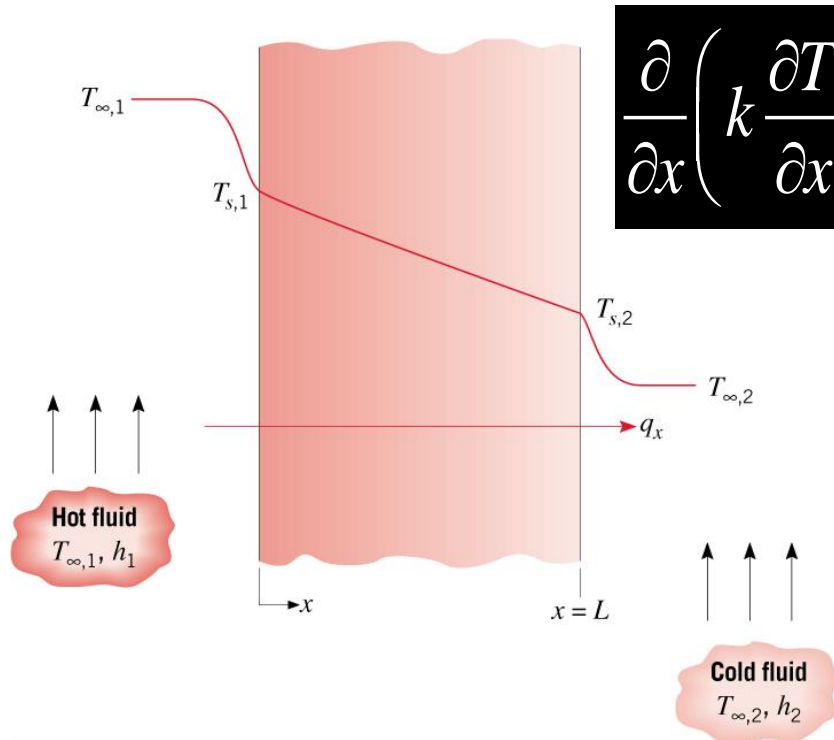


# **Cap. 3 – Condução de Calor Unidimensional e em Regime Permanente**

# Parede Plana: Condução Unidimensional em Coordenadas Cartesianas



$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

- **Simplificando:**

$$\frac{d}{dx} \left( k \frac{dT}{dx} \right) = 0$$

- **Cond. de Contorno:**
- **Distribuição de Temperatura** para  $k=\text{cte}$ :

$$T(x) = T_{s,1} + (T_{s,2} - T_{s,1}) \frac{x}{L}$$

- **Distribuição de Temperatura** para  $k=\text{cte}$ :

$$T(x) = T_{s,1} + (T_{s,2} - T_{s,1}) \frac{x}{L}$$

- **Fluxo de Calor e taxa de transferência de calor:**

$$q_x'' = -k \frac{dT}{dx} = \frac{k}{L} (T_{s,1} - T_{s,2}) \quad (3.5)$$

$$q_x = -kA \frac{dT}{dx} = \frac{kA}{L} (T_{s,1} - T_{s,2}) \quad (3.4)$$

# Resistência Térmica e circuito térmico equivalente

*Taxa transferida de calor (q) ~ corrente elétrica (i)*

*Temperatura (T) ~ diferença de potencial (ddp)*

*Resistência Térmica ( $R_t$ ) ~ resistência elétrica ( $R_e$ )*

$$R = \frac{V}{i} \text{ Lei de Ohm}$$

- Resistência Térmica  $\left( R_t = \frac{\Delta T}{q} \right)$  e circuito térmico equivalente:

Condução em parede plana:

$$R_{t,cond} = \frac{L}{kA}$$

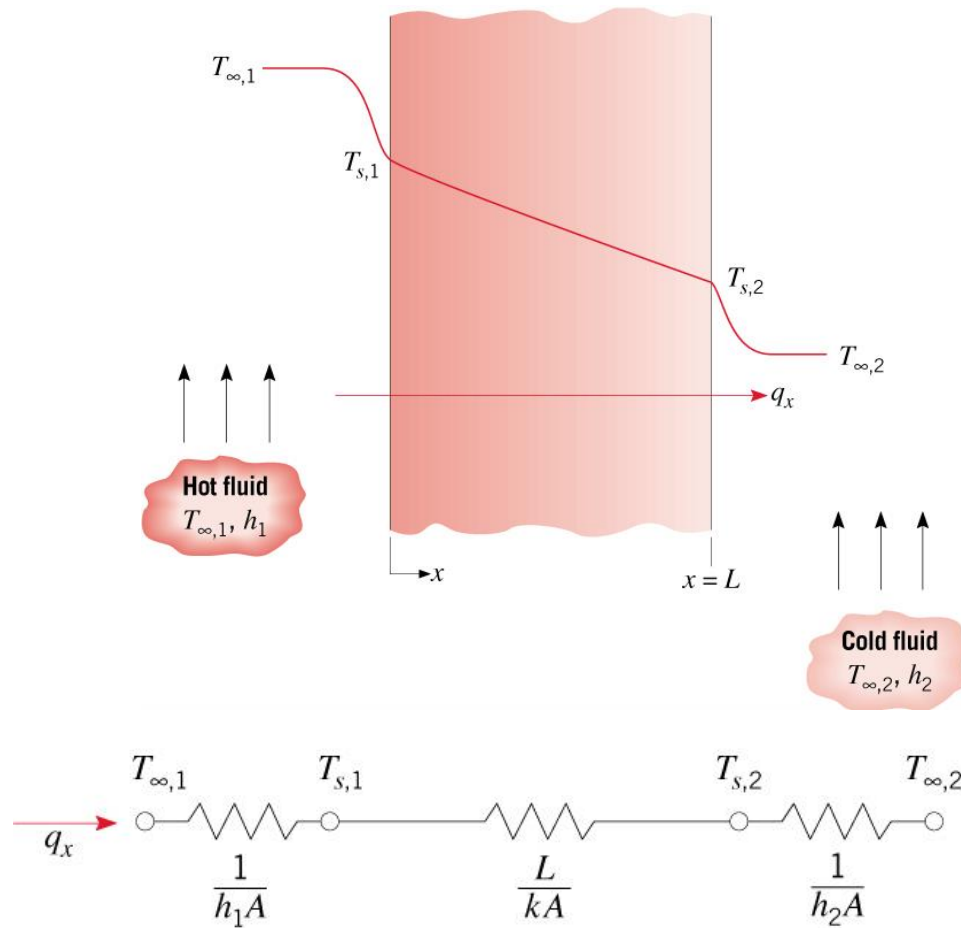
(3.6)

Convecção:

$$R_{t,conv} = \frac{1}{hA}$$

(3.9)

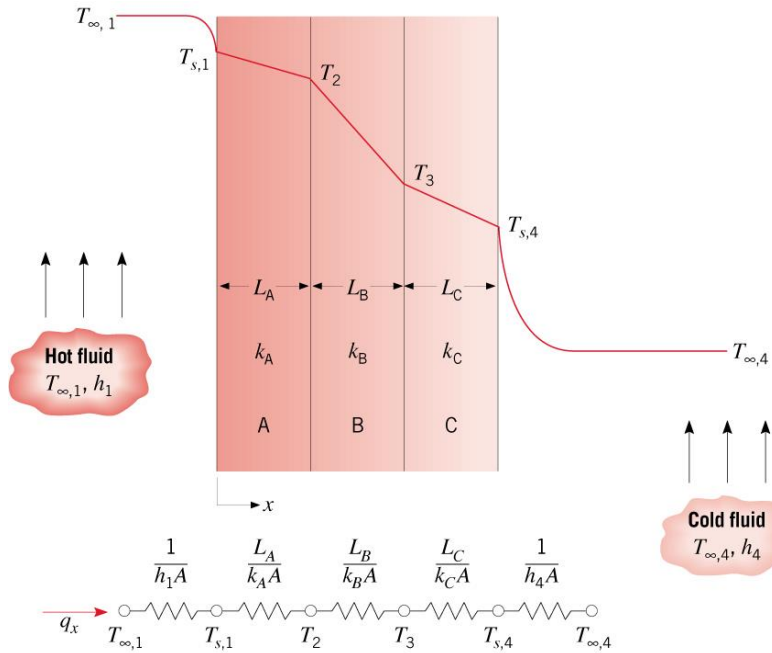
## Circuito térmico em parede plana:



$$R_{tot} = \frac{1}{h_1 A} + \frac{L}{k A} + \frac{1}{h_2 A} \quad (3.12)$$

$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}} \quad (3.11)$$

- Paredes planas compostas sem resistência de contato:



$$q_x = \frac{T_{\infty,1} - T_{\infty,4}}{\sum R_t} \quad (3.14)$$

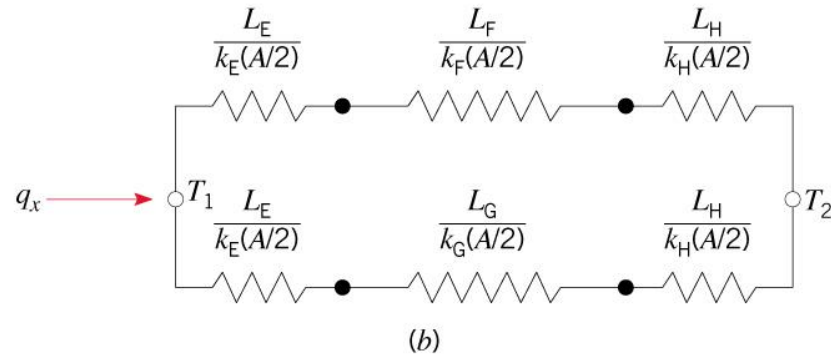
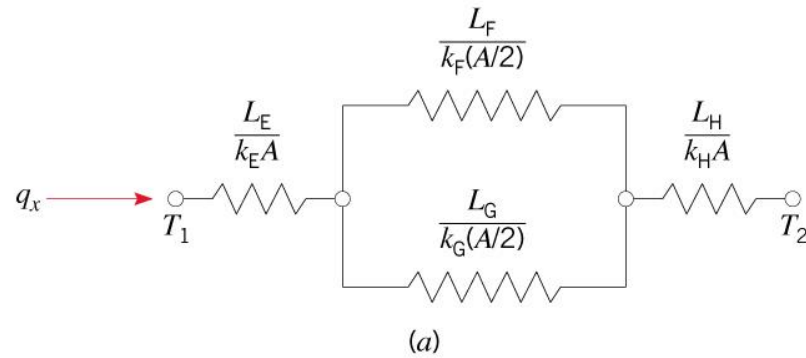
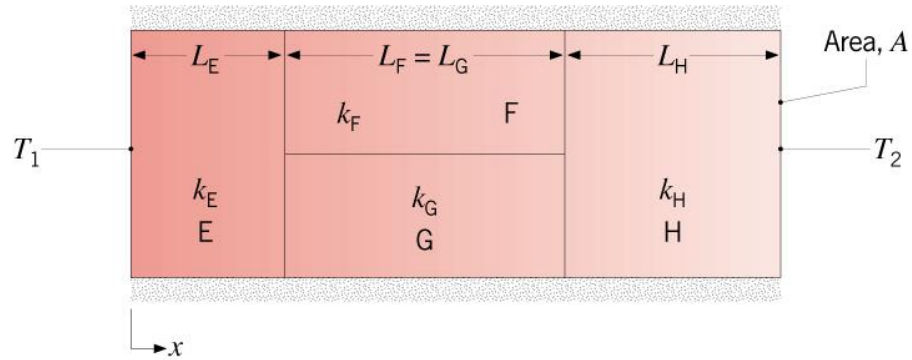
$$\sum R_t = R_{tot} = \frac{1}{A} \left[ \frac{1}{h_1} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_4} \right] = \frac{R''_{tot}}{A}$$

