scientific work changed and evolved as a direct result of—and from his reaction as a Quaker to—the political and technological changes that occurred during his lifetime. His life is an inspiration, showing how a mathematical scientist can respond to the problems of the world around him, while not necessarily being in accordance with the ways favored or promoted by the established organizations that direct and finance science.

Richardson's special contribution to all these fields was to apply quantitative and mathematical thinking to problems that were considered to be outside the scope of mathematics, and to have been so effective in it that his formulae and methods are still being used daily by working scientists and mathematicians. His personal stamp on the work is such that many of his results are still referred to by his name.

Notwithstanding his success in the use of mathematics, he was himself conscious of the dangers of applying mathematical ideas and techniques to complex human behavior and natural phenomena. As he aptly summarized in *The Mathematical Psychology of War* (Richardson 1919a):

> Mathematical expressions have, however, their special tendencies to pervert thought: the definiteness may be spurious, existing in the equations but not in the phenomena to be described; and the brevity may be due to the omission of the more important things, simply because they cannot be mathematized . . . . Against these faults we must constantly be on our guard . . . . It will probably be impossible to avoid them entirely, and so they ought to be realized and admitted . . . ."

Early Life, 1881–1903

Born October 11, 1881, of Quaker parents, David and Catherine Richardson of Newcastle, Lewis Fry Richardson was the youngest of seven children. David Richardson, who ran a prosperous tanning and leather business, had been trained in chemistry. His technical abilities enabled him to design new machinery and new production methods. Lewis showed early on an independent mind and an empirical approach, such as when he tested at the age of five the proposition learned from his elder sister that "money grows in the bank" (Ashford 1985). He buried some money in the garden and was disappointed to find that it did not grow. After a period of time spent at Newcastle Preparatory School, where his chief enjoyment was Euclid "as taught by Mr. Wilkinson" (Ashford 1985), he was sent, aged 12, to a Quaker boarding school, Bootham, at York. There he had excellent teaching, especially in science, which stimulated a great interest in natural history. He kept a diary of birds, insects, flowers, and weather. He collected 167 species of insects and made detailed studies of plants (he returned to collecting statistics—of wars—in the last phase of his research). Another important part of his character was developed by a teacher who "left me with the conviction that science only has to be subordinate to morals" (Ashford 1985).

His higher education began in 1898 with two years at Newcastle University (formerly Durham College of Science), where he took courses in mathematical physics, chemistry, botany, and zoology. He proceeded in 1900 to King's College, at the University of Cambridge, where he was taught physics in the natural sciences tripos by (among others) Professor JJ Thomson [discoverer of the ratio (*e/m*) of the charge (*e*) to the mass (*m*) of an electron] and graduated with a first-class degree in 1903.

Early Career, 1903–1913

During the first ten years (1903–1913) after graduating, LFR held a series of short research posts, a career not unfamiliar to today's scientists, first in a government research laboratory (the National Physical Laboratory), then in physics departments (at Aberystwyth and Manchester Universities), and finally in industry. Some of the research required in these posts clearly did not interest him (such as in metallurgy and metrology). However, it was while serving as a chemist with the National Peat Industry Limited, from 1906–1907, that LFR began his first pioneering research: in mathematics, not chemistry. He was faced with a problem: "Given the annual rainfall, how must the drains (i.e. channels in the peat moss) be cut in order to remove just the right amount of water?" (Ashford 1985). He found that the percolation of water through the peat could be described by the well-known (eighteenth-century) equation of Laplace,

$$\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0\right),$$

but that the boundaries around the region where the equation had to be solved did not have the nice shapes, e.g. circles and rectangles, studied by mathematicians and for which solutions were known. Although he realized that exact mathematical methods could be found for more complicated regions, these methods were difficult to derive and were not general enough for any shape, so faster and more general—if less accurate—methods were necessary. Though most of today's problems require the solution of more complicated equations, his prescient remarks on this dilemma are as relevant today as they were in 1908:

> Further than this, the method of solution must be easier to become skilled in than the usual methods (i.e. analytical solutions). Few have time to spend in learning their mysteries. And the results must be easy to verify—much easier than is the case with a complicated piece of algebra. Moreover, the time required to arrive at the desired result by analytical methods cannot be foreseen with any certainty. It may come out in a morning, it may be unfinished at the end of a month. It is no wonder that the practical engineer is shy of anything so risky (Richardson 1908).

LFR (Richardson 1908) showed first that a broad brush solution for the peat flow could be obtained by drawing lines of the flow freehand according to certain rules (which would require a good eraser, a soft pencil, and some patience until the lines satisfied these rules—an approach that was still being taught to engineering students in the 1960s). But a more accurate and systematic method for obtaining approximate solutions was to convert the differential equation, defining the smooth continuous changes of the variable (e.g. h, the height of a curve), into an approximate equation relating the small changes in the variable δh over small distances δx (or steps).

Then the rules of arithmetic (and algebra), rather than the special techniques of differential calculus for continuous functions (in which there are infinitely many steps of infinitesimal size on the staircase), can be used for the sequence. LFR (1927) explained that, paradoxically, this approach was a historical regression to a time before the invention of calculus.

Not surprisingly (to anyone familiar with the scientific and academic worlds), this new approach was at first too new for the referees who reviewed the paper for the *Philosophical Transactions of the Royal Society*. It was only after much deliberation and correspondence that it was published (Richardson 1910). He acknowledged the earlier, but for his purposes, less suitable approximate methods introduced by Runge (1895) in Göttingen.

More importantly for LFR's career, this approach to the approximate solution of differential equations was also too new for King's College, Cambridge, where LFR submitted this work, as a dissertation, in the competition for a Fellowship (i.e. a research and teaching post). Apparently (I am indebted to Professor Huppert for this information from the files of King's College, Cambridge), the opinions were sought from the mathematicians at Trinity College, who said this was approximate mathematics and they were not impressed. So LFR never returned to Cambridge and for the rest of his career he did not work in any of the main centers of academic research. At the time he did not regret this, commenting (apropos of Manchester) that he liked to work "somewhere where there are fewer people buzzing around" (Ashford 1895). But this isolation probably affected the development of his research and hindered its appreciation in the scientific community. Perhaps the lack of collaboration with colleagues explained why the presentation of his research was often idiosyncratic, and it probably also meant that he did not receive suggestions from other researchers as to how a line of research might profitably develop. It may explain why he moved from one subject to another suddenly, and often. Yet one must also recognize that the lack of the guiding influences of colleagues may have been a factor in the great originality and diversity of his research: GI Taylor (1958) commented, LFR was "a very interesting and original character who seldom thought on the same lines as his contemporaries and often was not understood by them."

In fact, these early papers helped pioneer the development of numerical methods for the solution of differential equations (Fox 1993), a subject he returned to later in his career (Richardson 1927, 1950), when with the arrival of calculating machines and, later, computers it was gaining greatly in importance. From this time, LFR saw his work as the essential first steps in solving the equations required for predicting weather (Charnock 1993).

