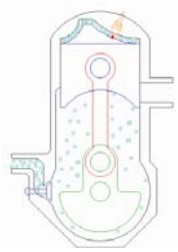
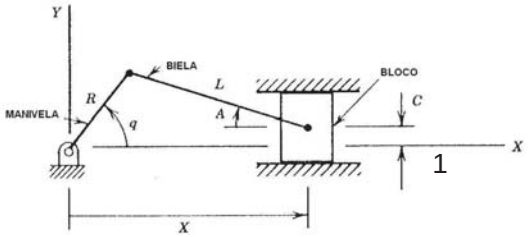
Mecanismos

Mecanismos com 1 GL

Mecanismo Biela-manivela



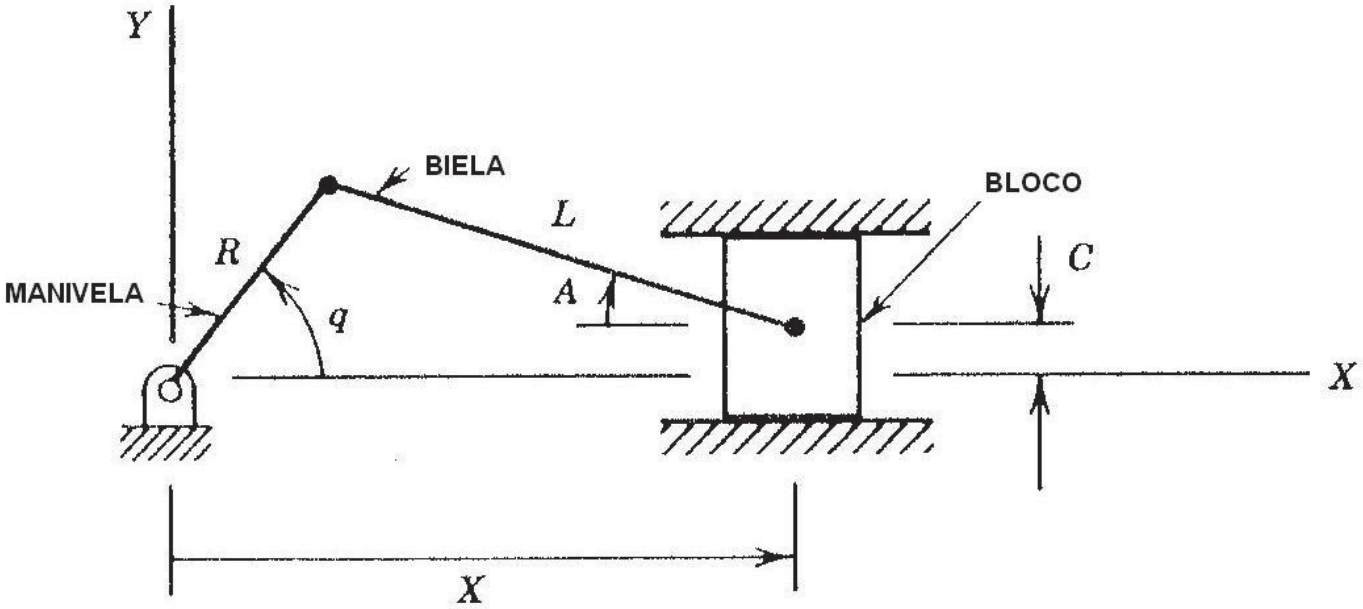
Prof. Jorge Luiz Erthal
jorgeerthal@gmail.com



Conteúdo

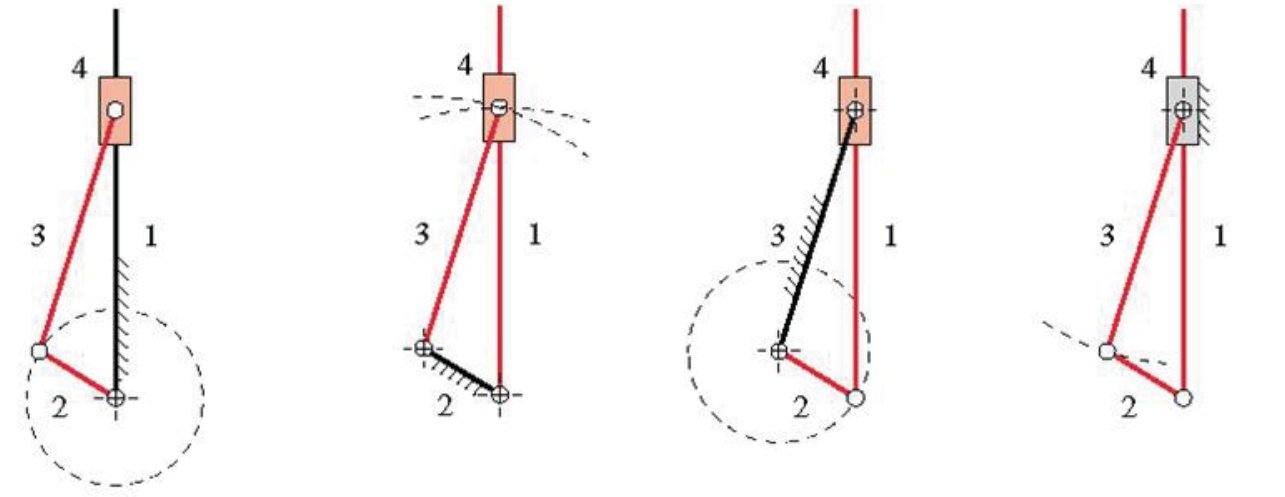
- Análise Geral
- Análise de pontos de interesse
- Variações no acionamento
  - Pontos singulares

Mecanismo Biela-Manivela



Ver arquivo MATLAB – bielamanivela_algebrico.m

Mecanismo Biela-Manivela



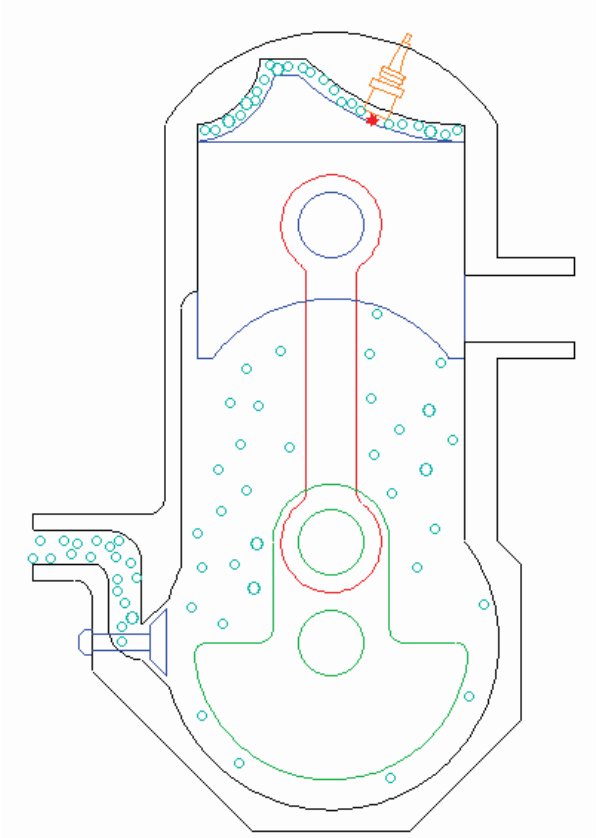
(a) Inversion # 1 slider block translates

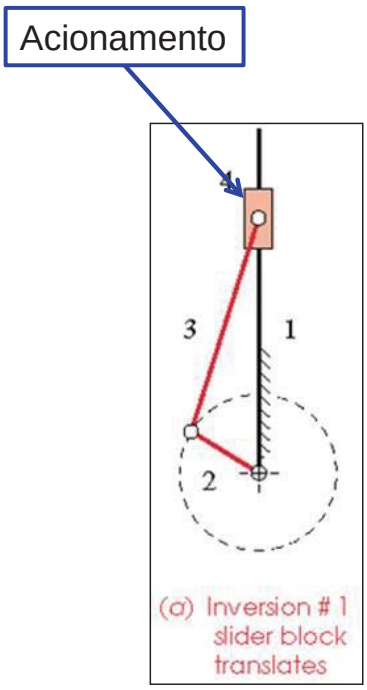
(b) Inversion # 2 slider block has complex motion