- Coeficiente Global de Transferência de Calor ( $U$ ) :

$$q_x = UA\Delta T_{overall} \quad (3.17)$$

$$R_{tot} = \frac{1}{UA} \quad (3.19)$$

- Parede Plana – Associação em Série/Paralelo:



- Resistência Térmica por unidade de área:

$$R''_{t,cond} = \frac{L}{k}$$

$$R''_{t,conv} = \frac{1}{h}$$

$$\text{Units: } R_t \leftrightarrow \text{K/W}$$

$$R''_t \leftrightarrow \text{m}^2 \cdot \text{K/W}$$

- Resistência térmica para radiação:

$$R_{t,rad} = \frac{1}{h_r A}$$

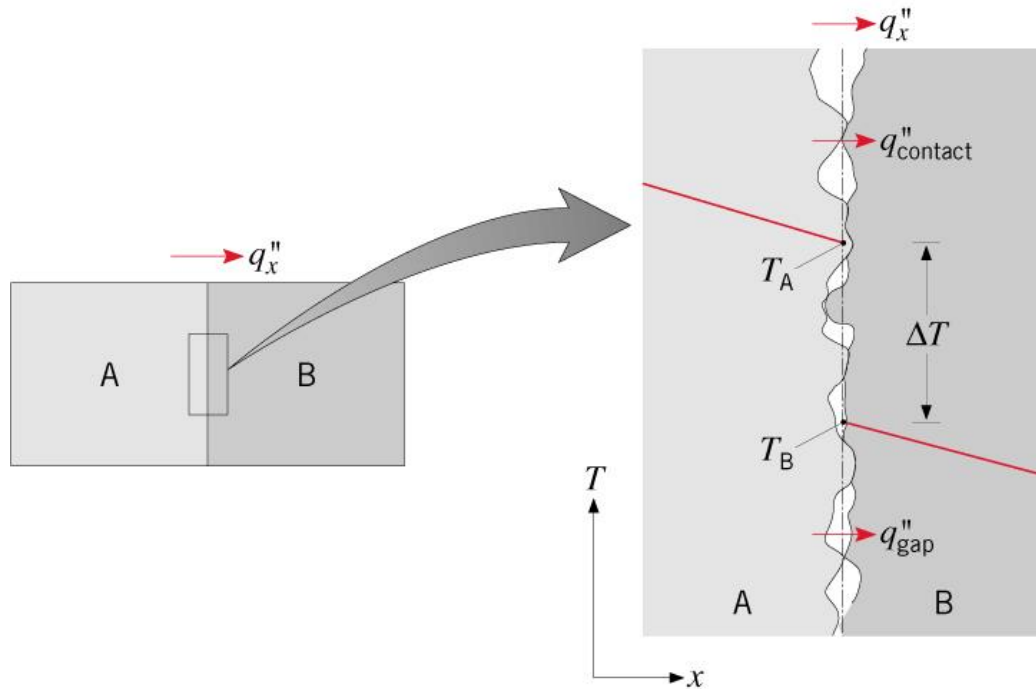
$$R''_{t,rad} = \frac{1}{h_r}$$

$$h_r = \varepsilon \sigma (T_s + T_{sur}) (T_s^2 + T_{sur}^2)$$

(1.9)



# Resistência de contato



$$R''_{t,c} = \frac{T_A - T_B}{q_x''}$$

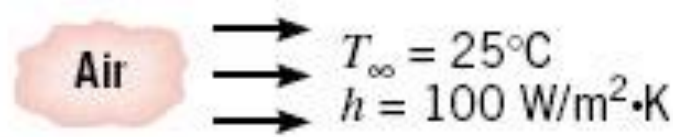
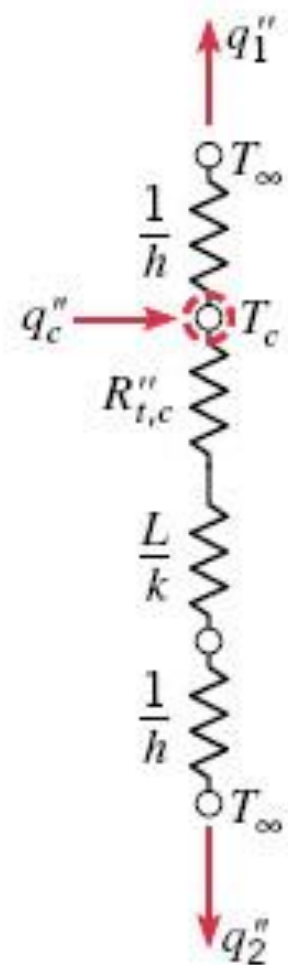
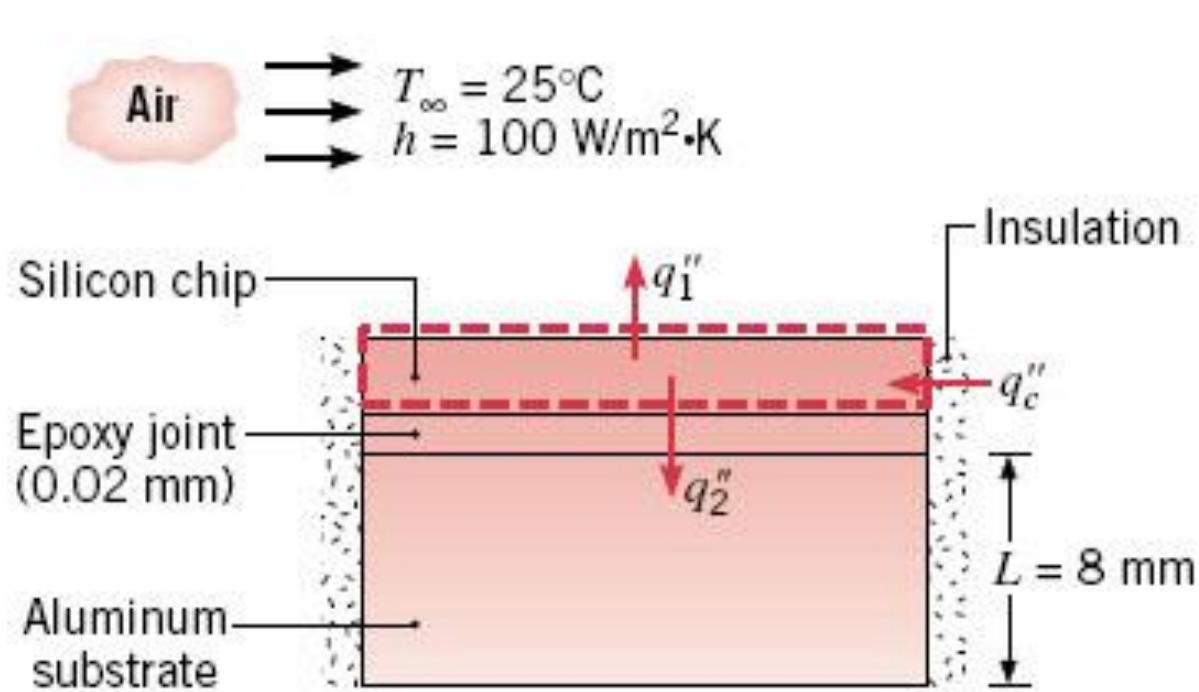
$$R_{t,c} = \frac{R''_{t,c}}{A_c}$$

Dependência da resistência de contato ?

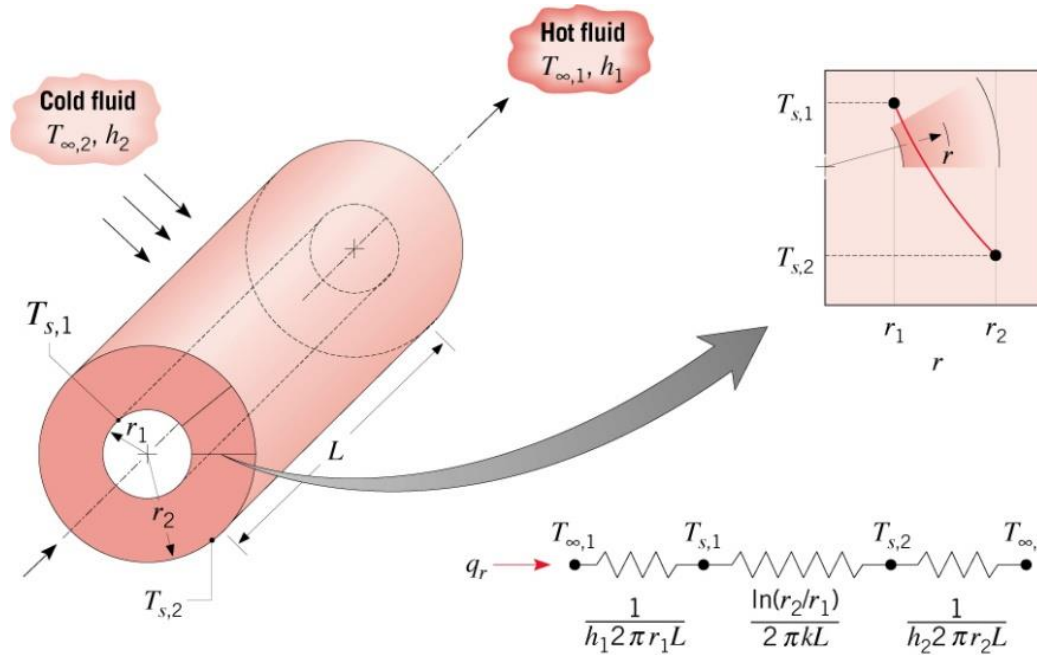
**TABLE 3.1** Thermal contact resistance for (a) metallic interfaces under vacuum conditions and (b) aluminum interface (10- $\mu\text{m}$  surface roughness,  $10^5 \text{ N/m}^2$ ) with different interfacial fluids [1]

Thermal Resistance,  $R''_{t,c} \times 10^4 \text{ (m}^2 \cdot \text{K/W)}$

<i>(a) Vacuum Interface</i>			<i>(b) Interfacial Fluid</i>	
Contact pressure	100 kN/m <sup>2</sup>	10,000 kN/m <sup>2</sup>	Air	2.75
Stainless steel	6–25	0.7–4.0	Helium	1.05
Copper	1–10	0.1–0.5	Hydrogen	0.720
Magnesium	1.5–3.5	0.2–0.4	Silicone oil	0.525
Aluminum	1.5–5.0	0.2–0.4	Glycerine	0.265



# Sistemas Radiais



$$\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{d}{dr} \left( kr \frac{dT}{dr} \right) = 0$$

- Distribuição de temperatura na parede para k=cte:

$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_{s,2}$$

(3.26)

- Distribuição de temperatura na parede para  $k=\text{cte}$ :