Despite the progress he had made in this field of his research during this early stage of his career, there were no funds to continue it; indeed, as already mentioned, he had to move between short-lived posts on different research projects. No clear direction for his future was emerging. Nevertheless, two of these brief projects did reveal his intention of eventually moving away from the physical to the behavioral sciences. He had said to himself as an undergraduate that "I would like to spend the first half of my life under the strict discipline of physics, and afterwards to apply that training to researches on living things." According to his own autobiographical notes, "I kept this programme a secret" (Ashford 1895).

In 1907 he sold his physics books and briefly went to work as an assistant to learn about statistical proof under Professor Karl Pearson at University College, London, an authority on mathematics, genetics, and the philosophy of science. In fact, LFR worked on stresses on masonry dams (1910) and helped prepare an index of the journal *Biometrika*, founded for the statistical study of biological problems. He had to leave within a few months because there were no funds for his proposed research into quantitative studies of heredity. This subject, together with eugenics, was a topic he took up between 1912 and 1913, when he was a lecturer at the Municipal School of Technology in Manchester [now the University of Manchester Institute of Science and Technology (UMIST)]. He published the first of his papers on the quantitative analysis of "living things" (Richardson 1913), entitled "On the measurement of mental nature and the study of adopted children" in *Eugenics Review* in 1913. It built on some earlier research of Pearson.

During this period of his life, LFR used to have holidays in Seaview, Isle of Wight (a holiday resort on the south coast of England), staying with a Cambridge friend, Stuart Garnett, son of William, who had worked under Professor James Clerk Maxwell as a demonstrator in the Cavendish Laboratory at the University of Cambridge in the 1870s. GI Taylor also used to visit the Garnetts. In 1909, LFR married Dorothy Garnett, the sister of Stuart. The family is described incidentally in a recent literary biography by Beauman (1993).

While LFR and Dorothy were on holiday in 1912, the large passenger liner Titanic collided with an iceberg in the Atlantic, off Newfoundland, in foggy weather and sank with great loss of life. LFR immediately had the idea that this kind of accident could be avoided if ships sent out a focused beam of sound and measured with a sensitive receiver the delay time of any echo. He tested this idea in Seagrove Bay, in a dinghy rowed by Dorothy (as she told the story), at different distances from Seaview Pier (now destroyed). He blew a penny whistle and by measuring the time for the return of the echo, amplifying it with an umbrella held over his shoulder, he calculated the distance. He found the method worked well and in October 1912 he filed a patent, the importance of which led the writers of a textbook on fluid mechanics (Drysdale et al 1936) to hope that Richardson's "method will ultimately eliminate the last of the serious dangers of navigation." This impromptu but highly original experiment was typical of many that LFR undertook with a minimum of expense but which usually led to important new ideas.

Incidentally, the sinking of the Titanic led to a scientific expedition to study the atmosphere and ocean off Newfoundland. The meteorologist was LFR's great contemporary, GI Taylor, whose early ideas on turbulence stemmed from his observations there. LFR's and Taylor's interests touched again in the next phase of LFR's career.

Meteorology—the First Phase, 1913–1916

On the recommendation of LFR's former colleagues at the National Physical Laboratory, he applied for and was appointed to the position of Superintendent of the Eskdalemuir Observatory in southern Scotland. The observatory had been set up in a remote location, primarily to record magnetic fields and seismic vibration but also to record meteorological measurements. LFR had no previous professional experience in meteorology, but as was explained to him in a letter from Napier Shaw, the chairman of the appointments committee (and a founder of scientific meteorology in Britain), he had been appointed to bring a more theoretical approach to the understanding of meteorology and a critical eye to the methods of measurement.

This was an attractive position for LFR: It came with a house and considerable freedom to pursue his major research interest, that of devising a method for calculating the weather a few hours or days ahead, using the relevant theoretical equations that describe the behavior of the atmosphere. Although most of the equations were known, a number of new aspects of the physics had to be estimated; also, there was the new task of transforming these equations into the approximate finite difference (or step-like) form. "In the bleak and humid solitude of Eskdalemuir," as he described it (Ashford 1895), he completed the first draft of his book, then entitled *Weather Prediction by Arithmetical Finite Differences*.

He also participated in some developments of seismography and new techniques for the detection of thunderstorms from their electrical and magnetic fields, using wireless and telephone lines. At the same time, he was performing the administrative duties of superintendent, which became more onerous after the outbreak of the First World War. Correspondence between LFR and Napier Shaw (Ashford 1985, p. 50) shows that LFR was uneasy about using his own and the observatory's scientific knowledge and equipment for military purposes, such as measuring the vibration in the ground caused by distant artillery fire. (In fact, these instruments were recently used to locate the site of the passenger plane crash near Lockerbie, Scotland.)

Whether this concern was the cause or not, LFR resigned on May 16, 1916 from the Meteorological Office to join the Friends' Ambulance Unit in France. He had asked earlier to be released for work with the Red Cross Unit of the Ambulance Corps at the outbreak of war but had been refused permission. Clearly, his parting with the Meteorological Office was not unamicable because he was allowed to rejoin it 1919 on his return from France.

Driving an Ambulance in France, 1916–1919

Although the Society of Friends (Quakers) always urged its members not to take part in war, in the 1914–1918 war a few Quakers of military age joined the armed forces. Many did not, and they either did humanitarian work connected with the war or refused to have anything to do with war activities. Many of the latter were imprisoned, though usually with other conscientious objectors (such as Bertrand Russell).

LFR followed the pacifist course of action and in 1916 joined the Friends' Ambulance Unit, which was financially supported by the Society of Friends. It was set up in 1914, following an initiative by Philip Noel Baker.

At this time, many families were divided in their response to the war. LFR's Cambridge friend and brother-in-law, Stuart Garnett, was killed while test-flying an aircraft in 1916 in the Royal Flying Corps, and Stuart's younger brother Kenneth, aged 25, died slowly during 1916–1917 after being injured with a shrapnel wound in the Battle of the Somme. LFR was exceptional among scientists (in Britain or Germany) in deliberately ceasing to do scientific research financed by the government during the war. This was the first major war in which leading scientists were called on by the armed forces and used to great effect, particularly in aerodynamics (GI Taylor at Cambridge, L Prandtl at Göttingen), ballistics (JE Littlewood at Cambridge), and the chemistry of explosives and gases (C Weizmann at Manchester).

After initial training, LFR was attached as a driver to the *Section Sanitaire Anglaise* (SSA 13), a group of 56 men with 22 ambulances working with the 14th French Army. They worked alongside the French military ambulance unit *Sections Sanitaires*. LFR was, in the words of the ambulance maintenance crew, "a careful and conscientious driver and managed to avoid careless driving through shell-holes" (Ashford 1985), transporting wounded soldiers, often under shell fire.

In his spare time in France he set up and designed various simple meteorological instruments and took readings. Also, he had brought along the first draft of his book on numerical weather prediction, and during a six-week period he worked on a specific calculation ("on a heap of hay in a cold wet rest billet") (Ashford 1985) to show how the numerical forecasting system might be used in practice. At one stage the manuscript was lost, during the battle of Champagne in April 1917, but fortunately it was rediscovered some months later under a heap of coal. The work was finally published in 1922, after the war, when LFR had taken up meteorological research again. In France he also did some laboratory experiments on the motions of water in a vessel on a rotating gramophone turntable; the apparatus was too crude to test the effects of thermal convection and rotation he was investigating.