(c) Inversion # 3 slider block rotates

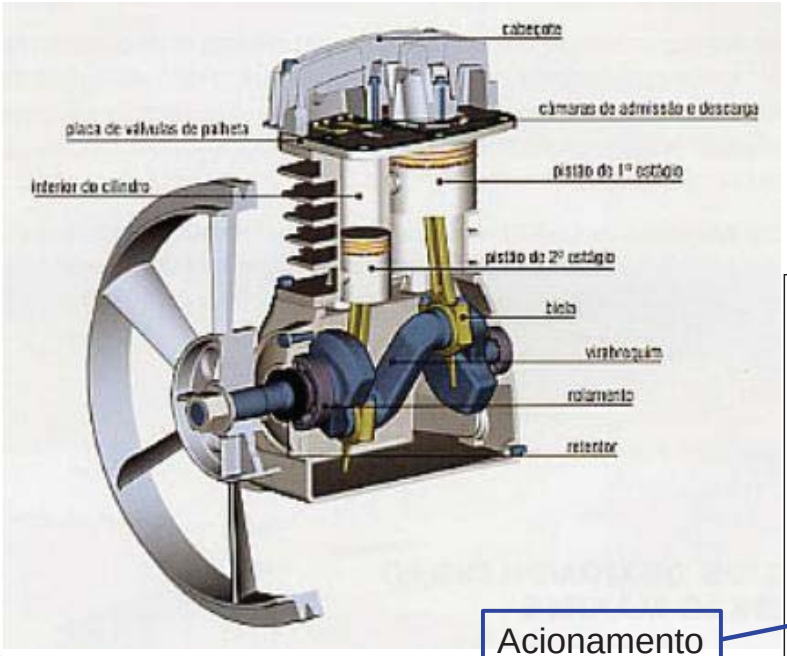
(d) Inversion # 4 slider block is stationary

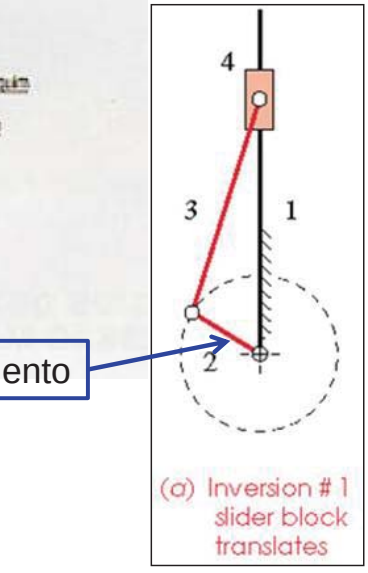
Caso 1 - Motor de combustão interna de 2 tempos