$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_{s,2} \quad (3.26)$$

- Fluxo de calor e taxa transferida de calor:

$$q_r'' = -k \frac{dT}{dr} = \frac{k}{r \ln(r_2/r_1)} (T_{s,1} - T_{s,2})$$

$$q_r' = 2\pi r q_r'' = \frac{2\pi k}{\ln(r_2/r_1)} (T_{s,1} - T_{s,2})$$

$$q_r = 2\pi r L q_r'' = \frac{2\pi L k}{\ln(r_2/r_1)} (T_{s,1} - T_{s,2})$$

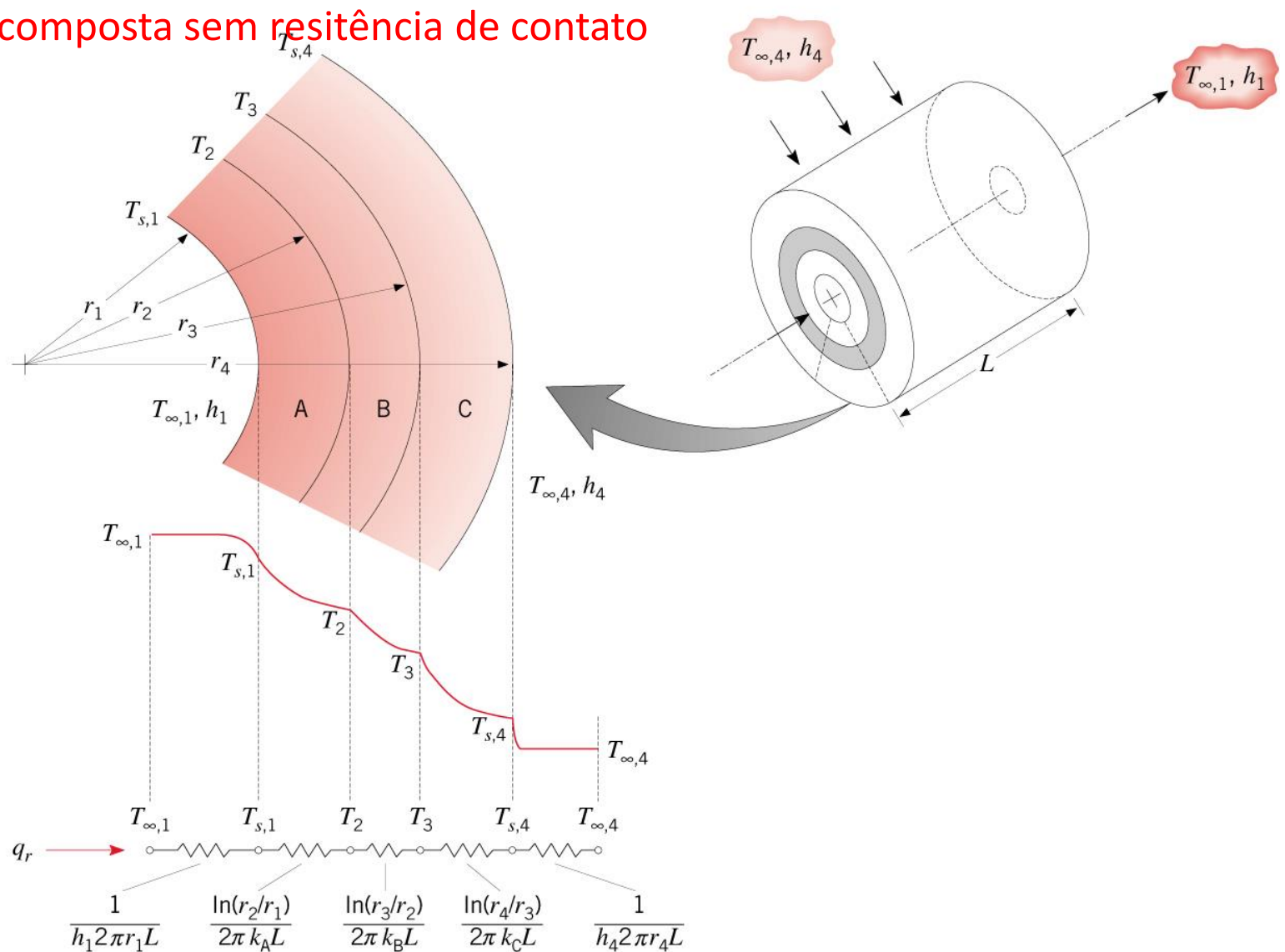
- Resistência Térmica de Condução

$$R_{t,cond} = \frac{\ln(r_2/r_1)}{2\pi L k} \quad \text{Units} \leftrightarrow \text{K/W} \quad (3.28)$$

$$R_{t,cond}' = \frac{\ln(r_2/r_1)}{2\pi k} \quad \text{Units} \leftrightarrow \text{m} \cdot \text{K/W}$$

É inapropriado trabalhar neste caso com  $R''$

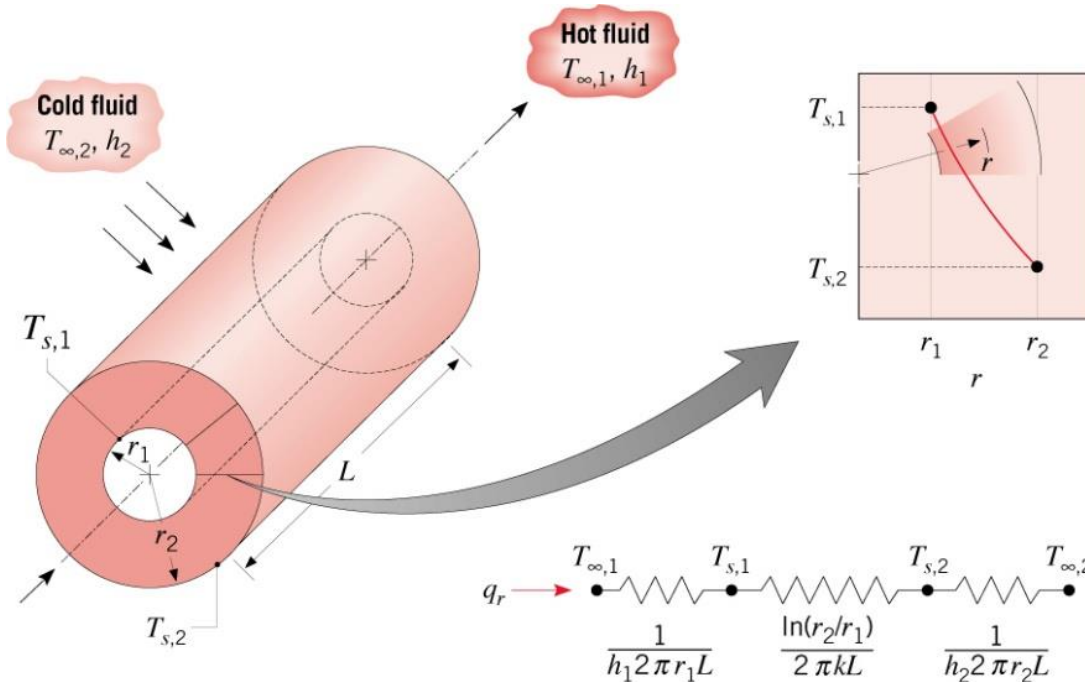
- Parede composta sem resistência de contato



$$q_r = \frac{T_{\infty,1} - T_{\infty,4}}{R_{tot}} = UA(T_{\infty,1} - T_{\infty,4})$$

$UA = R_{tot}^{-1}$  é independente do raio

# Espessura Crítica



$$q_r' = \frac{T_{\infty} - T_i}{R'_{tot}}$$

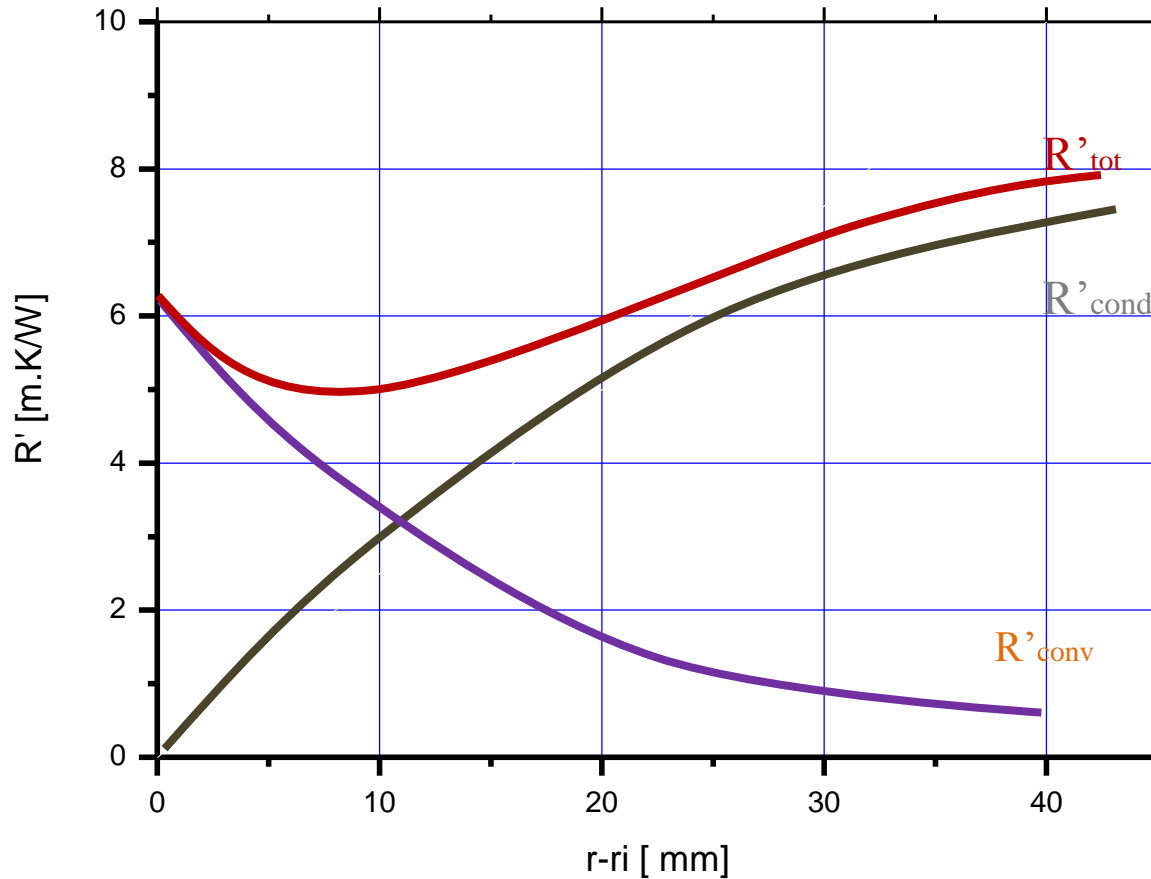
$$q_r'' = -k \frac{dT}{dr} = \frac{k}{r \ln(r_2/r_1)} (T_{s,1} - T_{s,2})$$

$$q_r' = 2\pi r q_r'' = \frac{2\pi k}{\ln(r_2/r_1)} (T_{s,1} - T_{s,2})$$

$$q_r = 2\pi r L q_r'' = \frac{2\pi L k}{\ln(r_2/r_1)} (T_{s,1} - T_{s,2})$$

$$R'_{tot} = \frac{\ln(r/r_i)}{2k\pi} + \frac{1}{2\pi r h}$$

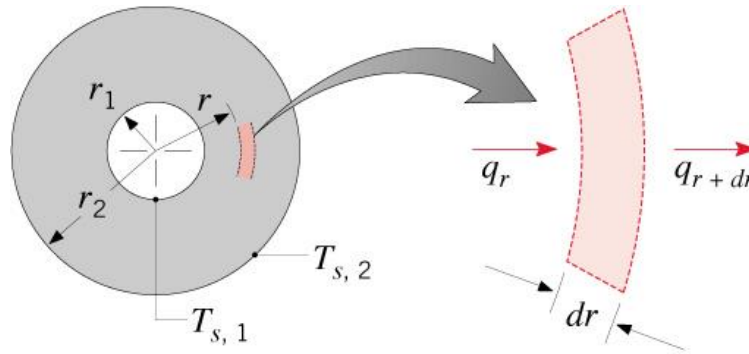
# Espessura crítica



$r=5$  mm  
 $k=0,055$  W/(m.K)  
 $h=5$  W/( $\text{m}^2 \cdot \text{K}$ )