The main effect of the war on his research and thinking was to direct him toward studying the causes of wars and how they may be prevented. He began to develop a mathematical model, the *Mathematical Psychology of War* (Richardson 1919a), for how the animosity between two mutually suspicious and well-armed nations might develop over time. He suggested that the animosity of each of the two sides (the Entente Cordiale, i.e. Britain and France, on one side, and Germany on the other) could be expressed in terms of numbers derived from measurements. This was therefore a mathematical quantity and could be represented in equations by the symbols A_E and A_G, respectively, which showed how A_E depends on A_G and vice versa, i.e. $dA_E/dt = kA_G$; $dA_G/dt = kA_E$. Although mathematics was already in use in various social sciences, it had not previously been used for modeling war behavior. However, it had begun to be used for modeling tactics of war, especially on the basis of the fundamental, and still used, equation of FW Lanchester (1916)—another unorthodox Englishman who is known equally well for innovations in automobile engineering, but now less well for his pioneering research on aerodynamics.

From Meteorology to Mathematical Psychology, 1919–1929

From France LFR wrote to Napier Shaw of the Meteorological Office asking whether he could return there, with the specific aim of working on upper-air sounding experiments with a view to making "weather predictions by a numerical process . . . a practical system" (Ashford 1985). In his letter, LFR floated the idea that with a large technical staff of 12–15, he could make rapid progress in overcoming some of the outstanding observational and mathematical problems. If this was not possible, LFR suggested an alternative proposal of a modest research position for himself and an assistant. Shaw replied that the large scheme might be possible technically and even financially, but that other forecasting schemes requiring funds had also been proposed, such as V and J Bjerknes' graphical scheme based on the analysis of fronts that had been developed at Bergen during the 1914–1918 war, and which subsequently became the basis for understanding weather systems and qualitative weather forecasting throughout the world (Friedman 1989). Interestingly, V Bjerknes, who later collaborated with LFR, had also hinted at the possibility of numerical weather forecasting in 1912 but did not pursue it himself.

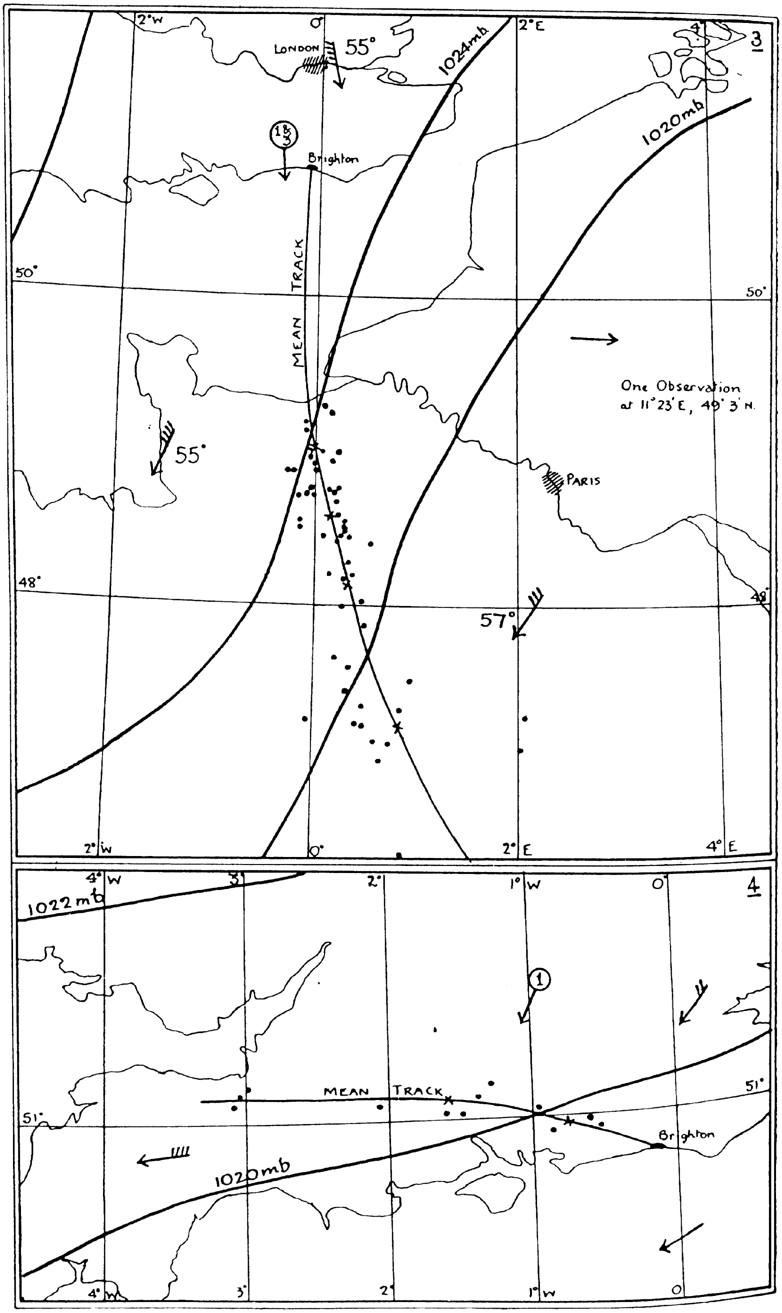
With a view to providing the upper-air data that was a necessary input for the numerical model, and also to test the model, LFR developed at Benson three different kinds of instruments for atmospheric measurements up to several kilometers above the ground. They were all characteristically original, but in fact none was continued after he left Benson (Charnock 1993). He developed a complicated method for measuring the upper-wind speed by shooting metal spheres of various diameters upward at small angles to the vertical (Richardson 1923, 1924a,b). He observed over slightly rolling terrain a nocturnal jet with a maximum velocity 70% greater than the geostrophic wind speed. A nice LFR touch to the experiments was the arrangement whereby, to protect themselves and record the point of impact, the shooters stood in a shelter underneath a large metal sheet and fired the gun upward through a central hole (see the photograph in Ashford 1985, p.129, of Dines in a straw hat under the Richardson shelter).

A major task in developing the underlying theory for the numerical method of forecasting was to improve the theory of turbulence and turbulent mixing in the lowest 2 km of the atmosphere. The turbulent eddies determine how rapidly the upper-level atmospheric winds can slip over the Earth's surface and how much heat and moisture can be carried upward or downward to the earth's surface. The earlier research by Schmidt in Austria, Boussinesq in France, and Taylor in England had shown that these eddies behave in some respects like molecules in a gas, so it was meaningful to define an eddy viscosity or eddy conductivity K (Richardson 1922, p.67). However, whereas the viscosity or conductivity of a gas is independent of the velocity or the kind of motion of the gas and independent of the scale over which there are variations in velocity or temperature, and has the same value everywhere (provided the gas is at the same temperature and pressure), none of these properties is true of a turbulent flow.

LFR's research between 1919 and 1926 led to great advances in understanding these special properties of turbulent eddies and in providing a novel kind of physical explanation.

