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of Nozzle Contours**

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A Strategy for the Optimal Design of Nozzle Contours*

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Abstract

A strategy is proposed and analyzed for the aerodynamic design of optimally contoured, high-enthalpy, hypersonic nozzles. The approach involves expressing the desired contour as an optimal *convex* combination of trial configurations. The methods used are given a firm theoretical foundation. This includes mathematical uniqueness results that show what exit conditions guarantee uniform flow in a neighborhood of the nozzle exit. Also, convergence results are verified for the design scheme. Based upon this theoretical foundation, a modular, robust, axisymmetric nozzle design code is implemented. This design package is used successfully to design a nozzle that accelerates a turbulent, viscous perfect gas to a uniform Mach Number 4.0 flow in a neighborhood of the exit plane.

Nomenclature

A Matrix defined for the quadratic programming problem in Eq. (6)

b Vector defined for the quadratic programming problem in Eq. (6)

c_j, c_j^l Coefficients used to define nozzle contours, f and f^l

C^k Space of functions with k continuous derivatives

e Total energy per unit mass

e Vector with all unit components

f Nozzle contour function

f^l Approximation to an optimal nozzle contour obtained after l iterations

f_j Trial configuration or basis function used to define a nozzle contour

h, h_0 Total enthalpy per unit mass and its constant value at a nozzle exit

L Lagrangian function defined in Eq. (8)

\mathcal{L} Lagrangian function defined in Eq. (11)

M, M_0 Mach Number and its constant value at a nozzle exit

M Mach Number distribution for a given contour

M_i Component of M for the i th grid cell

M' (Fréchet) derivative of the Mach Number distribution with respect to contour

M'_i Component of M' for the i th grid cell

p, p_0, p_∞ Local static, stagnation, and free-stream static pressure

Q Vector of conservation variables

s, s_0 Entropy per unit mass and its constant value at a nozzle exit

$T, T_0,$ Local static, stagnation, free-stream static, and wall temperature

$T_\infty, T_{\text{wall}}$

u, v, w, u_∞ Cartesian velocity components and free-stream velocity

U, V, W Matrices used for the singular value decomposition of Eq. (9)

x, y, z Cartesian coordinates

x Vector defined for the quadratic programming problem in Eq. (6)

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