# Coordenadas Esféricas



$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) = 0$$

- Distribuição de temperatura para  $k=\text{cte}$ :

$$T(r) = T_{s,1} - (T_{s,1} - T_{s,2}) \frac{1 - (r_1/r)}{1 - (r_1/r_2)}$$

- Fluxo de calor, taxa transferida de calor e Resistência Térmica:

$$q_r'' = -k \frac{dT}{dr} = \frac{k}{r^2 \left[ (1/r_1) - (1/r_2) \right]} (T_{s,1} - T_{s,2})$$

$$q_r = 4\pi r^2 q_r'' = \frac{4\pi k}{(1/r_1) - (1/r_2)} (T_{s,1} - T_{s,2}) \quad (3.35)$$

$$R_{t,cond} = \frac{(1/r_1) - (1/r_2)}{4\pi k} \quad (3.36)$$

$$q_r = \frac{\Delta T_{overall}}{R_{tot}} = UA\Delta T_{overall}$$

$$UA = R_{tot}^{-1} \leftrightarrow \text{Constant}$$

$$U_i = (A_i R_{tot})^{-1} \leftrightarrow \text{Depends on } A_i$$

CONDUÇÃO UNIDIMENSIONAL EM  
REGIME PERMANENTE COM  
GERAÇÃO DE CALOR

## Geração de energia

$$\dot{q} = \frac{\dot{E}_g}{\nabla} = \frac{I^2 R_e}{\nabla}$$

(3.38)

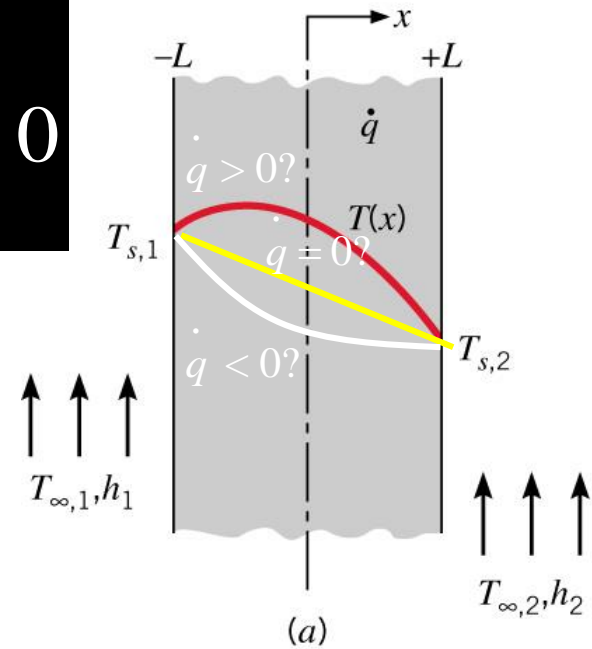
$$\dot{q} \propto \exp(-\alpha x)$$

# A Parede Plana

- Para  $k=\text{cte}$

$$\frac{d}{dx} \left( k \frac{dT}{dx} \right) + \dot{q} = 0 \rightarrow \frac{d^2 T}{dx^2} + \frac{\dot{q}}{k} = 0$$

$$T(x) = - \left( \frac{\dot{q}}{2k} \right) x^2 + C_1 x + C_2$$



$$T(x) = \frac{\dot{q}L^2}{2k} \left( 1 - \frac{x^2}{L^2} \right) + \frac{T_{s,2} - T_{s,1}}{2} \frac{x}{L} + \frac{T_{s,2} + T_{s,1}}{2}$$

Condição de simetria ou face isolada :

- Distribuição de temperatura :

$$T(x) = \frac{\dot{q} L^2}{2k} \left( 1 - \frac{x^2}{L^2} \right) + T_s$$

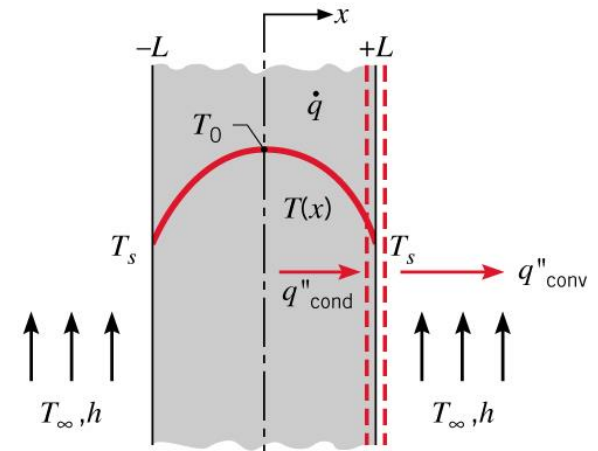
$T_s$

$$-\dot{E}_{out} + \dot{E}_g = 0$$

$$-hA_s(T_s - T_\infty) + \dot{q} A_s L = 0$$

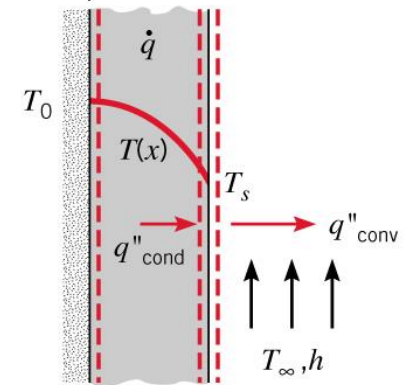
$$T_s = T_\infty + \frac{\dot{q} L}{h}$$

(3.42)



(b)

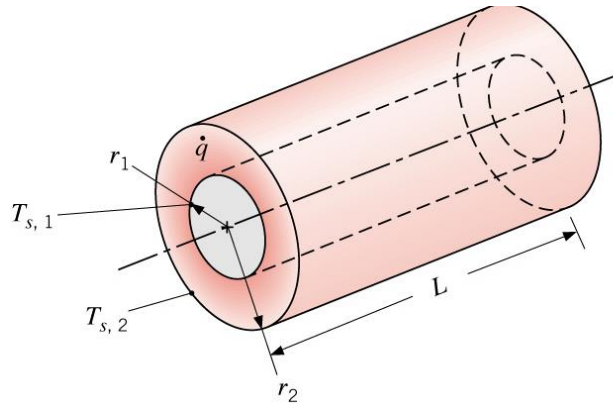
(3.46)



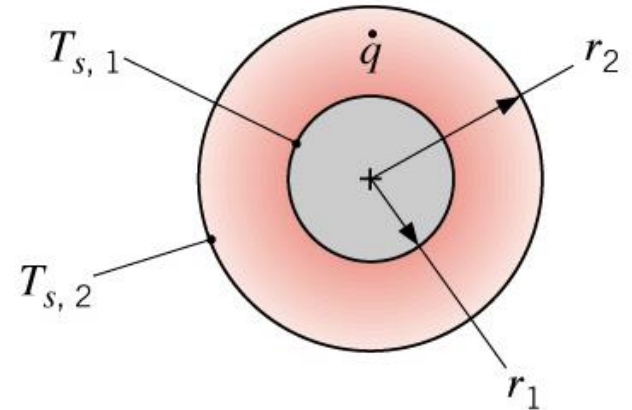
(c)

# Sistemas Radiais

Tubo cilíndrico

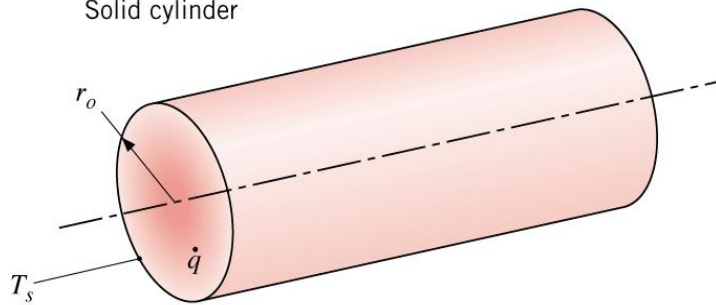


Casca esférica

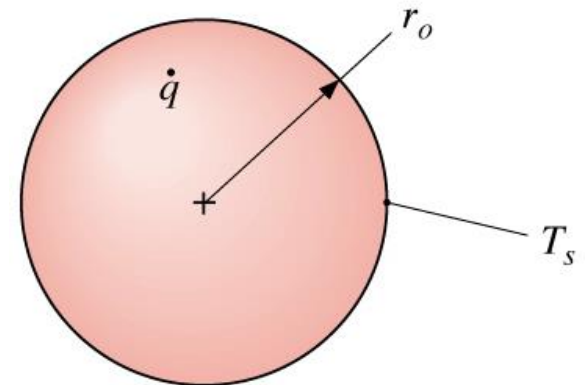


Cilindro Sólido

Solid cylinder



Esfera Sólida



Cilindro

$$\frac{1}{r} \frac{d}{dr} \left( kr \frac{dT}{dr} \right) + \dot{q} = 0$$

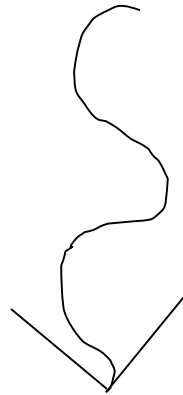
Esfera

$$\frac{1}{r^2} \frac{d}{dr} \left( kr^2 \frac{dT}{dr} \right) + \dot{q} = 0$$

# Geometria cilíndrica

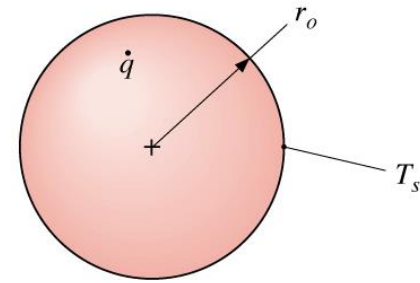
$$\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r} \frac{d}{dr} \left( kr \frac{dT}{dr} \right) + \dot{q} = 0$$





# Esfera com convecção na superfície



$$\frac{1}{r^2} \frac{d}{dr} \left( kr^2 \frac{dT}{dr} \right) + \dot{q} = 0$$

Distribuição de Temperatura

$$kr^2 \frac{dT}{dr} = -\frac{\dot{q} r^3}{3} + C_1$$

$$T = -\frac{\dot{q} r^2}{6k} - \frac{C_1}{r} + C_2$$

$$\frac{dT}{dr} \Big|_{r=0} = 0 \rightarrow C_1 = 0$$

$$T(r_o) = T_s \rightarrow C_2 = T_s + \frac{\dot{q} r_o^2}{6k}$$

$$T(r) = \frac{\dot{q} r_o^2}{6k} \left( 1 - \frac{r^2}{r_o^2} \right) + T_s$$

Temperatura na superfície

BF

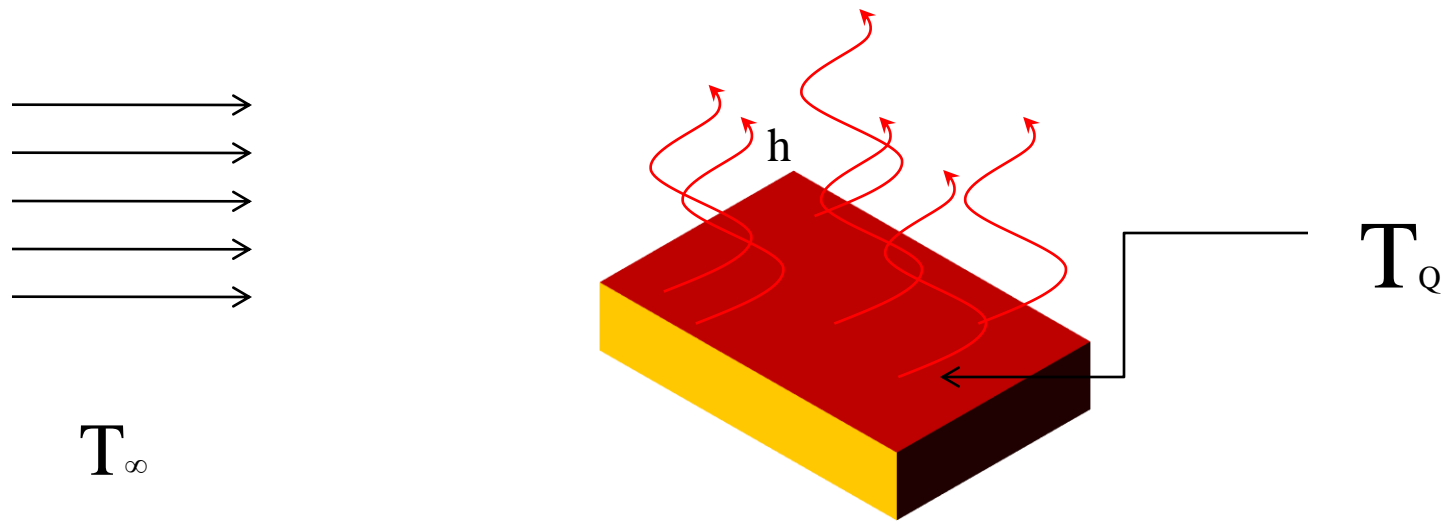
$$-\dot{E}_{out} + \dot{E}_g = 0 \rightarrow T_s = T_\infty + \frac{\dot{q} r_o}{3h}$$