At Benson he began a series of experiments to study how material randomly disperses in the turbulent eddies in the atmosphere, by measuring the widths of smoke plumes and the distances between floating objects (from seeds to balloons) released into the wind. Some of these experiments provided more data that largely confirmed previous results, such as how K increases with height above the ground in the atmosphere and how it increases with wind speed (Richardson 1920). Later he conducted experiments with balloons that traveled several hours away from their release point (Richardson 1926b). Some of these experiments were in the characteristically informal LFR style; some balloons were released at publicity events in Hyde Park or in a balloon competition on Brighton Beach. Many of them traveled over France, Belgium, and Holland (Figure 1). Labels attached to the balloons were returned by people who found the balloons after they landed. Then the balloon trajectories were plotted. The results showed that the longer the length of the travel, the greater the variability or dispersion. As is not unusual in science, once Richardson had his own data, he found earlier data by other scientists that were consistent with the pattern he had begun to observe. The most novel experiments (Richardson 1929) involved releasing seeds (with Dorothy and his son Stephen on Hindhead Common) and small balloons simultaneously at different initial separations. Both sets of experiments were used by LFR (1926a) to derive the general law that the rate of increase of the square of the separation (i.e. the rate of diffusion) between objects diffusing in a turbulent stream grows in proportion to the separation raised to the power 4/3—the famous "four-thirds" law. This showed conclusively that turbulence is not quite like the molecules of a gas, that it contains eddies with many length scales, and that different methods of analysis are necessary. Many of the questions he raised in 1926 are still not satisfactorily resolved, though his insights are still instructive.

Figure 1 The results of Richardson's experiments (*top*, Sept. 9, 1922; *bottom*, June 1, 1923) (see Richardson 1926a,b). The data points correspond to places from which people returned postcards from downed balloons that had been launched at Brighton—a classic Richardson experiment that helped give rise to the four-thirds law.


3
2°W
0°
2°E
4°
LONDON
55°
1024mb
1020mb
1&3
Brighton
MEAN TRACK
50°
50°
One Observation
at 11°23'E, 49°3'N.
55°
PARIS
48°
48°
57°
2°W
0°
2°E
4°E
4
4°W
3°
2°
1°W
0°
1022mb
1
MEAN TRACK
51°
51°
Brighton
1020mb
4°W
3°
2°
1°W
0

In a well-known satirical verse, Jonathan Swift, author of *Gulliver's Travels*, had contemptuously compared poets' use of each others' work to the behavior of fleas. Casting the fleas as eddies, LFR (1922, p. 66) adapted the verse to describe his conception of turbulence: "Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity—in the molecular sense."

LFR's next important insight into the nature of turbulence also arose from experiments at Benson. He observed how the fluctuation in wind speed increased or decreased depending on the difference between the wind speeds at two heights (in this case l m and 26 m) and depending on the difference between the temperatures at these two heights, so that in the evening as the ground temperature fell and the temperature difference increased, the fluctuations decreased. In his analysis he derived estimates for the relative contributions to the energy of the turbulence, on the one hand from the buoyancy forces caused by eddies moving between levels of higher and lower temperatures (which might reduce or amplify the turbulence) and on the other hand from the accelerations of the eddies moving between levels of high- and low-wind speed (based on the earlier statistical analysis of Osborne Reynolds at Manchester 30 years earlier, and his and GI Taylor's experiments on turbulent fluctuations). The ratio of these two contributions is now called the Richardson number, *Ri*, a name given by W Paeschke; LFR showed that if *Ri* was greater than one, the turbulence was suppressed. Dimensionless numbers, like theorems, commemorate physical scientists; but the Richardson number is probably unique in that it takes positive and negative values, depending on whether the atmosphere is stable or unstable.

This insight into the occurrence and strength of turbulence was soon acknowledged in the scientific world, probably because it crystallized ideas that had already been suggested by GI Taylor and others. Indeed, Prandtl (1931) incorporated Richardson's criterion into his textbook on fluid dynamics only a few years later. However, LFR did not live long enough to see his concepts of eddy structure and eddy diffusion of clouds of material (Richardson 1926a, 1929a) recognized for their importance in the development of turbulence theory by Kolmogorov (1941) and Oboukhov (1941) in Moscow (see Yaglom 1994), and for their seminal contribution to diffusion theory by Batchelor (1952) in Cambridge.

The third main element in LFR's program of research for the improved methods of forecasting was his studies of radiation and the thermodynamics in the atmosphere. At Benson he focused on water in clouds (1919b), and the thermodynamics of moving and radiating parcels of air. Later, between 1927 and 1928, studying the reflection of radiation from the Earth's surface, he was able to mount his instruments on a small aircraft of the De Havilland Aircraft

Company, which measured the reflectivity (or albedo) of woods, fields, and suburbs between London and St. Albans (Richardson 1930). With GC Simpson, LFR (1928a) recognized earlier than other meteorologists the importance of reflection on the "as yet imperfectly understood effect on local climate" (Ashford 1985).

These studies were part of an international program, organized at the Madrid meeting of the International Union of Geodesy and Geophysics (I.U.G.G.) in 1924, which involved comparative measurements of the albedo in different countries using the same photometer instruments. This was one of the instruments that LFR had developed while in France with the Friends' Ambulance Unit. (The cost in 1927 was £14.10.)

While at Benson in 1919, LFR sent the completed manuscript for his book *Weather Prediction by Numerical Process* to Cambridge University Press for printing, but it did not appear in print until 1922. The book gathered together the results of LFR's scientific and mathematical studies of atmospheric processes and showed how these formed the basis of a numerical method for weather prediction (Charnock 1993). But the book is best known for the great failure of the method when it came to be applied in a particular calculation of the meteorology over a 6-h period over Germany in 1910, using the data from the maps of V Bjerknes. (This was the calculation he had performed in France.) So it was a surprising and brave decision to include this calculation. The large and erroneous rise in pressure by 145 millibars in 6 h (about 50 times too large) was caused by the natural short time scale, gravity wave oscillations of the equations that are filtered out in modern weather prediction methods, as Lynch (1993) recently found when he recomputed LFR's numerical scheme. Amazingly, he found in LFR's laborious hand calculations very few errors.

In the final chapter, LFR briefly discussed the practical organization of such forecasting, using only humans computing with the aid of slide rules and calculating machines. He imagined a large hall, similar to a concert hall, with the chief forecaster acting like a conductor organizing the information flowing in space to him from all parts of the hall. But his estimate for the number of computers required to race "the weather for the whole globe" (Richardson 1922) was 64,000, which meant the hall would be more like a football stadium. (At that time, he never imagined that the calculating machines could eventually do the whole job unaided.)

He saw substantial practical and economic advantages flowing from improved forecasting, mainly to agriculture. He did not mention other possible activities, one of which, aviation, has repaid this advantage by providing most of the financial support for meteorology from that time on.