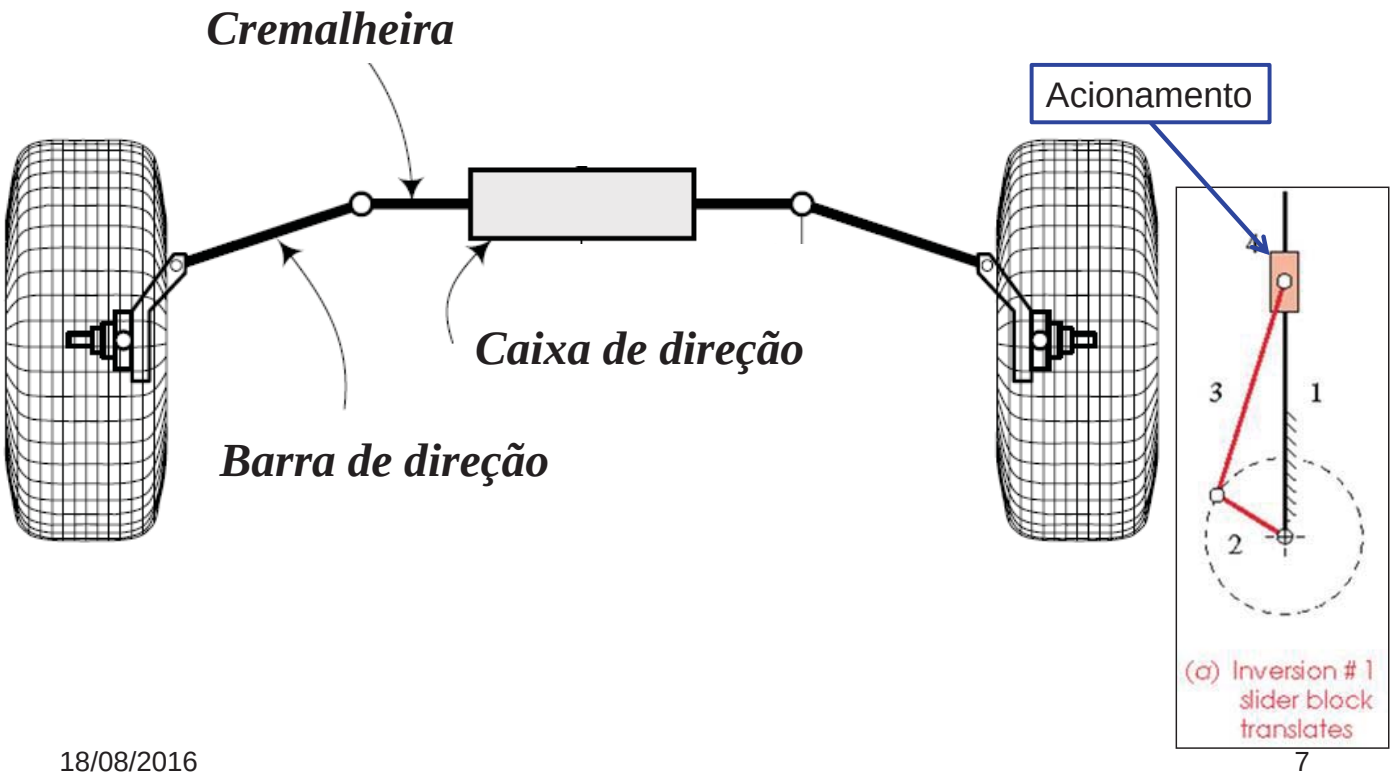


Caso 1 - Compressor alternativo

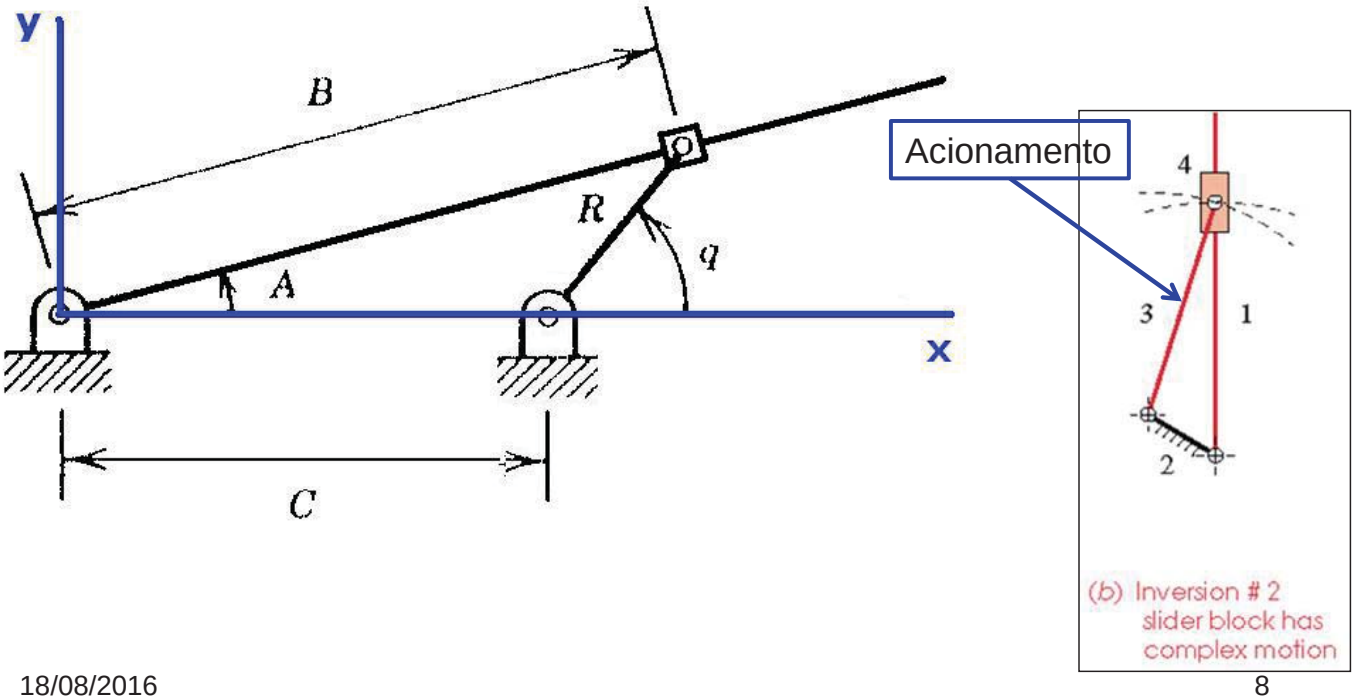




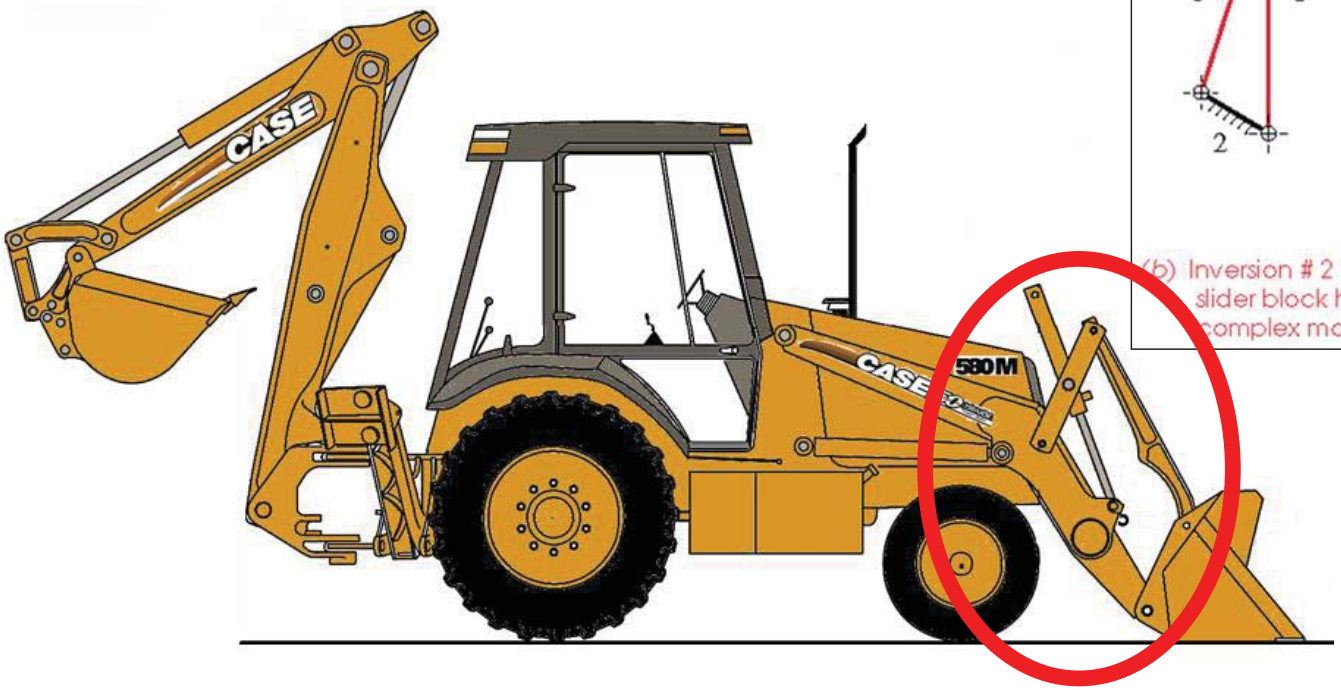
Caso 1 - Sistema de direção com pinhão e cremalheira