BE:

$$\dot{E}_{in} - \dot{E}_{out} = 0 \rightarrow q_{cond}(r_o) = q_{con} \rightarrow T_s = T_\infty + \frac{\dot{q} r_o}{3h}$$

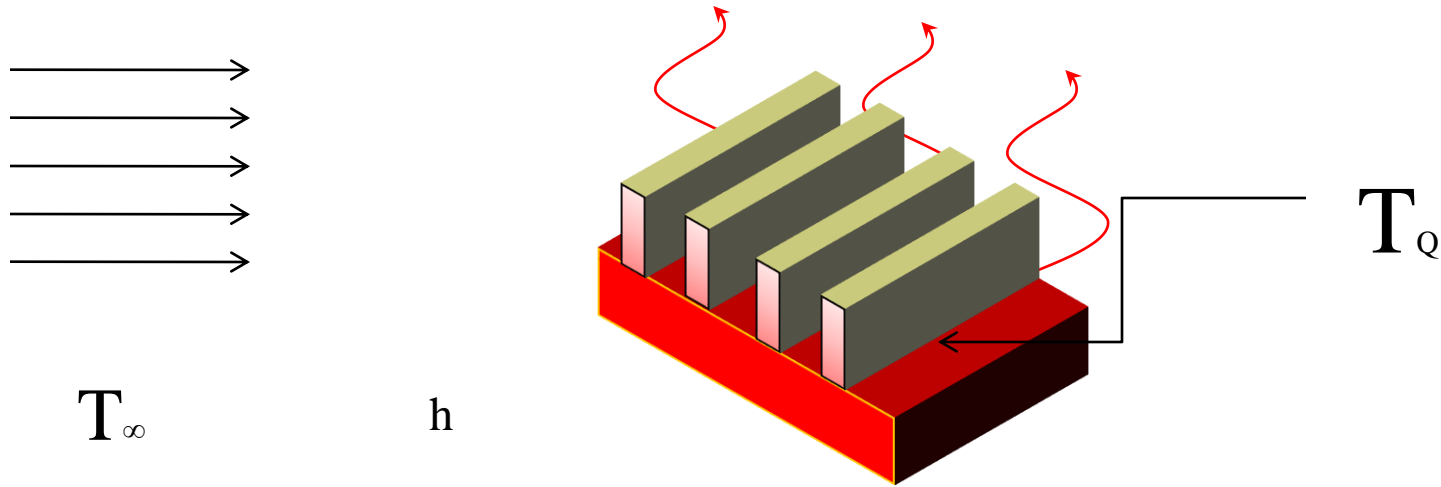
Aletas

# Aletas

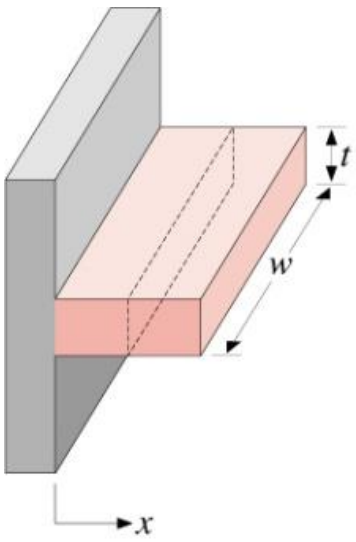


$$q = hA(T_0 - T_\infty)$$

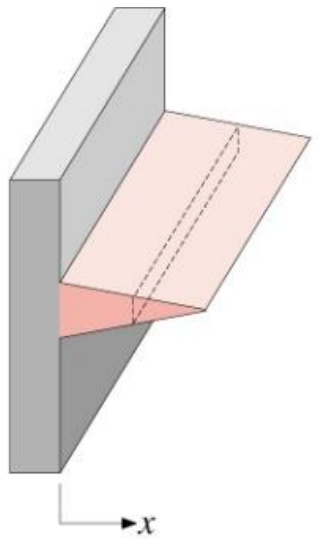
$$q = hA(T_Q - T_\infty)$$



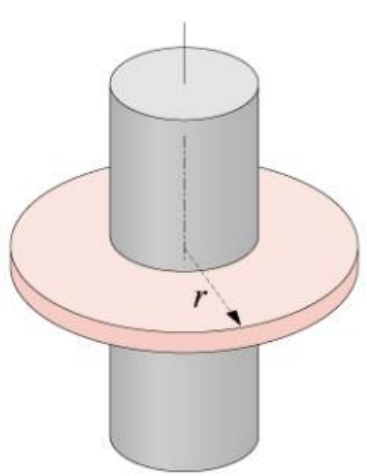
# Configurações típicas – uma aleta



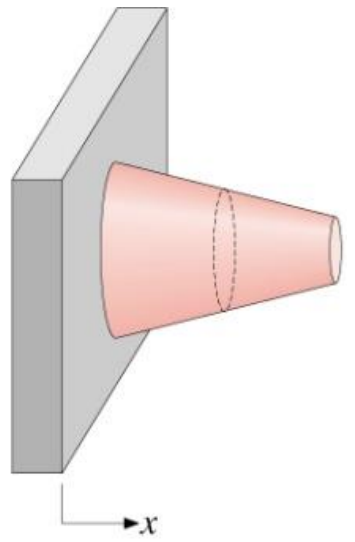
(a)



(b)

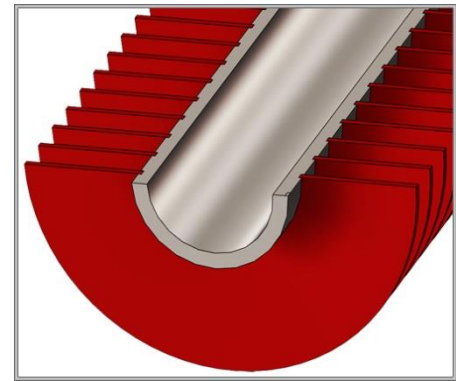
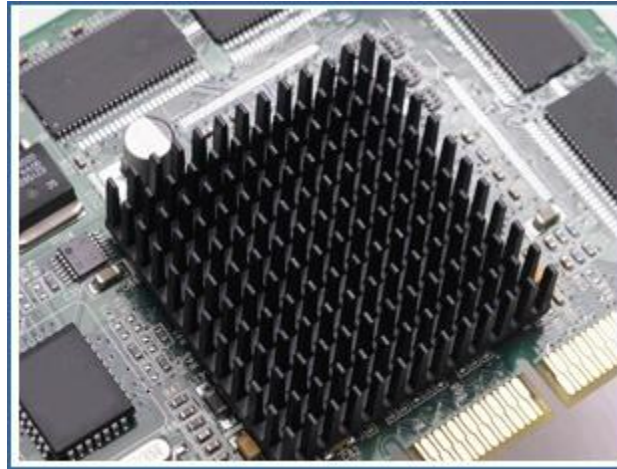
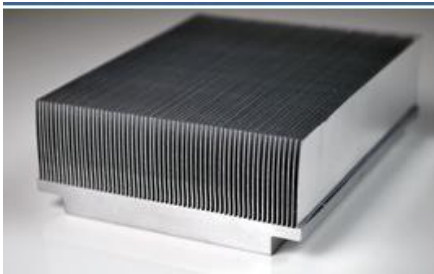
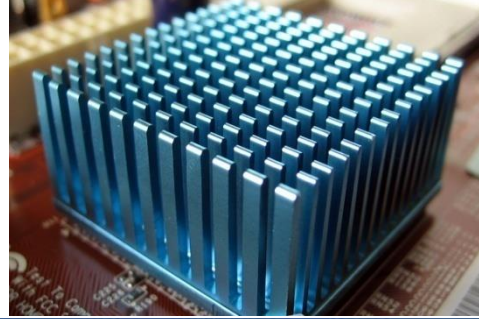
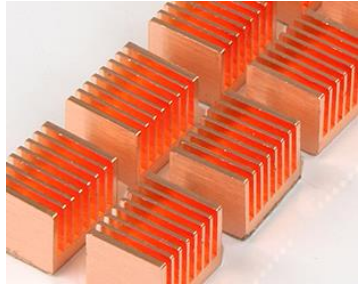


(c)



(d)