The reviewers of the book (quoted extensively in Ashford 1985) were impressed by its "originality, its coordinated treatment of the dynamical processes," and agreed that it provided the basis for systematic quantitative forecasting. But both they and the forecasters within meteorological organizations did not believe that this provided a practical approach at that time. LFR's ideas and methods later formed part of the program led by Charney, Fjörtoft, and von Neumann (1950) to introduce numerical weather forecasting to the United States, which became a practical possibility in the 1940s and 1950s with the arrival of electronic computers. But the results from numerical weather prediction only really began to dominate those derived by the Bergen approach for 1- to 3-day forecasts in the middle 1980s. LFR's book continues to be referenced, and his vision is often described in the popular press around the world when journalists attempt to explain the mysteries of numerical weather forecasting.

LFR's time at Benson came to a painful end in 1920, as a result of his pacifist convictions. The Meteorological Office had been financed as a scientific organization, with the Royal Society appointing its governing council. During the war of 1914–1918, the armed forces found that meteorology was so important that they set up their own separate meteorological services. The government decided in 1918 that these should be brought into one organization. Its first recommendation was that this should be a government scientific organization, analogous to the National Physical Laboratory, but this was overruled by the strong Air Minister at the time, Winston Churchill. He persuaded the Government that the Meteorological Office should be incorporated into the Air Ministry, which controlled the Royal Air Force (Dr Burton, private communication). LFR followed these developments with great anxiety, but when the final decision came in July 1920, he resigned, effective September 1, 1920. There were many expressions of regret by his colleagues in the Meteorological Office, with whom he continued to collaborate on scientific matters for the rest of his life.

Because of the likely outcome of the government's decision on the Meteorological Office, LFR had applied for a vacancy in May 1920 as a lecturer at Westminster Training College (then near Westminster Abbey), teaching physics and mathematics to prospective school teachers up to the level of a bachelor's degree.

In the beginning of his period at Westminster, from 1920 to about 1925, his main research interests continued to be in meteorology. He was secretary of the Royal Meteorological Society from 1920–1924, attending meetings and helping instigate the new periodical for reporting meteorology and research, *Memoirs of the Royal Meteorological Society*. He contributed the first paper to the first issue (1926b), on atmospheric dispersion. He attended two major international symposia, at Bergen in 1921 and Leipzig in 1927, which helped establish the new internationally agreed-upon methods of forecasting by analyzing the movements of fronts and helped promote new methods of meteorological research.

V Bjerknes and others expressed surprise that a scientist of Richardson's stature did not have a secure position to continue his meteorological research at a high level (Ashford 1985). LFR was elected to a Fellowship of the Royal Society of London in 1926; the citation read "Distinguished for his knowledge of physics and eminent in the application of mathematics to physical problems of the atmosphere and other structures. Author of *Weather Prediction by Numerical Process* (Camb. Univ. Press), and of numerous papers of great originality, both in scientific idea and experimental method . . . ."

In about 1926, LFR changed his field of research to psychology, with the main objective of applying the ideas and methods of mathematics and physics to this field. He had read McDougall's (1905) book, *Physiological Psychology*, and he began his research by attempting to express its results in mathematical form. The main themes of his work over the next eight years can be summarized as (*a*) establishing that many different sensations are quantifiable (a commonplace idea now, but then highly controversial); (*b*) finding methods of measuring and quantifying these sensations and relating them to external physical conditions or stimuli [nowadays the field of applied psychology—in the introduction to his paper (1929c), LFR states "Psychology will never be an exact science unless psychic intensities can be measured. Some authorities say that such measurement is impossible."]; and (*c*) modeling these sensations with mathematical equations or in terms of analogous processes in physics [the aspect emphasized here; Poulton (1993) discusses his major contributions to psychology].

To quantify the sensation of touch, he asked the subject to estimate the distance between two pin pricks (the stimulus) touching his/her finger—a standard neurological test. He devised the kind of graph that is now standard in experimental and applied psychology by plotting a quantitative measure of sensation (in this case millimeters) against a physical stimulus (also measured in millimeters). Although the graphs differed between the subjects (his colleague, Ross, and his wife, Dorothy), both had a shape that curved upward and then leveled out (1928b).

By recording the intensity of his own mental image when thinking of a word (or the opposite of a given word), timing the decay of the images and then plotting the result, he found that many kinds of mental images also produced a set of curves that were similar to each other (1929b). These results were the basis of LFR's more general ideas about how thoughts arise, slowly decay, and then sometimes repeat themselves.

In another important study of sensation produced by a stimulus, his subjects, who included the college organist, had to estimate the loudness produced in a pair of headphones by a measured electric current (1930b). He established for the first time that the ratio of the loudness of the signal to that of a standard signal varied logarithmically with the stimulus, a result that helped lay the basis for the quantitative science of acoustics and for acoustical engineering.

As a postscript to this work, it is interesting to note the recognition since the 1940s of the importance of the quantitative study of sensation (e.g. Poulton 1988), not only for clinical psychology and the cure of suffering and disabilities, but also for the design and control of environments where people live and work, from the provision of correct light and sound levels in ships and offices to the planning of humanly acceptable wind environments in cities where both meteorology and psychology are important (e.g. Hunt et al 1975).

In his psychology studies, LFR did not assume that mental and other biological processes are analogous to simple and completely predictable physical or mathematical systems, such as a pendulum or an oscillating atom, but rather that they are analogous to systems with irregular behavior, such as neon lamps, which are dormant until some pulse of charge—or the physiological equivalent—is applied to bring the system above a threshold level. The mathematics of irregular behavior was only just beginning at that time, such as the discovery by van der Pol (1926) of nonlinear differential equations whose solutions show oscillations at one frequency, which then slowly or suddenly change to another frequency. Richardson (1937) wrote in an excited vein that these kinds of differential equations might be good models for psychological and biological processes. This idea is now well established, but LFR also saw a wider cultural dimension to this discovery. He suggested that since mental processes are not quite periodic, this might explain the popularity of syncopated music. Although LFR's musical tastes inclined to classical chamber music and certainly not to popular concerts, he was presumably making an elliptical reference to jazz, which had invaded Europe in the 1920s and 1930s.

Mathematics of Conflict and Fractals, 1929–1943

In 1929 LFR moved again, when Westminster Training College ceased to provide scientific education up to the level of a bachelor's degree.

LFR obtained the post of Principal at the Technical College in Paisley, an industrial town near Glasgow, which used to be famous for its textiles and the paisley pattern. It was not then (or now) usual for a Fellow of the Royal Society to take on such a post, but at that time there was no academic position available in the universities for someone wishing to pursue research in meteorology, or a new kind of quantitative psychology. (He had turned down the offer of a post in New Zealand earlier.)

Despite being in Paisley, away from the main centers of scientific activity and lecturing for 16 h a week, LFR continued his research. In 1935 he turned again to the study of the causes and the prevention of wars and other conflicts. This was the last turning point in his scientific career; just as he had largely given up meteorology in 1926, he now ceased further work in psychology. In the comparatively tranquil years that followed, from 1919–1935, he did not pursue his earlier research on conflict or attempt to publish his earlier work. He and others had hopes that the League of Nations in Geneva would be effective in preventing further wars.

However, the League's Disarmament Conference in Geneva from 1932–1934 was not successful, and rearmament in many countries followed. This led LFR "to reconsider and republish" his earlier ideas (see Sutherland & Nicholson 1993).