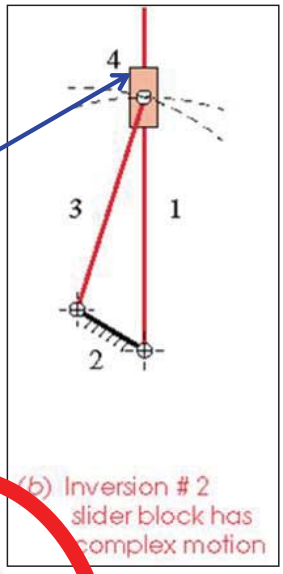


Caso 2 - Exemplo 2-1

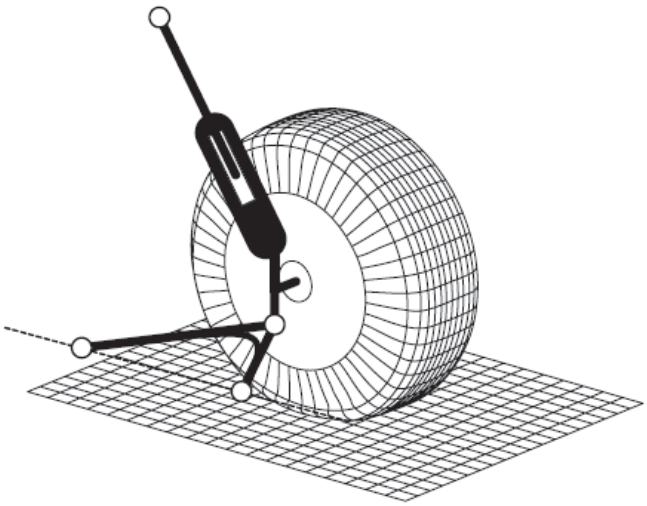


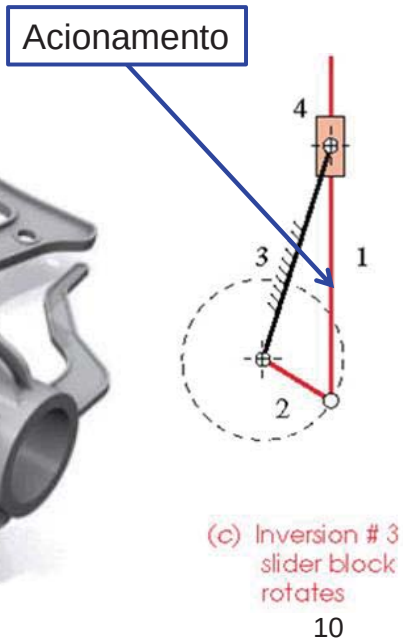
Caso 2 - Pá carregadeira



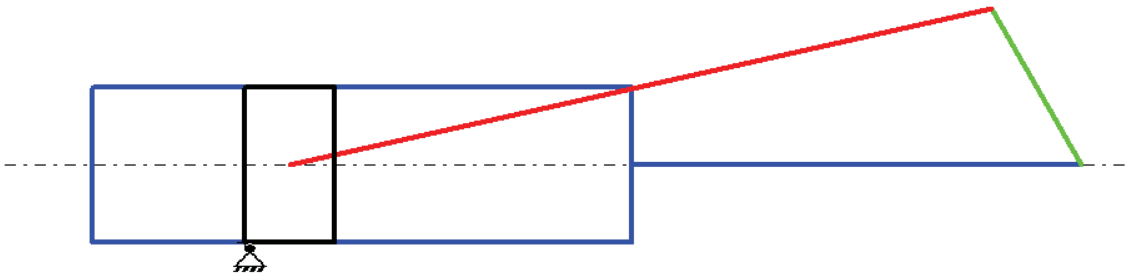


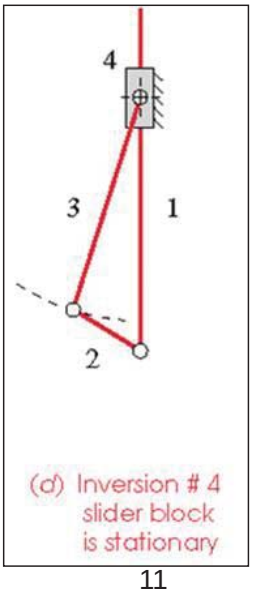
Caso 3 - Suspensão MacPherson



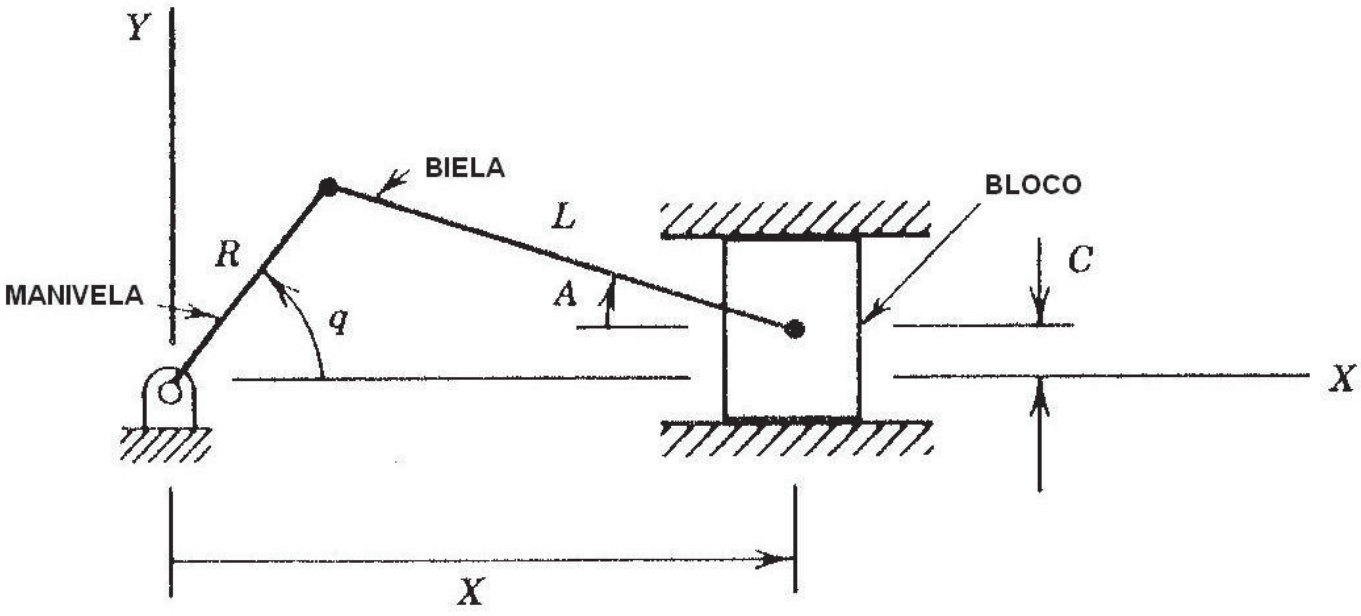


Caso 4





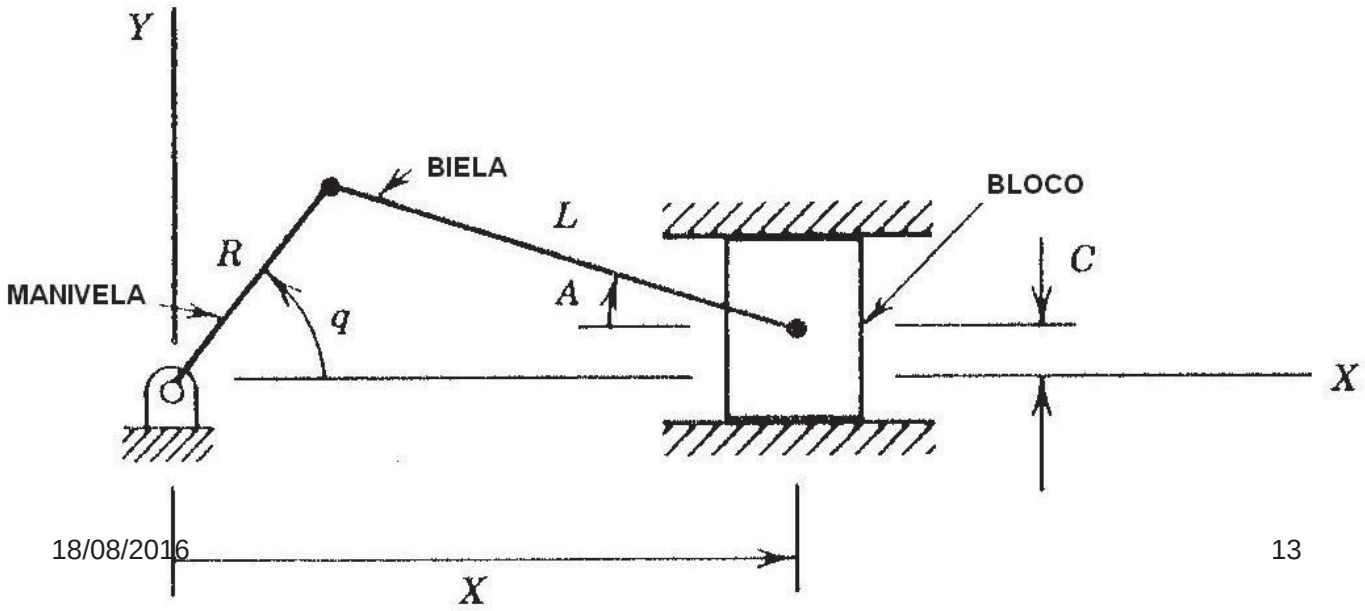
1-Cálculo do número de graus de liberdade



1-Cálculo do número de graus de liberdade

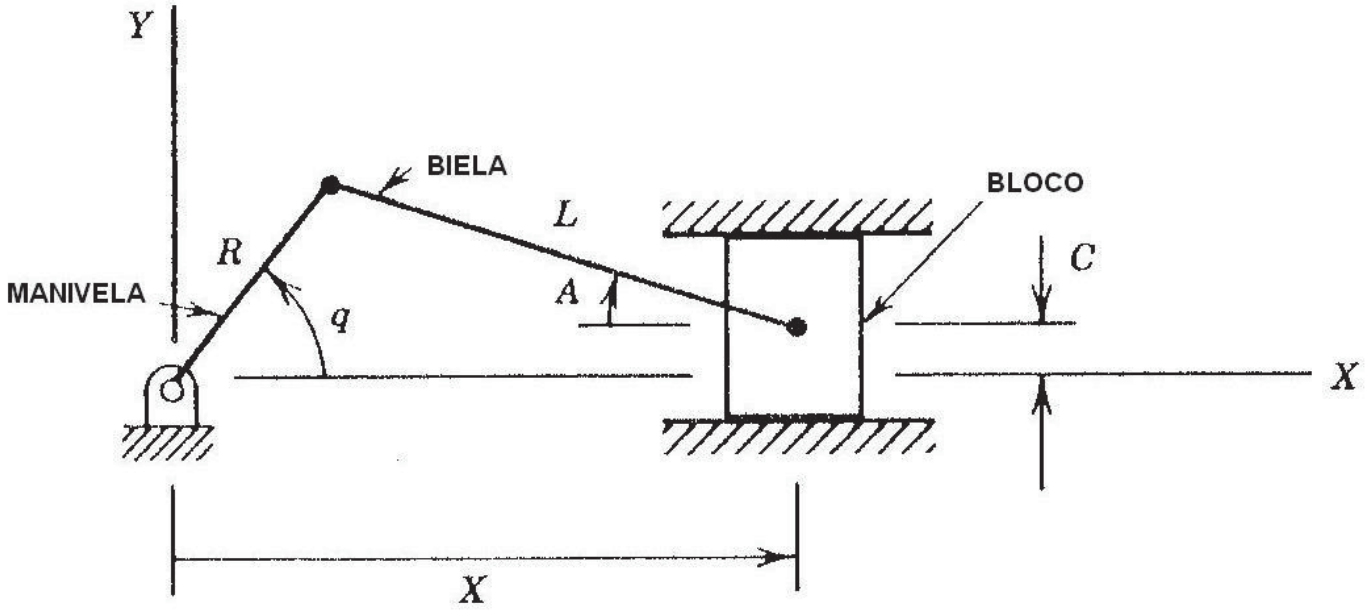
$N := 4 \qquad P_1 := 4 \qquad P_2 := 0$