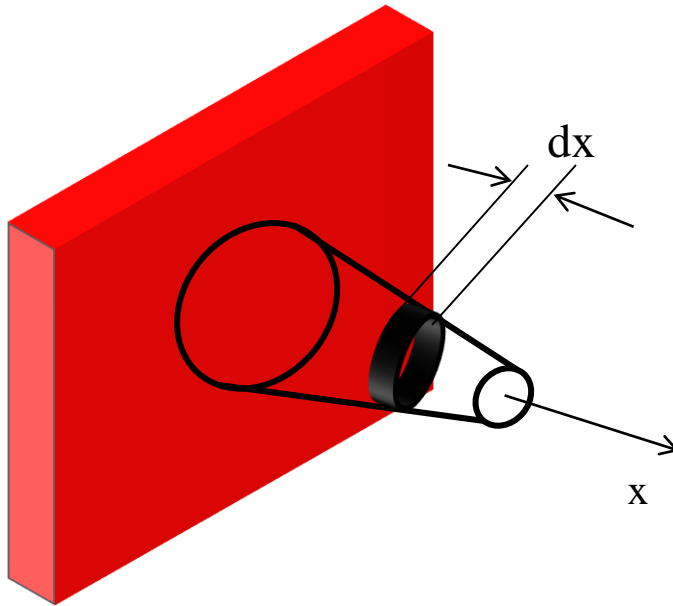
# Conjunto de aletas



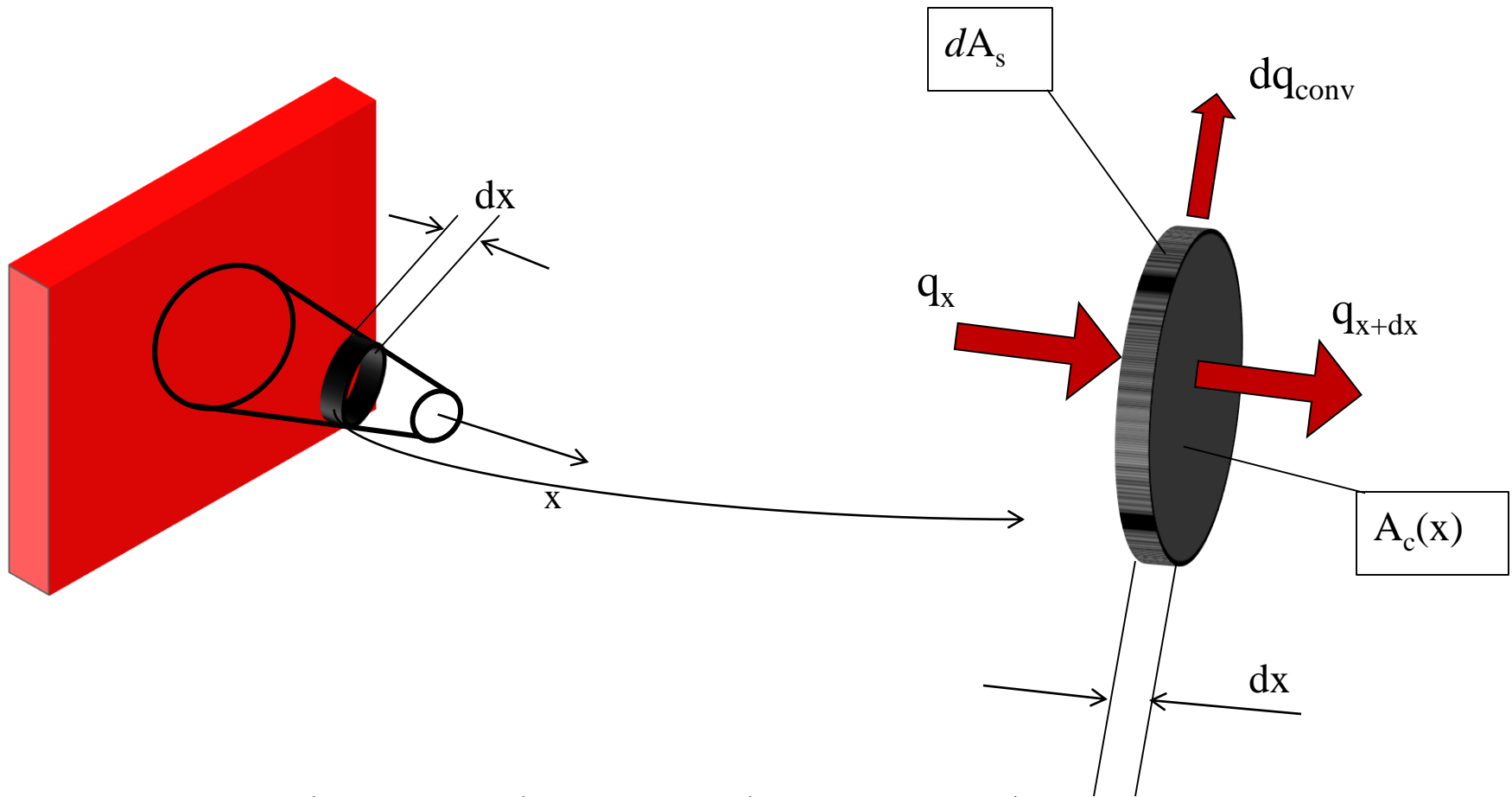
# Aletas: Formulação

## Hipóteses:

- Condução unidimensional pela aleta (espessura fina em relação ao comprimento)
- Regime permanente
- Propriedades constantes ( $k$ ,  $h$ )
- Sem geração (como primeira análise)
- Desprezar o efeito da radiação (como primeira análise)



# Formulação Aletas



$$\frac{d^2T}{dx^2} + \left( \frac{1}{A_c} \frac{dA_c}{dx} \right) \frac{dT}{dx} - \left( \frac{1}{A_c} \frac{h}{k} \frac{dA_s}{dx} \right) (T - T_\infty) = 0$$



# Formulação de Aletas: Seção Constante

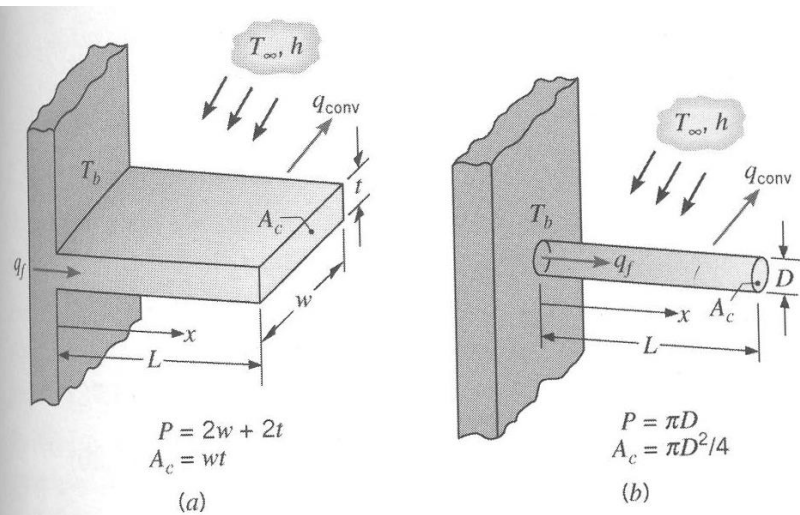


Fig. 3.16 Aletas planas de seção transversal uniforme. (a) Aleta retangular. (b) Aleta em forma de pino.

$$\frac{d^2T}{dx^2} + \left( \frac{1}{A_c} \frac{dA_c}{dx} \right) \frac{dT}{dx} - \left( \frac{1}{A_c} \frac{h}{k} \frac{dA_s}{dx} \right) (T - T_\infty) = 0$$

$$\frac{dA_s}{dx} = \frac{P dx}{dx} = P$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c} (T - T_\infty) = 0$$

$$m^2 \equiv \left( \frac{hP}{kA_c} \right)$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

# Solução de Aletas: Condições de contorno

Base ( $x = 0$ )

$$\theta(0) = T_b - T_\infty \equiv \theta_b$$

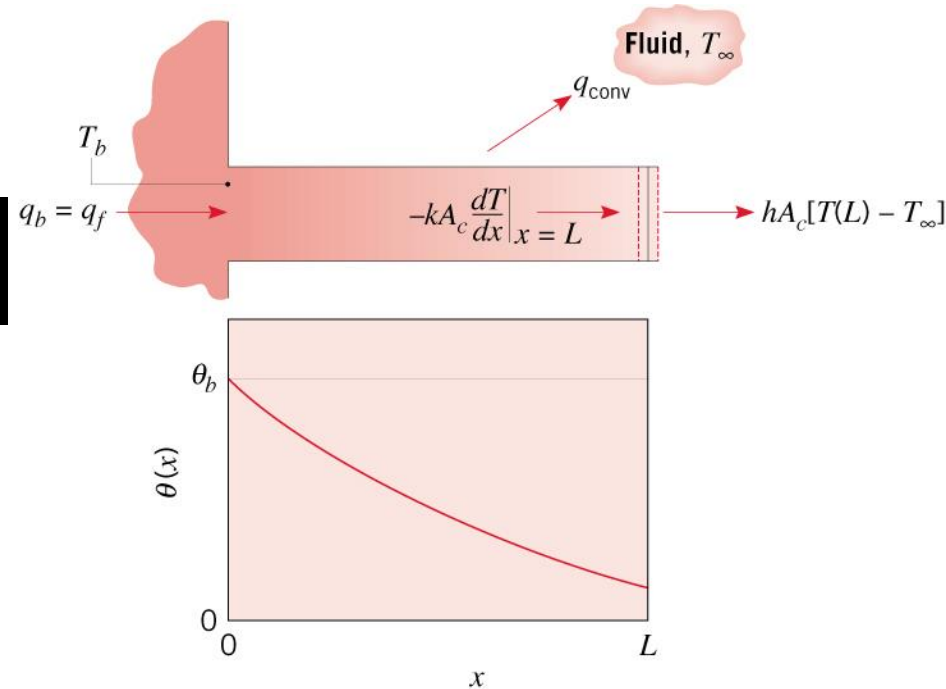
Ponta ( $x = L$ )

A. **Convection:**  $-k d\theta / dx |_{x=L} = h\theta(L)$

B. **Adiabatic:**  $d\theta / dx |_{x=L} = 0$

C. **Fixed temperature:**  $\theta(L) = \theta_L$

D. **Infinite fin ( $mL > 2.65$ ):**  $\theta(L) = 0$



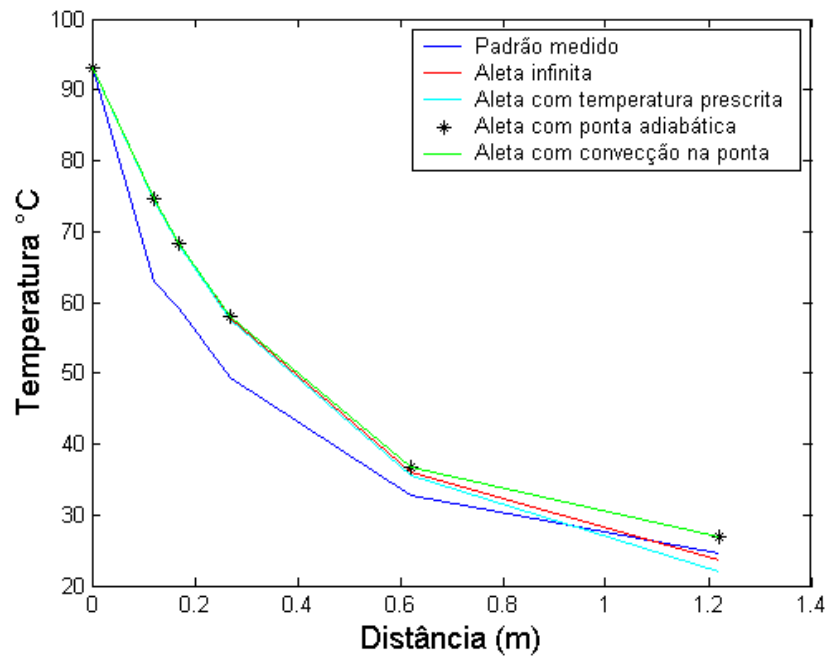
• Taxa transferida de calor pela aleta:

$$q_f = -kA_c \frac{d\theta}{dx} \Big|_{x=0} = \int_{A_f} h\theta(x) dA_s$$

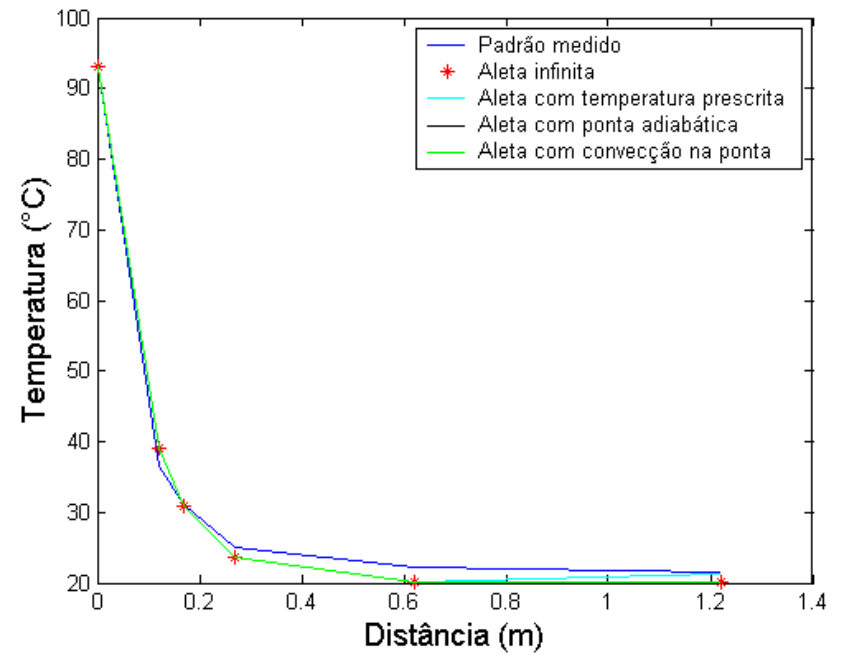
# Soluções de Aletas

TABELA 3.4 Distribuição de temperatura e perda de calor para aletas de seção transversal uniforme