The first of the major themes of LFR's research in this field was the development of a suitable mathematical model for the tendencies of nations to prepare for war. He worked out the implications of the model for previous wars using historical data and then made predictions for 1935 onward using the model and recent economic and defense expenditure data. In a letter to *Nature* (Richardson 1935), he presented the first order coupled ordinary differential equations he had conceived in the First World War and pointed out that their solutions were consistent with the rapid, exponential growth of armaments by all sides before 1914. But it worried him that the equations also showed that the unilateral disarmament of Germany after 1918, enforced by the Allied Powers, combined with the persistent level of armaments of the victor countries would lead to the level of Germany's armaments growing again. In other words, the post-1918 situation was not stable. From the model he concluded that great statesmanship would be needed to prevent an unstable situation from developing, which could only be prevented by a change of policies (which he expressed as the need for new terms in the equations).

Subsequently, he published a full account of the theory, together with the data, in a monograph, "Generalized Foreign Politics" (Richardson 1939).

LFR (1951) returned to this mathematical model of arms races after the Second World War, when a new nuclear arms race was beginning between the United States and the USSR. He began: "There have only been three great arms-races. The first two of them ended in wars in 1914 and 1939; the third is still going on." He considered whether there were any new factors that ought to be considered that could lead to an end of the arms race without fighting.

Although an arms race tends to ever-growing armaments, small differences between countries tend to be amplified, which may make the "losing" nation submissive and stop increasing its arms. LFR introduced this effect into the equation as a submissiveness factor and showed mathematically that the arms race would cease. He asked, "Could events really happen thus? As far as I know, they never yet have done so." (But one might ask whether after 1989 the answer is now yes, even if the reasons do not exactly correspond to those in LFR's model?) (Sutherland & Nicholson 1993).

In fact, during his retirement, first at Paisley and later in Kilmun, LFR did not extend much further his studies of instability of peace or causes of wars. Perhaps it was because of the inevitability of a Second World War occurring in his lifetime that, during the late 1930s, he changed the direction of his research on wars to the study of their statistics: when and where they occurred, which kinds of people were involved, and which kinds of geographical factors made them more or less prevalent. Fluid dynamicists reading this review will be intrigued to learn how these studies helped give rise to the practical use of fractals.

As in his meteorological and psychological research, he began by collecting novel empirical data. Then he constructed new theoretical approaches, drawing on an even broader range of disciplines, in particular, psychoanalysis, geography, history, and politics, as well as deepening his use of mathematics.

But data are usually gathered with some idea as to how they will be used. In his study of wars, LFR investigated the Freudian thesis (see Freud 1933) that "from a psychological point of view a war, a riot, and a murder, though differing in many important aspects, social, legal and ethical, have at least this in common that they are all manifestations of the instinct of aggressiveness" (Richardson 1948a). This was the "justification for looking to see whether there is any statistical connection between war, riot and murder." Thus, starting in about 1939, LFR began the massive task of accumulating these data, solely from history books, encyclopedias, newspaper files, and in some cases correspondence. LFR's best-known finding from his own data, which he first reported in another of his excited letters to *Nature* in 1941, is that the number of quarrels (after 1800) decreased in frequency directly in relation to their magnitude, defined as the logarithm of the numbers killed on all sides of the quarrel. His data ranged from smaller-scale conflicts, such as banditry in Manchuria in 1935 and gang fights in Chicago (where the usual numbers were less than 10 and their logarithm less than 1), to world wars where the numbers were about 10 million (with a logarithm of 7). He found that the same formula was also applicable within a restricted range of magnitude and argued (essentially on the basis, familiar to fluid dynamicists, of self similarity) that therefore there is some general law for all kinds of conflict.

The data also showed that the average time between wars of different magnitudes (within this period) was random but did have a particular (Poisson) statistical distribution (Richardson 1960, p. 128). Thus, for example, you can estimate the most likely number of years before the next war or deadly quarrel of a given magnitude will occur (always assuming the future is similar to the past)—analogous to the amount of time spent waiting to cross a road. Recently, in the popular science book *Cosmos*, Professor Carl Sagan (1980) of Cornell University extrapolated (perhaps tendentiously) from LFR's result to predict that it will probably be about 1000 years before there is a conflict so large (magnitude 10) that the entire world population (10,000 million) will be annihilated.

Deadly quarrels, like other complex phenomena, occur randomly but nevertheless are determined by different factors that can be investigated and quantified; in LFR's words, "chaos restricted by geography and modified by infectiousness" (1960, p.285).

His data had shown that in different countries and in different populations there were different tendencies for quarrels to occur.

Other investigators might have sought historical, geographical, or sociological explanations but, as with his work on psychology, LFR sought explanations of his empirical and statistical discoveries by developing models drawn from physics. He attempted to explain the occurrence of wars in terms of how populations are distributed in concentrated groups within regions, and also how the shapes, length, and contiguities of the regions affect the propensity to war of populations within these regions.

One aspect of this topographical analysis, which led to a lasting contribution to mathematics and fluid mechanics, was the study of curves that are highly irregular and wiggly. LFR's discovery was not fundamentally a new result, but the way he found it using empirical methods continues to illuminate this aspect of research of mathematics and its application in many fields (see Drazin 1993). In dividing up countries and regions into hexagonal boxes of different shapes for constructing models of population distribution, he needed to calculate the lengths of the frontiers from the sum of the lengths of the sides of the boxes on the frontier. An embarrassing doubt arose as to whether actual frontiers were so intricate that their length could not be approximated by the perimeters of simple geometrical figures circumscribing the frontier. A special investigation was made to settle this question (Richardson 1961).

"(B)y walking a pair of dividers along a map of the frontier so as to count the number of equal sides of a polygon, the corners of which lie on the frontier," he made measurements. By summing the steps, an approximation to the total measured length of the frontier was calculated for each value of the step length between the points of the divider. LFR pointed out that Archimedes used the same method in his approximate calculation of the circumference of the circle.

For a straight line or a smooth curve, it is found that the perimeter is independent of the step length between the dividers. However, for wiggly or convoluted coast lines, as the step length decreases, more and more of the irregularities are included in the measurement of total length. So the measured perimeter is greater the smaller the step length. For a very wiggly coast line, such as the west coast of Britain, this variation is significant whereas for the rounded coast of South Africa it is not. Thus, the relation between the perimeter and the step length produces a quite new mathematical measure of wiggliness. LFR used this study to classify and correct the simple relation between the length of a frontier and the number of "hexagonal boxes" of a given area in the country in which the population was counted. This work was only published posthumously (Richardson 1961).

Since then there have been many other studies that have shown how the natural (as well as the mathematical) world is full of curves and surfaces that cannot be represented by smooth mathematical functions. LFR's pioneering discovery [which he had foreshadowed by his question "Does the wind have a velocity?" in his paper (Richardson 1926a), and which, I recently learned, he had discussed with WH McRea on an open-topped London bus in the 1930s] is now as widely known as any of his other work, largely because of the full account in Benoit Mandelbrot's celebrated book, *The Fractal Geometry of Nature* (1982). Ashford (1985, p. 260) reported how Mandelbrot, who is a specialist in this field of mathematics, accidentally discovered LFR's work on fractals when he was clearing out old papers and happened to glance at the appendix of the *General Systems Year Book* of 1961, where this work was posthumously published.