$F := 3 \cdot (N - 1) - 2 \cdot P_1 - P_2 \qquad F = 1$

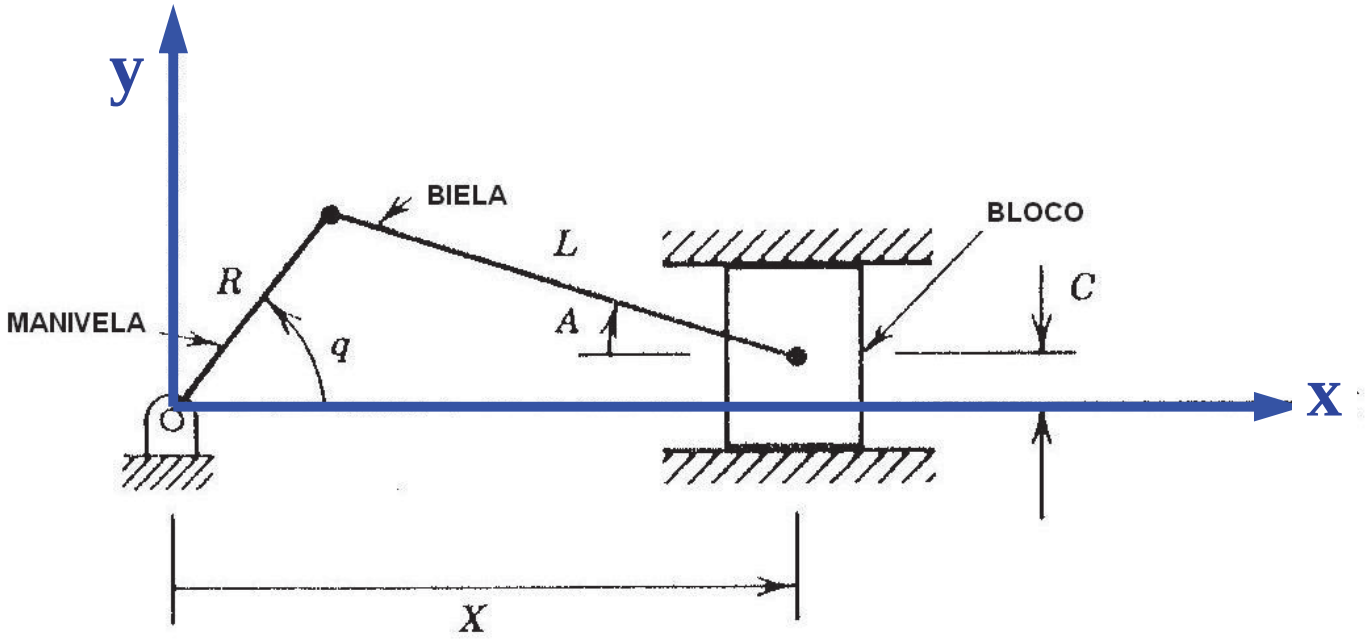


2-Decomposição dos pares superiores

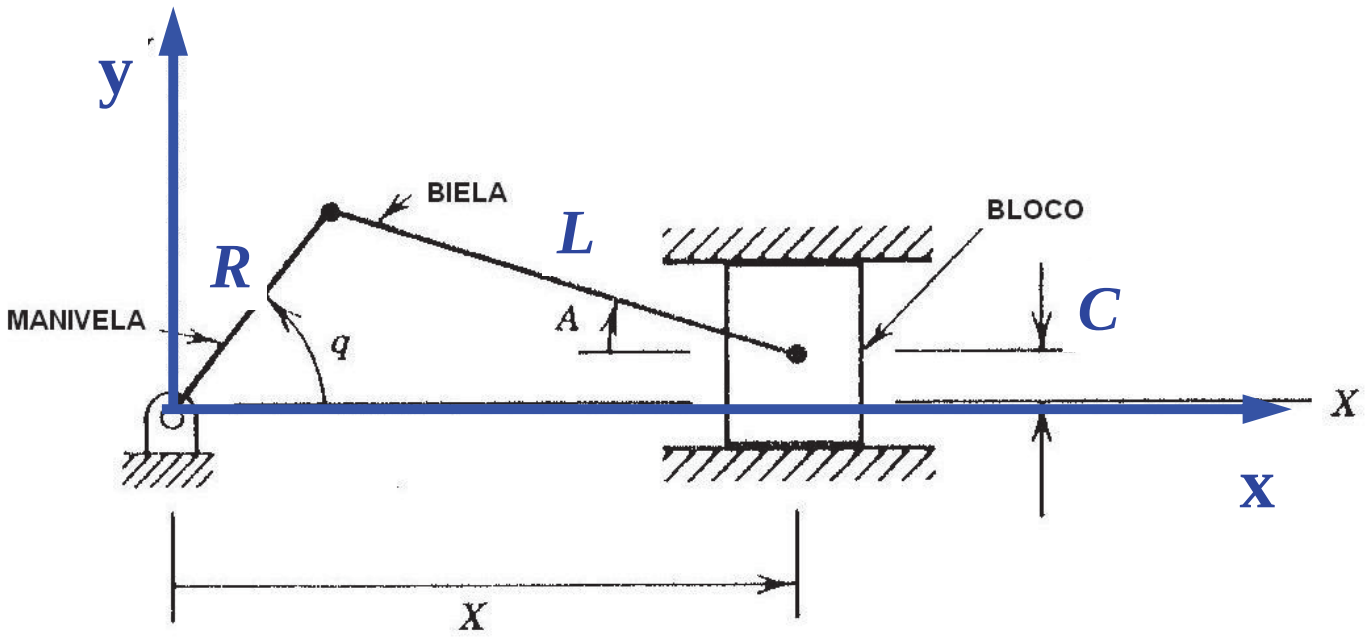
Não há pares superiores



3-Definição do sistema GLOBAL de coordenadas



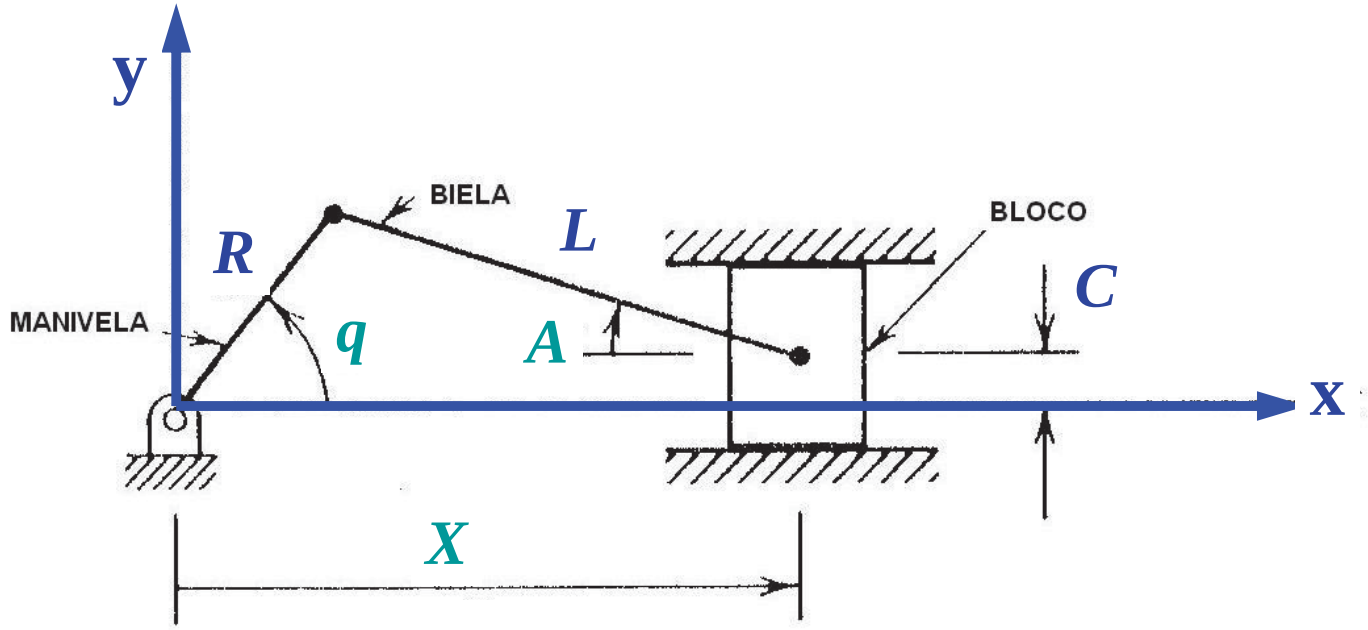
4-Identificação das medidas constantes



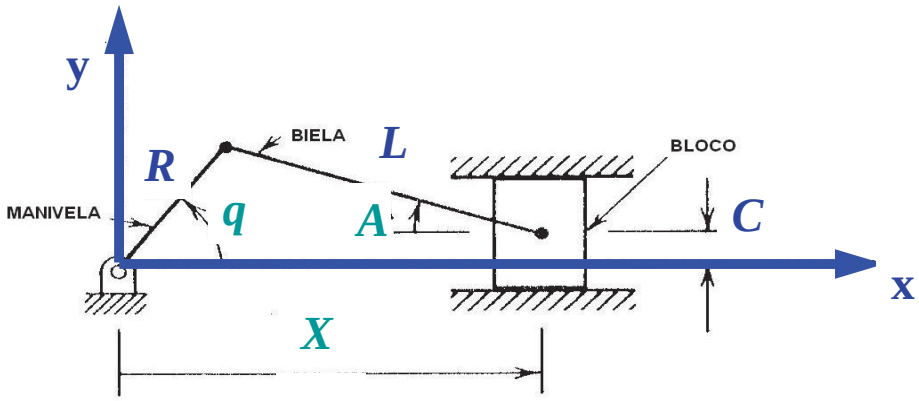
5-Definição das variáveis primárias e secundárias

variável primária – q

Variáveis secundárias – A, X



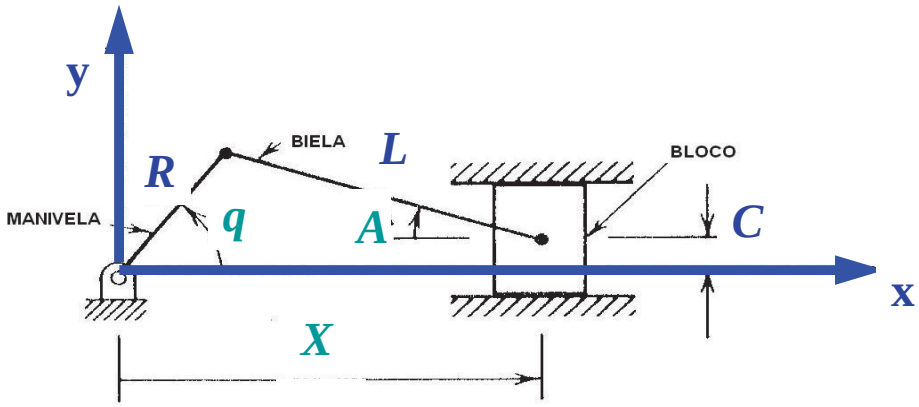
6-Montagem das equações cinemáticas de posição



$$R.\cos(q) + L.\cos(A) - X = 0$$

$$R.sen(q) - L.sen(A) - C = 0$$

7-Solução do sistema de equações



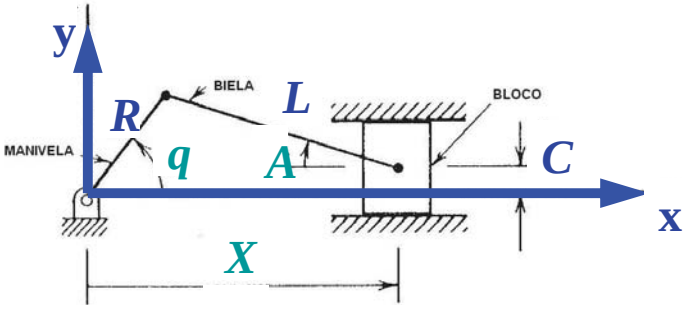
$$A(q) = \text{asen}\left(\frac{R.\text{sen}(q) - C}{L}\right)$$

$$X(q) = R.\cos(q) + L.\cos(A(q))$$

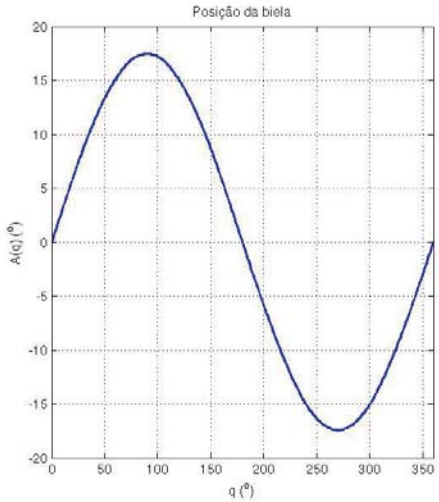
7-Solução para uma faixa de variação de q

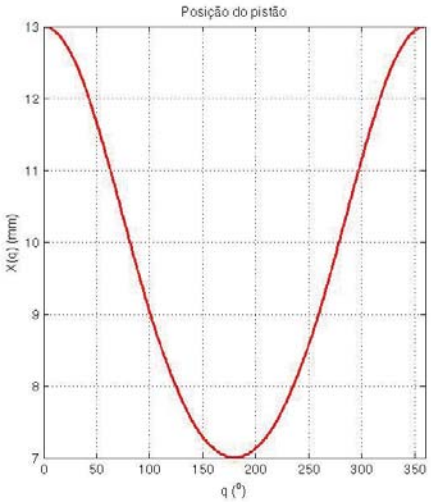
$$A(q) = \text{asen}\left(\frac{R.\text{sen}(q) - C}{L}\right)$$

$$X(q) = R.\cos(q) + L.\cos(A(q))$$



R=3
L=10
C=0