CASO	CONDIÇÃO DA EXTREMIDADE ( $x = L$ )	DISTRIBUIÇÃO DE TEMPERATURA $\theta/\theta_b$	TAXA DE TRANSFERÊNCIA DE CALOR DA ALETA $q_a$
A	Transferência de calor por convecção: $h\theta(L) = -kd\theta/dx _{x=L}$	$\frac{\cosh m(L-x) + (h/mk) \sinh m(L-x)}{\cosh mL + (h/mk) \sinh mL}$ (3.70)	$M \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$ (3.72)
B	$d\theta/dx _{x=L} = 0$ Adiabática	$\frac{\cosh m(L-x)}{\cosh mL}$ (3.75)	$M \tanh mL$ (3.76)
C	Temperatura prescrita: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b) \sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.77)	$M \frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.78)
D	Aleta infinita ( $L \rightarrow \infty$ ) $\theta(L) = 0$	$e^{-mx}$ (3.79)	$M$ (3.80)
$\theta \equiv T - T_\infty$ $\theta_b = \theta(0) = T_b - T_\infty$		$m^2 \equiv hP/kA_c$ $M \equiv \sqrt{hPkA_c} \theta_b$	



Cobre



Inox

# Parâmetros de Performance de Aleta

- Efetividade da Aleta:

$$\varepsilon_f = \frac{q_f}{hA_{c,b} \theta_b} = \frac{\text{taxa de transferência de calor com Aleta}}{\text{taxa de transferência de calor sem Aleta}}$$

Para uma aleta infinita

$$\varepsilon_f = \frac{\sqrt{hPkA_c} \theta_b}{hA_c \theta_b} = \sqrt{\frac{kP}{hA_c}}$$

$\varepsilon_f \uparrow$  with  $\downarrow h, \uparrow k$  and  $\downarrow A_c / P$

$$\frac{q}{q_{\max}} = \frac{M \cdot \text{tgh}(mL)}{\underbrace{M \cdot \text{tgh}(m \cdot \infty)}_{=M \cdot 1}} = \text{tgh}(mL) = 0,98 \rightarrow mL = 2,3 \rightarrow L = \frac{2,3}{m}$$

- Eficiência da Aleta:

$$\eta_f = \frac{q_f}{q_{m\acute{a}x}} = \frac{q_f}{hA_{s,a}\theta_b}$$

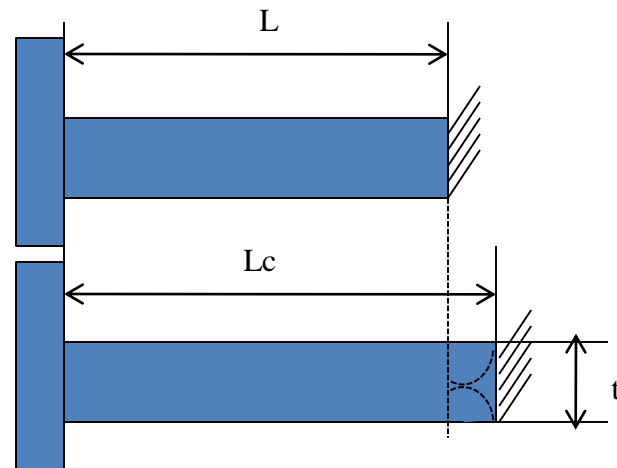
Para uma aleta plana, com seção reta uniforme e com ponta adiabática:

$$\eta_f = \frac{q_f}{hA_{s,a}\theta_b} = \frac{M \operatorname{tgh} mL}{hA_{s,a}\theta_b} = \frac{\operatorname{tgh} mL}{mL} = \begin{cases} = 0, & L \rightarrow \infty \\ = 1, & L \rightarrow 0 \end{cases}$$

Se a ponta não for adiabática

$$L_c = L + t/2$$

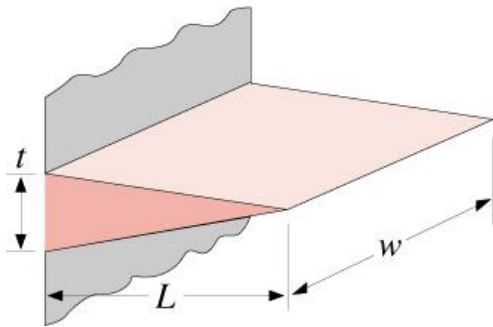
$$\eta_f = \frac{\operatorname{tgh} mL_c}{mL_c}$$



Para uma aleta plana, com seção reta uniforme e com ponta adiabática:

$$mL_c = \left( \frac{hP}{kA_c} \right)^{1/2} L_c = \left( \frac{h2w}{kwt} \right)^{1/2} L_c = \left( \frac{2h}{kL_c t} \right)^{1/2} L_c^{3/2} = \left( \frac{2h}{kA_p} \right)^{1/2} L_c^{3/2}$$

Para uma aleta triangular:

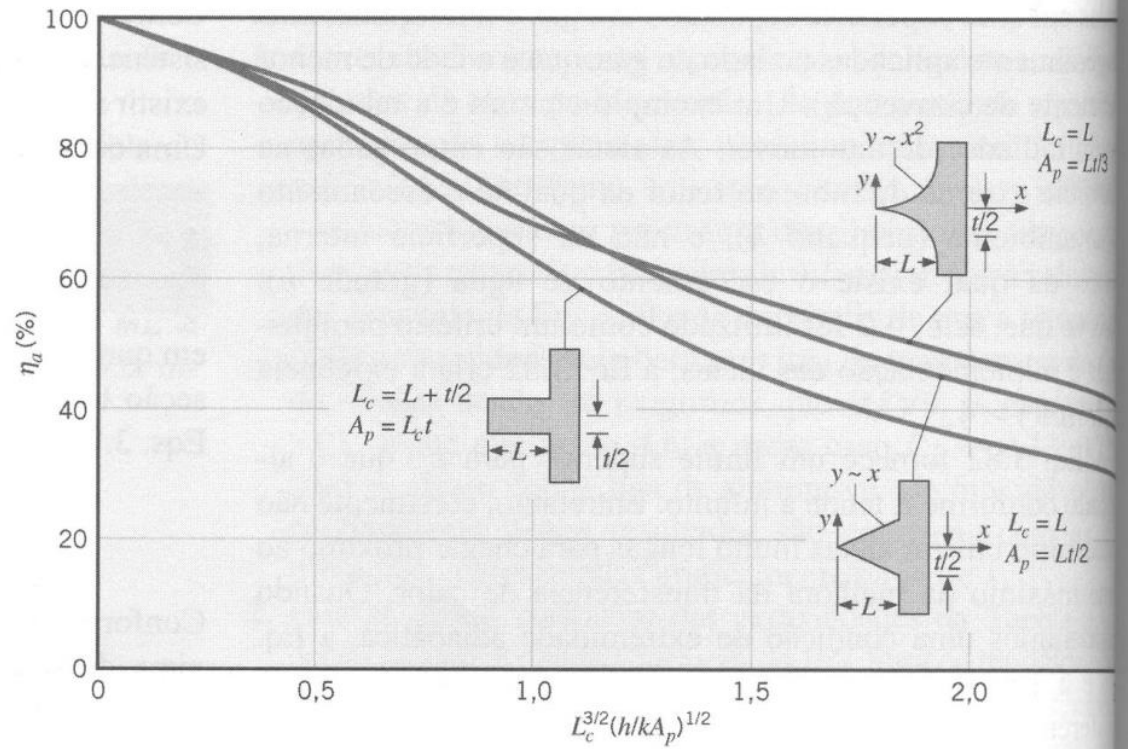


$$A_f = 2w \left[ L^2 + (t/2)^2 \right]^{1/2}$$

$$A_p = (t/2)L$$

$$\eta_f = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)}$$

Fig. 3.18 Eficiência de aletas (perfis retangular, triangular e parabólico).





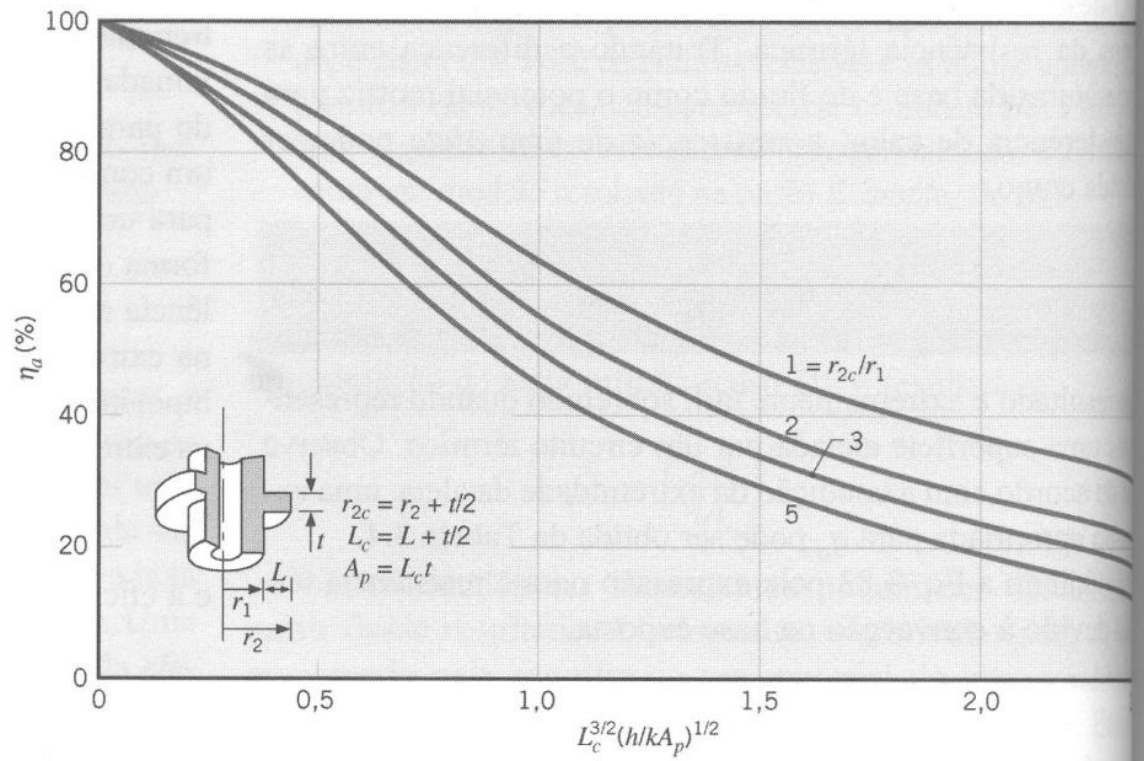


Fig. 3.19 Eficiência de aletas anulares com perfil retangular.