Retirement and His Final Research at Kilmun, 1943–1953

In 1943, LFR and his wife moved to their last home, Hill House at Kilmun, situated among a small group of houses on the shore of Holy Loch below high moorland. This is about 25 miles from Glasgow. This was where he received the famous visit by H Stommel, from Woods Hole, which was the opportunity to return to the question of how pairs, or a cloud, of particles separate in a turbulent flow, such as he saw from his house every day on the waters of the Holy Loch. They threw parsnips from a small pier into Loch Long, and using a remarkable measuring instrument they confirmed that the rate of spreading increased as the distance between the pairs of parsnips increased, consistent with the four-thirds law (Richardson & Stommel 1948). [LFR was certainly very pleased with this research, which he explained to my brother and me (aged 10 and 11) on our visit to him in 1953.]

He also returned to his research on the numerical solutions of differential equations and the associated study of the solution of sets of linear equations. He made use of important developments in the latter field, as he commented (Richardson 1950), "my two previous accounts of the method can now be much improved, by alliance with the great science of algebra; a coalescence suggested to me in 1948 by Arnold Lubin."

It appears that by the 1940s LFR was aware that it would be possible to make use of electrical circuits and electronic valves either to model differential equations (the analogue approach) or to act as logical devices to perform arithmetical operations. This would enable differential equations, such as those needed for weather predictions, to be calculated automatically without thousands of human computers, as he called them. In fact, as David Eversley (Eversley 1988) has recorded, in his "laboratory" at his home at Kilmun, LFR began making an analog computer using variable resistances and on/off electric valves to represent different inputs such as wind strength, barometric pressure, temperature at different levels in the atmosphere, and so on. During visits there, Eversley helped him solder the circuits. None of this was mentioned in his papers of the 1940s, in which he continued to use the word computer to refer to a human, and not to a machine.

However, at this time, J von Neumann and J Charney at Princeton were beginning to use the electronic digital computer ENIAC to calculate the weather using equations and methods close to those set out by LFR in his 1922 book, *Weather Prediction by Numerical Process*. Charney sent to LFR a copy of his paper (Charney et al 1950) containing the first results that had been published in *Tellus* in November 1950. LFR wrote to congratulate him and his collaborators on their remarkable progress, commenting that this was "an enormous scientific advance on the single, and quite wrong result, . . . in which the calculations of Richardson (1922) ended" (Ashford 1985).

Besides continuing his tireless scientific life at Kilmun, LFR also worked hard in the house and garden, because his pension left little money for luxuries or for employing others to help. He remained a great experimental innovator all his days. He constructed an amazing heating system of pipes and wires in the central hallway of his house; he used very smelly chemical tests before choosing paints for decorating the home; he had a great system for making jam (to economize on the rationed allowance of sugar); and he devised, with Dorothy, ingenious ways of consuming the vegetable harvests as they ripened, such as dishes of broad beans for breakfast, lunch, and tea at a certain period in August. However, during September 1953, he felt "very old and tired," as he wrote in a characteristically disputatious letter to Quincy Wright in the United States in which he again expressed his long-held disbelief in the stability of a balance of power. He died in his sleep on September 30, 1953.

Concluding Remarks

This brief account and personal view of Lewis Fry Richardson's life and work shows the extraordinary originality of his research, the intense care he took of it (for example, ensuring the tracing paper did not shrink when a light was brought close to it), and the novelty of its presentation. I hope that the present and future generations of students of fluid mechanics will continue to read his papers and find them as stimulating as I have. Some of their ideas are still worth pursuing and continue to give insight into current research controversies; they even contain useful empirical data that have still not been scientifically digested.

Just to list his formulae is one way of seeing the significance of his work: Richardson's method for extrapolation; the Richardson number (*Ri*) for turbulent flows in the presence of buoyancy forces; the Richardson four-thirds laws for the diffusion of clouds of particles in turbulent flows; his scales for pain and hearing; Richardson's equation for the causes of war; the scale for the magnitude of wars [which has recently been applied to natural disasters (Keller & Al-Madhari 1993)] and the formula for their frequency; and finally the index (now called a fractal dimension) as a measure of the irregularity of curves. LFR is not alone in having the fame and usefulness of his research result as much from the way it has synthesized others' findings as from its own particular contribution.

In Richardson's case, there is no question about the permanent value of the many special insights he gave to numerical analysis and fluid mechanics; his methodology for numerical weather prediction has essentially been adopted by modern practitioners (e.g. Hollingsworth 1994). Equally, in experimental psychology, LFR helped introduce quantitative techniques that are now used in many applications—wherever the sensory perception of physical conditions or influences is a crucial factor, such as in medicine, environmental health and planning, industrial hygiene, and ergonomics.

However, it is much harder to evaluate his work on how hostile attitudes and actions of nations change when wars are likely, or his results on the statistics of international and civil warfare and their relation to geographical factors. LFR's quantitative approach has profoundly influenced academic studies of conflict. I understand it is also used as an antithesis to the usual historical approach based on the study of the individual "eddies" of the past. As to whether LFR's work has influenced government policy in international affairs, all one can say with certainty is that the planning and conduct of wars continue to be significantly influenced by various kinds of mathematical modeling, some of which resemble the methods introduced by LFR, Lanchester, and others between 1915 and 1935 (Bennett 1988).

One cannot help feeling that the author of the 1951 paper "Could an arms-race end without fighting?" would be mightily pleased with the recent remarkable ending of the nuclear arms race by the major powers. But at the same time, the Quaker scientist who discovered the laws describing the connection between more and longer frontiers and the frequency of different levels of conflicts would have noted, with sorrow, that recent history gives no hint that his inexorable formula for smaller scale conflicts has ceased to apply.

ACKNOWLEDGMENTS

Much of the introduction of this article was based on the biographical account by Ashford (1985) and Gold (1954). It is also based on recollections by various members of Lewis and Dorothy Richardson's family, as well as my own. A fuller version of this article and other reviews of LFR's mathematical and

meteorological work by L Fox, H Charnock, and PG Drazin are included in the collected works of LFR (Drazin & Sutherland 1993).

Visit the *Annual Reviews home page* at http://www.AnnualReviews.org.