Equações das velocidades

Derivadas das equações de posição em relação ao tempo

$$-R\cdot\sin(q)\cdot\dot{q} - L\cdot\sin(A)\cdot\dot{A} - \dot{X} = 0$$

$$R\cdot\cos(q)\cdot\dot{q} - L\cdot\cos(A)\cdot\dot{A} = 0$$

$$\begin{pmatrix} -L\cdot\sin(A) & -1 \\ -L\cdot\cos(A) & 0 \end{pmatrix}\cdot\begin{pmatrix} \dot{A} \\ \dot{X} \end{pmatrix} + \begin{pmatrix} -R\cdot\sin(q) \\ R\cdot\cos(q) \end{pmatrix}\cdot\dot{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Matriz jacobiana

Vetor das velocidades secundárias

Vetor dos coeficientes constantes

Velocidade primária

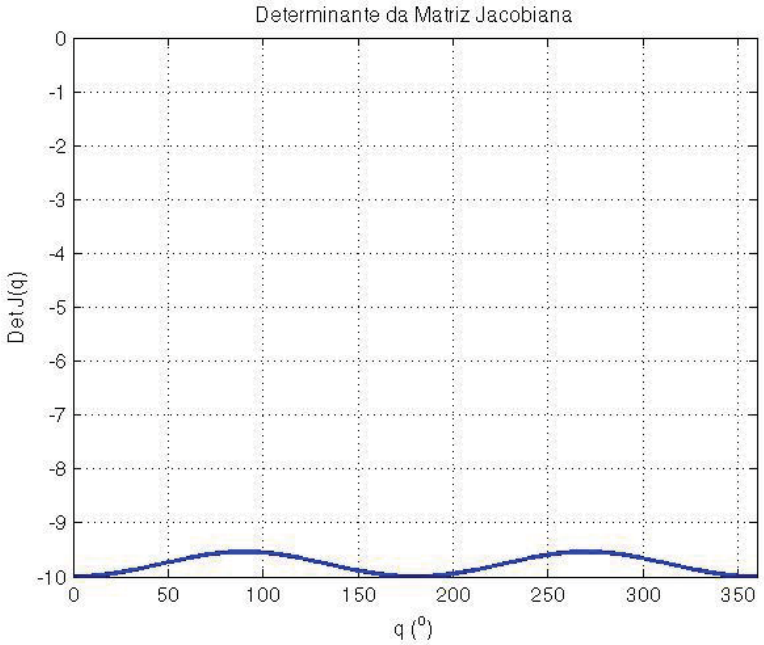
$$J\cdot\dot{S} + Q\cdot\dot{q} = 0$$

Determinante da matriz Jacobiana

$$J(q) = \begin{bmatrix} -L.sen(A(q)) & -1 \\ -L.\cos(A(q)) & 0 \end{bmatrix}$$

$$detJ(q) = -L.\cos(A(q))$$

R=3
L=10
C=0



Solução para as velocidades secundárias

$$\begin{pmatrix} \dot{A} \\ \dot{X} \end{pmatrix} = -\begin{pmatrix} 0 & -\frac{1}{L\cdot\cos(A(q))} \\ -1 & \tan(A(q)) \end{pmatrix}\cdot\begin{pmatrix} -R\cdot\sin(q) \\ R\cdot\cos(q) \end{pmatrix}\cdot\dot{q}$$