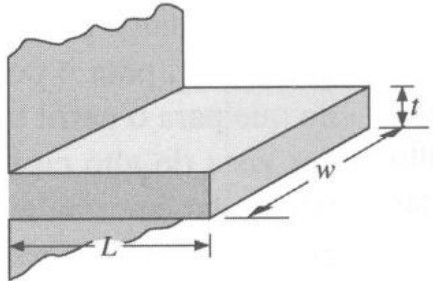
**Aletas Planas**

*Retangular<sup>a</sup>*

$$A_a = 2wL_c$$

$$L_c = L + (t/2)$$

$$A_p = tL$$

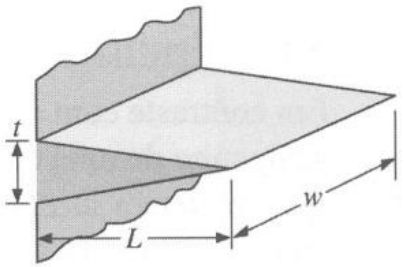


$$\eta_a = \frac{\tanh mL_c}{mL_c} \tag{3.89}$$

*Triangular<sup>a</sup>*

$$A_a = 2w[L^2 + (t/2)^2]^{1/2}$$

$$A_p = (t/2)L$$



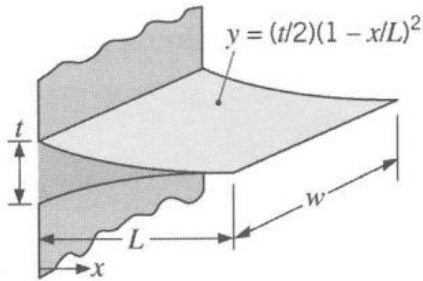
$$\eta_a = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)} \tag{3.93}$$

*Parabólica<sup>a</sup>*

$$A_a = w[C_1L + (L^2/t)\ln(t/L + C_1)]$$

$$C_1 = [1 + (t/L)^2]^{1/2}$$

$$A_p = (t/3)L$$



$$\eta_a = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1} \tag{3.94}$$

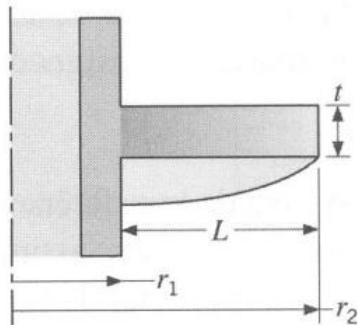
## Aleta Circular

*Retangular<sup>a</sup>*

$$A_a = 2\pi (r_{2c}^2 - r_1^2)$$

$$r_{2c} = r_2 + (t/2)$$

$$V = \pi (r_2^2 - r_1^2)t$$



$$\eta_a = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})} \quad (3.91)$$

$$C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$$

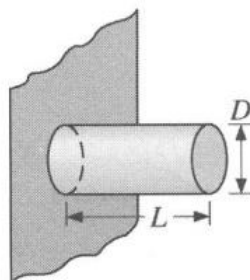
## Aletas em Forma de Pino

*Retangular<sup>b</sup>*

$$A_a = \pi DL_c$$

$$L_c = L + (D/4)$$

$$V = (\pi D^2/4)L$$

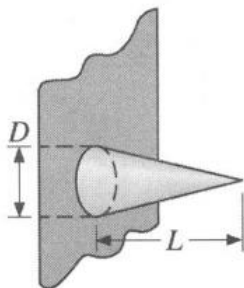


$$\eta_a = \frac{\tanh mL_c}{mL_c} \quad (3.95)$$

*Triangular<sup>b</sup>*

$$A_a = \frac{\pi D}{2} [L^2 + (D/2)^2]^{1/2}$$

$$V = (\pi/12)D^2L$$



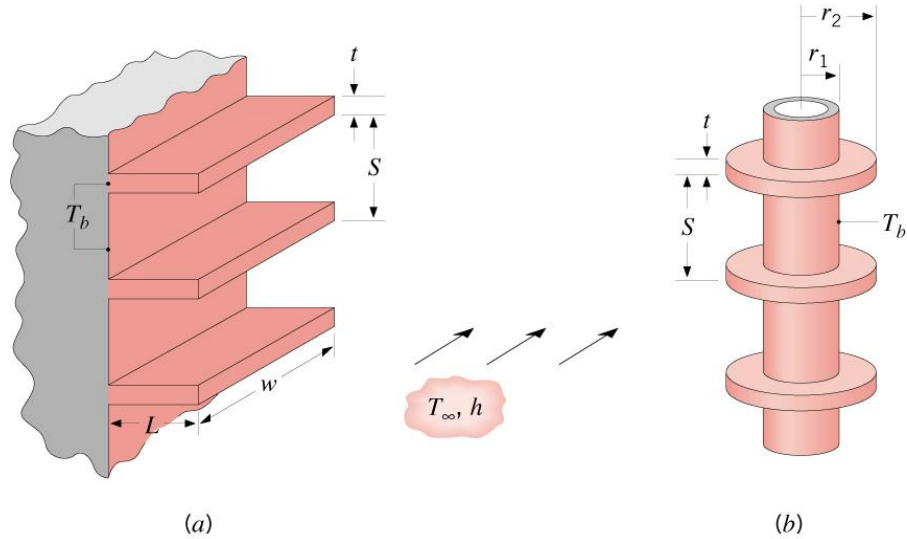
$$\eta_a = \frac{2 I_2(2mL)}{mL I_1(2mL)} \quad (3.96)$$

- Resistência da Aleta:

$$R_{t,f} \equiv \frac{\theta_b}{q_f} = \frac{1}{hA_f\eta_f}$$

(3.92)

# Conjunto de Aletas



– Área total da Superfície:

$$A_t = NA_f + A_b$$

Núm. de aletas      Área da base exposta

$$A_b = A_t - NA_f$$

(3.99)

– taxa transferida de calor total:

$$q_t = N\eta_f hA_f \theta_b + hA_b \theta_b \equiv \eta_o hA_t \theta_b = \frac{\theta_b}{R_{t,o}}$$

(3.100)

– Eficiência Global da Superfície e resistência da aleta:

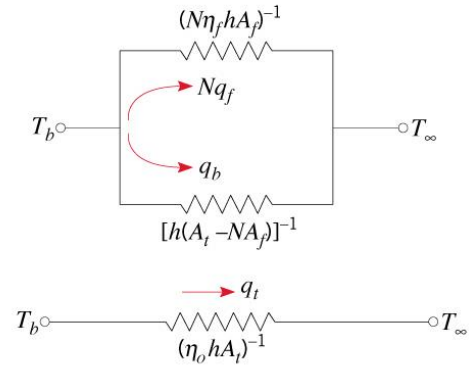
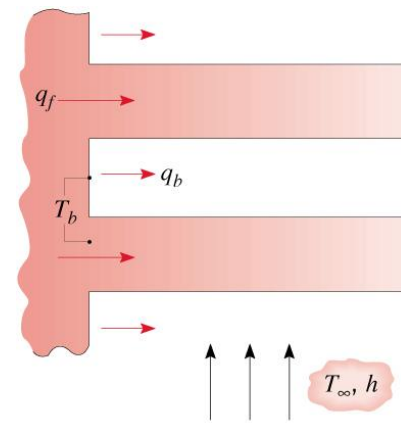
$$\eta_o = 1 - \frac{NA_f}{A_t} (1 - \eta_f)$$

(3.102)

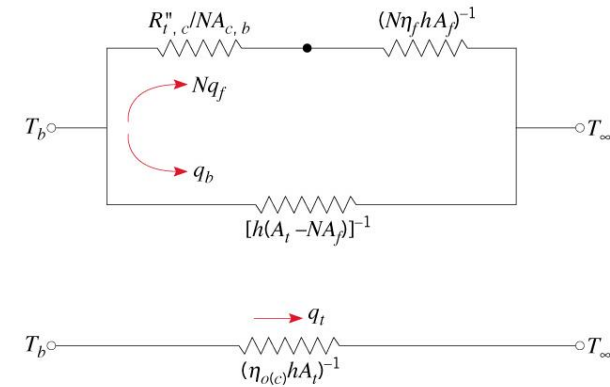
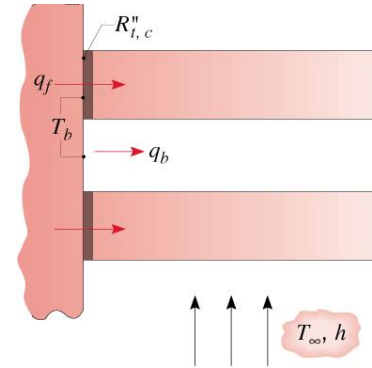
$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o hA_t}$$

(3.103)

# Circuito Térmico Equivalente:



(a)



(b)

• Efeito da resistência térmica de contato:

$$\eta_{o(c)} = 1 - \frac{NA_f}{A_t} \left( 1 - \frac{\eta_f}{C_1} \right)$$

$$C_1 = 1 + \eta_f h A_f \left( R''_{t,c} / A_{c,b} \right)$$

$$R_{t,o(c)} = \frac{1}{\eta_{o(c)} h A_t}$$

(3.105a)

(3.105b)

(3.104)

### EXEMPLO 3.9

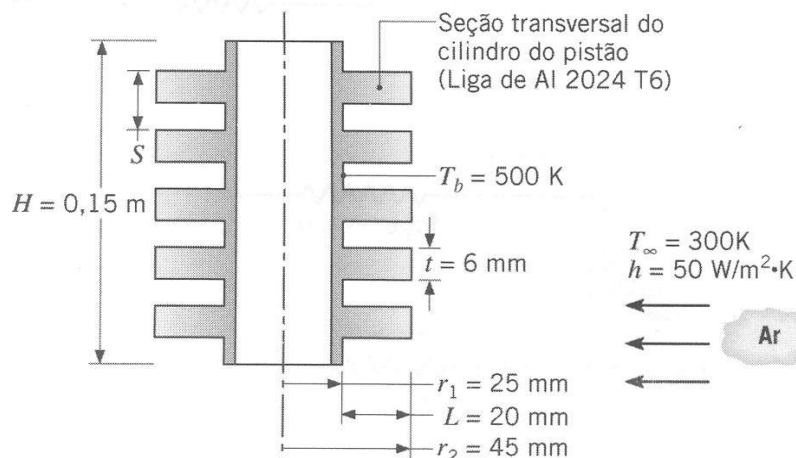
O cilindro do pistão do motor de uma motocicleta é construído em liga de alumínio 2024-T6, com uma altura  $H = 0,15 \text{ m}$  e diâmetro externo  $D = 50 \text{ mm}$ . Sob condições típicas de operação, a superfície externa do cilindro está a uma temperatura de  $500 \text{ K}$  e encontra-se exposta ao ar ambiente a  $300 \text{ K}$ , com um coeficiente de convecção  $50 \text{ W/m}^2 \cdot \text{K}$ . Aletas anulares são fundidas integralmente com o cilindro para aumentar a transferência de calor para a vizinhança. Considere cinco dessas aletas, com espessura  $t = 6 \text{ mm}$ , comprimento  $L = 20 \text{ mm}$  e espaçadas igualmente. Qual o aumento na transferência de calor devido ao uso das aletas?

#### SOLUÇÃO

**Dados:** Condições de operação do cilindro aletado de uma motocicleta.

**Achar:** O aumento na transferência de calor associado ao uso das aletas.

**Esquema:**



#### Considerações:

1. Condições de regime estacionário.
2. Condução unidimensional radial nas aletas.
3. Propriedades constantes.
4. Troca por radiação com a vizinhança desprezível.
5. Coeficiente de convecção uniforme na superfície externa (com ou sem aletas).

**Propriedades:** Tabela A.1, alumínio 2024-T6 ( $T = 400 \text{ K}$ )  
 $k = 186 \text{ W/m} \cdot \text{K}$ .

**Análise:** Com as aletas, a taxa de transferência de calor é dada pela Eq. 3.101

$$q_t = hA_t \left[ 1 - \frac{NA_a}{A_t} (1 - \eta_a) \right] \theta_b$$

onde  $A_a = 2\pi (r_{2c}^2 - r_1^2) = 2\pi [(0,048\text{m})^2 - (0,025\text{m})^2] = 0,0105\text{m}^2$  e, da Eq. 3.99,  $A_a = NA_a + 2\pi r_1 (H - Nt) = 0,0105\text{m}^2 + 2\pi (0,025\text{m}) [0,15\text{ m} - 0,03\text{ m}] = 0,0716\text{ m}^2$ . Com  $N = 5$ ,  $L_c = 0,023\text{ m}$ ,  $A_p = 1,380 \times 10^{-4}\text{m}^2$ , obtém-se  $L_c^{3/2} (h/kA_p)^{1/2} = 0,15$ . Logo, da Fig. 3.19, a eficiência das aletas  $\eta_a \approx 0,95$ . Com as aletas, a taxa da transferência total de calor é então