Literature Cited

Ashford O. 1985. *Prophet or Professor: The Life and Work of L.F. Richardson.* Bristol: Hilger

Batchelor GK. 1952. Diffusion in a field of homogeneous turbulence. II. The relative motion of particles. *Proc. Cambridge Philos. Soc.* 48:345–62

Beauman N. 1993. *Morgan: A Biography of E. M. Forster.* London: Hodder & Stoughton

Bennett P, ed. 1988. *Mathematical Modelling of Conflict and Its Resolution. Proc. IMA Conf. Cambridge, Dec. 1986.* Oxford: Clarendon

Charney JG, Fj'rtoft R, von Neumann J. 1950. Numerical integrations of the barotropic vorticity equations. *Tellus* 2:237–54

Charnock H. 1993. Meteorology. See Drazin & Sutherland 1993, 1:29–36

Drazin PG. 1993. Fractals. See Drazin & Sutherland 1993, 1:45

Drazin P. G., Sutherland I, eds. 1993. *Collected Papers of Lewis Fry Richardson*, Vols. 1, 2. Cambridge: Cambridge Univ. Press

Drysdale CV, Ferguson A, Geddes AEM, Gibson AH, Hunt FRW, et al. 1936. *The Mechanical Properties of Fluids*. London: Blackie

Eversley D. 1988. Unpublished letter to Mrs. V Bottomley, MP

Freud S. 1933. *Civilization, Society and Religion. Penguin Freud Library,* Vol. 12. London: Penguin

Friedman RM. 1989. *Appropriating the Weather—V Bjerknes and the Construction of Modern Meteorology.* New York: Cornell Univ. Press

Gold E. 1954. Lewis Fry Richardson. *Obit Note Fellows R. Soc.* 9:217–35

Hollingsworth A. 1994. Validation and diagnosis of atmospheric models. *Dyn. Atmos. Oceans* 20:227–46

Hunt JCR, Poulton EC, Mumford JE. 1975. The effects of wind on people; new criteria based on wind tunnel experiments. *Build. Environ.* 11:15–28

James W. 1901. *Principles of Psychology.* London: Macmillan

Keller AZ, Al-Madhari A. 1993. Use of disaster magnitude scale to analyse natural disasters. In *Natural Disasters Protecting Vulnerable Communities*, ed. PA Merriman, CWA Browitt. London: Thomas Telford

Kolmogorov AN. 1941. The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers. *C. R. Acad. Sci. URSS* 30:301

Lanchester FW. 1916. *Aircraft in Warfare, the Dawn of the Fourth Arm.* London: Constable

Lynch P. 1993. Richardson's forecast factory: the $64,000 question. *Meteorol. Mag.* 122:69–70

Mandelbrot B. 1982. *The Fractal Geometry of Nature.* San Francisco: Freeman

McDougall W. 1905. *Physiological Psychology.* London: Dent

Oboukhov AM. 1941. Spectral energy distribution in a turbulent flow. *Izv. Acad. Nauk SSR Geogr. Geofiez.* 5:453–66

Poulton EC. 1988. *Bias in Quantifying Judgement.* London: Erlbaum

Poulton EC. 1993. The quantifying of mental events and sensations. See Drazin & Sutherland 1993, 2:29–32

Prandtl, L. 1931. Abriss der Strömungslere. Brunswick: Vieweg

Richardson LF. 1908. A freehand graphic way of determining stream lines and equipotentials. *Proc. Phys. Soc. London* 21:88–124

Richardson LF. 1910. The approximate arithmetical solution by finite differences of physical problems involving differential equations, with an application to the stresses in a masonry dam. *Phil. Trans. R. Soc. London Ser. A* 210:307–57

Richardson LF. 1913. The measurements of mental 'nature' and the study of adopted children. *Eugenics Rev.* 4:391–94

Richardson LF. 1919a. *Mathematical Psychology of War.* Oxford: Hunt

Richardson LF. 1919b. Measurement of water in clouds. *Proc. R. Soc. London Ser. A* 96:19–31

Richardson LF. 1920. Some measurements of atmospheric turbulence. *Philos. Trans. R. Soc. London Ser. A* 221:1–28

Richardson LF. 1922. *Weather Prediction by Numerical Process.* Cambridge: Cambridge Univ. Press

Richardson LF. 1923. Wind above the night-calm at Benson at 7 a.m. *Q. J. R. Met. Soc.* 49:34

Richardson LF. 1924a. Attempts to measure air temperature by shooting spheres upward. *Q. J. R. Met. Soc.* 50:19–22

Richardson LF. 1924b. The aerodynamic resistance of spheres, shot upward to measure the wind. *Proc. Phys. Soc. London* 36:67–80

Richardson LF. 1926a. Atmospheric diffusion shown on a distance-neighbour graph. *Proc. R. Soc. London Ser. A* 110:709–37

Richardson LF (with D Proctor). 1926b. Diffusion over distances ranging from 3 km to 86 km. *Mem. R. Met. Soc.* 1:1–16

Richardson LF. 1927. The deferred approach to the limit. I. *Philos. Trans. R. Soc. London Ser. A* 226:299–349

Richardson LF. 1928a. Report on photometers for a survey of the reflectivity of the earth's surface. *Int. Geod. Geophys. Meet. Sect. Prague 1927,* 1–18

Richardson LF. 1928b. Thresholds when sensation is regarded as quantitative. *Br. J. Psychol.* 19:158–66

Richardson LF. 1929a. A search for the law of atmospheric diffusion. *Beitr. Phys. Frei. Atmos.* 15:24–29

Richardson LF. 1929b. Imagery, conation and cerebral conductance. *J. Gen. Psychol.* 2:324–52

Richardson LF. 1929c. Quantitative mental estimates of light and colour. *Br. J. Psychol.* 20:27–37

Richardson LF. 1930a. The reflectivity of woodland, fields and suburbs between London and St. Albans. *Q. J. R. Met. Soc.* 56:31–38

Richardson LF (with JS Ross). 1930b. Loudness and telephone current. *J. Gen. Psychol.* 3:288–306

Richardson LF. 1935. Mathematical psychology of war. *Nature London* 135:830–31

Richardson LF. 1937. Hints from physics and meteorology as to mental periodicities. *Br. J. Psychol.* 28:212–15

Richardson LF. 1939. Generalized foreign politics. *Br. J. Psychol. Monogr. Suppl., No. 23. 91 pp.*

Richardson LF. 1941. Frequency of occurrence of wars and other fatal quarrels. *Nat. London* 148:784

Richardson LF. 1948a. War-moods: I and II. *Psychometrika* 13:147–74; 197–232

Richardson LF. 1950. A purification method for computing the latent columns of numerical matrices and some integrals of differential equations. *Philos. Trans. R. Soc. London Ser. A* 242:439–91

Richardson LF. 1951. Could an arms-race end without fighting? *Nat. London* 168:567–68

Richardson LF. 1960. *Statistics of Deadly Quarrels.* Pittsburgh: Boxwood

Richardson LF. 1961. The problem of contiguity: an appendix of statistics of deadly quarrels. *Gen. Syst. Yearb.* 6:139–87

Richardson LF, Stommel H. 1948. Note on eddy diffusion in the sea. *J. Met.* 5:238–40

Runge C. 1895. Uber die numerische Auflosüng von Differential-gleichungen. *Math. Ann.* 46

Sagan C. 1980. *Cosmos*. New York: Random House

Sutherland I. 1993. The causation of wars. See Drazin & Sutherland 1993, 2:32–42

Taylor G. I. 1958. The present position in the theory of turbulent diffusion. *Adv. Geophys.* 6:101–11

van der Pol B. 1926. On relaxation-oscillations. *Philos. Mag.* 61:978–92

Yaglom AM. 1994. A.N. Kolmogorov as a fluid mechanician and founder of a school in turbulence research. *Annu. Rev. Fluid Mech.* 26:1–22