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with $\mu \neq 0$. A solution for G is

$$G(t) = G_0 e^{-\mu^2 t/a} \quad (5.17)$$

To construct a solution for F , we must also construct $k(x, y)$. After a bit of trial and error, we found the following solution for $\mu = 1$:

$$\begin{aligned} F(x, y) &= e^x \cos y \\ k(x, y) &= e^x \sin y - x \end{aligned} \quad (5.18)$$

This solution can be scaled to ensure it satisfies the requirements outlined in Section 5.2.1.

5.2.5.2 Steady incompressible flow with no source term

Manufactured solutions can also be constructed for nonlinear systems of homogeneous equations. We illustrate using the equations for two-dimensional, steady, incompressible, laminar flow:

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ \frac{\partial}{\partial x}(u^2 + h) + \frac{\partial}{\partial y}(uv) &= \nu \nabla^2 u \\ \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2 + h) &= \nu \nabla^2 v \end{aligned} \quad (5.18)$$

with $h = P/\rho$ and ν a constant. To satisfy the continuity equation, let $\phi = \phi(x, y)$ and set

$$\begin{aligned} u &= -\frac{\partial \phi}{\partial y} \\ v &= +\frac{\partial \phi}{\partial x} \end{aligned} \quad (5.19)$$

The momentum equations then become

$$\begin{aligned} \frac{\partial h}{\partial x} &= R \\ \frac{\partial h}{\partial y} &= Q \end{aligned} \quad (5.20)$$

where

$$R = \frac{\partial \phi}{\partial x} \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial \phi}{\partial y} \frac{\partial^2 \phi}{\partial x \partial y} - v \frac{\partial}{\partial y} (\nabla^2 \phi) \quad (5.21)$$

$$Q = \frac{\partial \phi}{\partial y} \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial \phi}{\partial x} \frac{\partial^2 \phi}{\partial x \partial y} - v \frac{\partial}{\partial x} (\nabla^2 \phi)$$

In order for h to exist, we must have

$$\frac{\partial R}{\partial y} = \frac{\partial Q}{\partial x} \quad (5.22)$$

This means that ϕ must satisfy

$$v \nabla^4 \phi - \frac{\partial \phi}{\partial x} \frac{\partial}{\partial y} (\nabla^2 \phi) + \frac{\partial \phi}{\partial y} \frac{\partial}{\partial x} (\nabla^2 \phi) = 0 \quad (5.23)$$

To construct a manufactured solution, choose ϕ so that $\nabla^2 \phi = \mu$ (a constant). Then the last equation is automatically satisfied. The velocity components u and v are then computed from the derivatives of ϕ . By computing R and Q from ϕ , one can find h by integration. For example, let

$$\phi(x, y) = e^x \cos y - e^y \sin x \quad (5.24)$$

Then $\mu = 0$ because $\nabla^2 \phi = 0$. From the derivatives of ϕ , we find

$$u(x, y) = e^x \sin y + e^y \sin x \quad (5.25)$$

$$v(x, y) = e^x \cos y - e^y \cos x$$

The functions R and Q are then

$$R(x, y) = -e^{2x} - e^{x+y} [\sin(x+y) - \cos(x+y)] \quad (5.26)$$

$$Q(x, y) = -e^{2y} - e^{x+y} [\sin(x+y) - \cos(x+y)]$$

and finally,

$$h(x, y) = -\frac{1}{2} e^{2x} - \frac{1}{2} e^{2y} + e^{x+y} \cos(x+y) \quad (5.27)$$