$$\begin{pmatrix} \dot{A} \\ \dot{X} \end{pmatrix} = \begin{pmatrix} \frac{R\cdot\cos(q)}{L\cdot\cos(A(q))} \\ -R\cdot\sin(q) - R\cdot\tan(A(q))\cdot\cos(q) \end{pmatrix}\cdot\dot{q}$$

$$\dot{S} = -J^{-1}\cdot Q\cdot\dot{q}$$

$$\dot{S} = K\cdot\dot{q}$$

$$Ka(q) := \frac{R\cdot\cos(q)}{L\cdot\cos(A(q))}$$

$$Kx(q) := -R\cdot\sin(q) - R\cdot\tan(A(q))\cdot\cos(q)$$

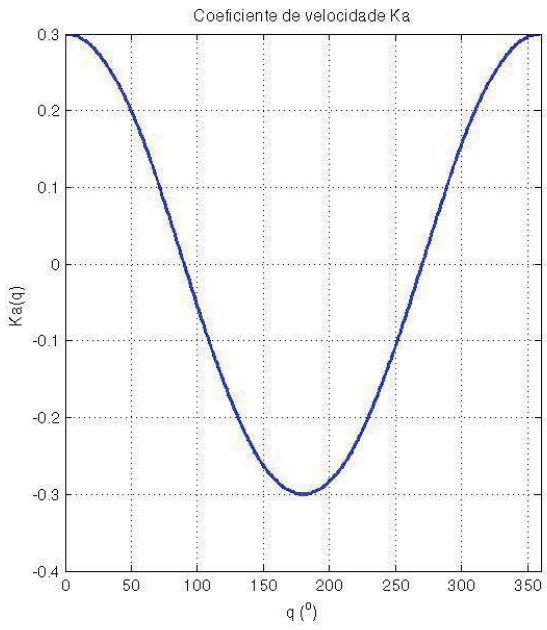
$$\dot{A}(q) := Ka(q)\cdot\dot{q}$$

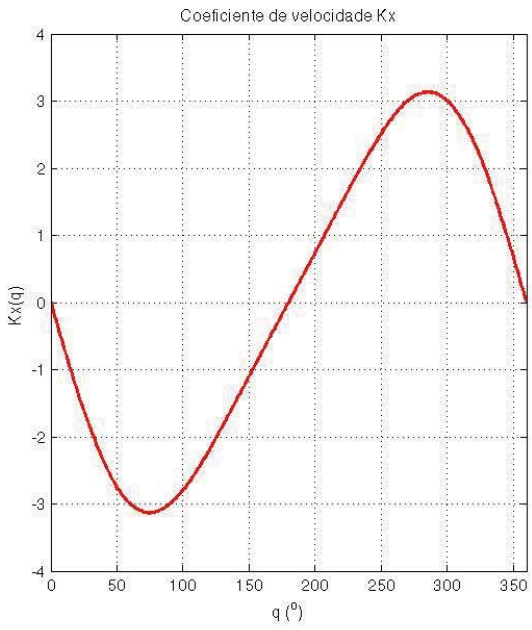
$$\dot{X}(q) := Kx(q)\cdot\dot{q}$$

Coeficientes de velocidade

$$K_a = \frac{R.\cos(q)}{L.\cos(A(q))}$$

$$K_x = -R.\sin(q) - R.\tan\big(A(q)\big).\cos(q)$$





R=3 L=10 C=0

Acelerações secundárias

$$\ddot{A}(q) = Ka(q)\cdot\ddot{q} + La(q)\cdot\dot{q}^2$$

$$\ddot{X}(q) = Kx(q)\cdot\ddot{q} + Lx(q)\cdot\dot{q}^2$$

$$La(q) = \frac{d}{dq}Ka(q) \qquad Lx(q) = \frac{d}{dq}Kx(q)$$

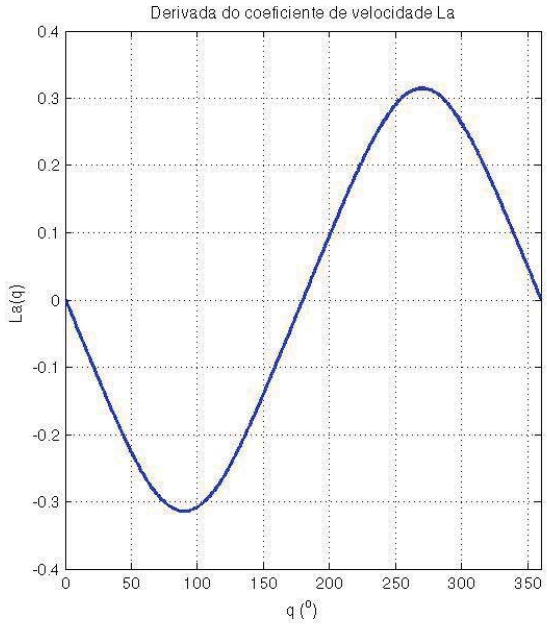
$$La(q) := -\frac{R\cdot\sin(q) - Ka(q)^2\cdot L\cdot\sin(A(q))}{L\cdot\cos(A(q))}$$

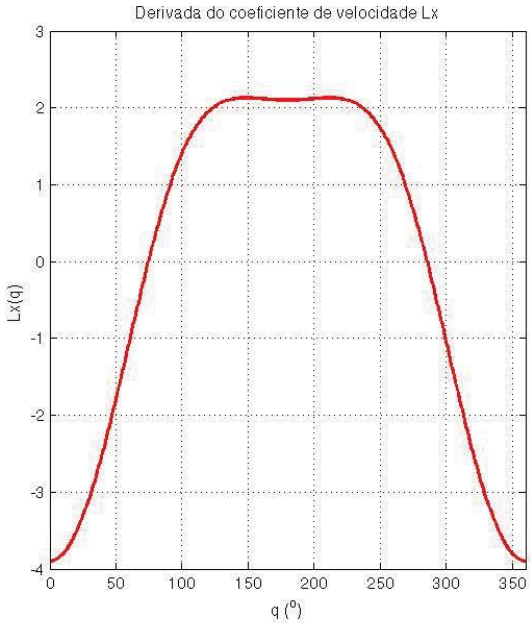
$$Lx(q) := \tan(A(q))\cdot\left(R\cdot\sin(q) - Ka(q)^2\cdot L\cdot\sin(A(q))\right) - R\cdot\cos(q) - Ka(q)^2\cdot L\cdot\cos(A(q))$$

Derivadas dos coeficientes de velocidade

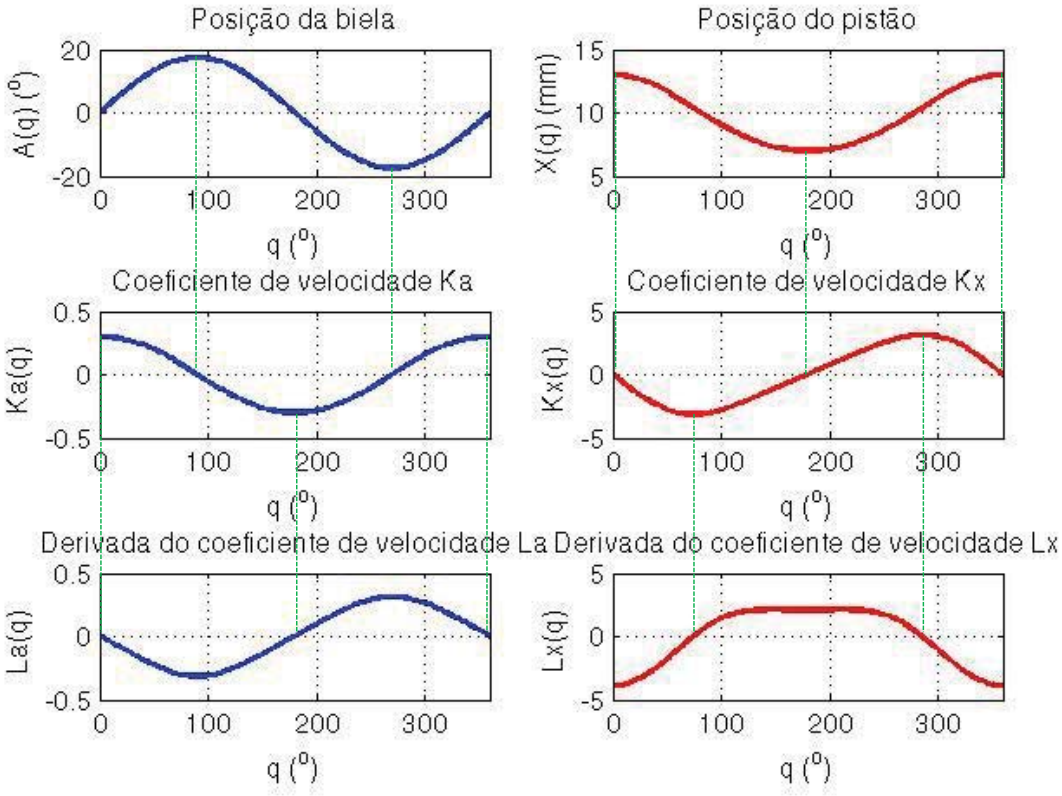
$$L_a(q) = \frac{R.\sin(q) - K_a(q)^2.L.\sin(A(q))}{L.\cos(A(q))}$$

$$L_x(q) = \tan\big(A(q)\big).\big(R.\sin(q) - K_a(q)^2.L.\sin\big(A(q)\big)\big) - R.\cos(q) - K_a(q)^2.L.\cos(A(q))$$



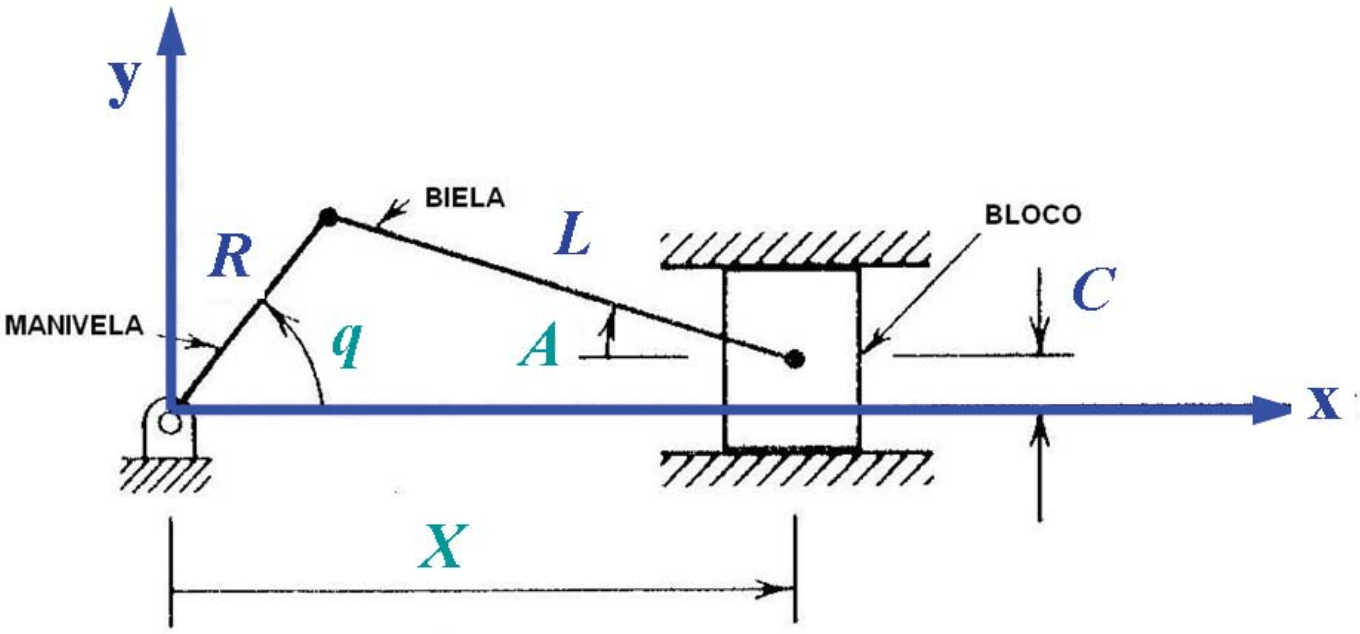


Análise Geral - Resultados

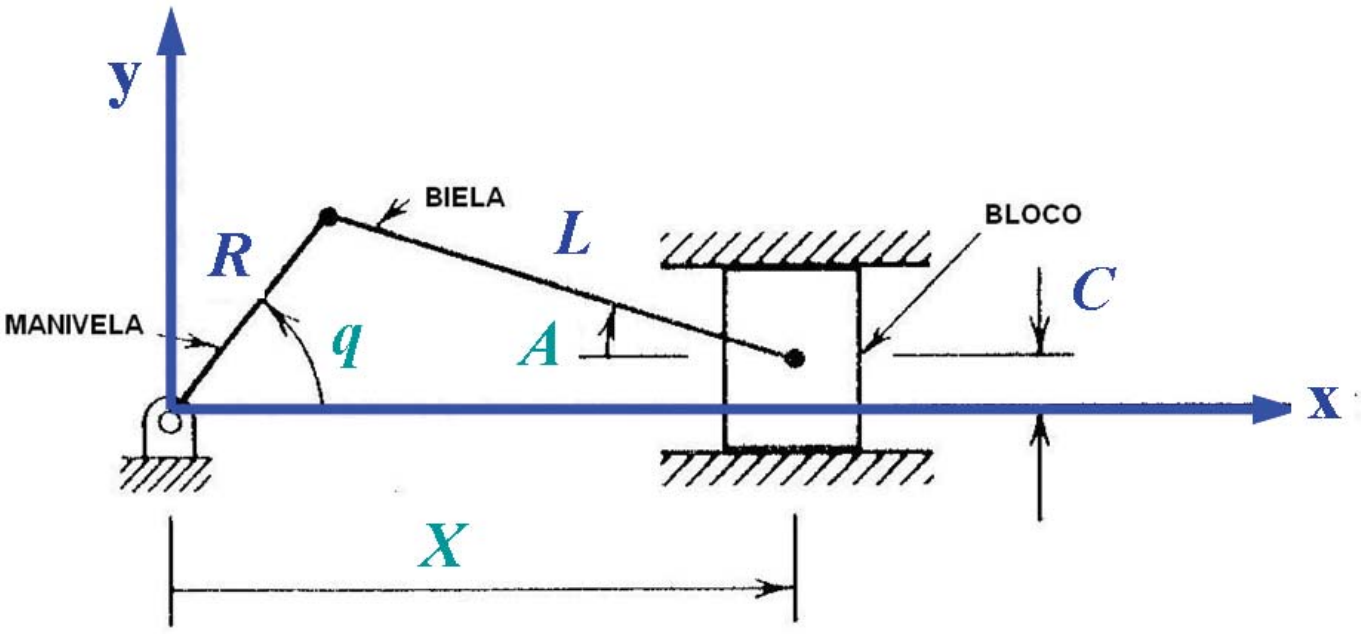


R=3 L=10 C=0

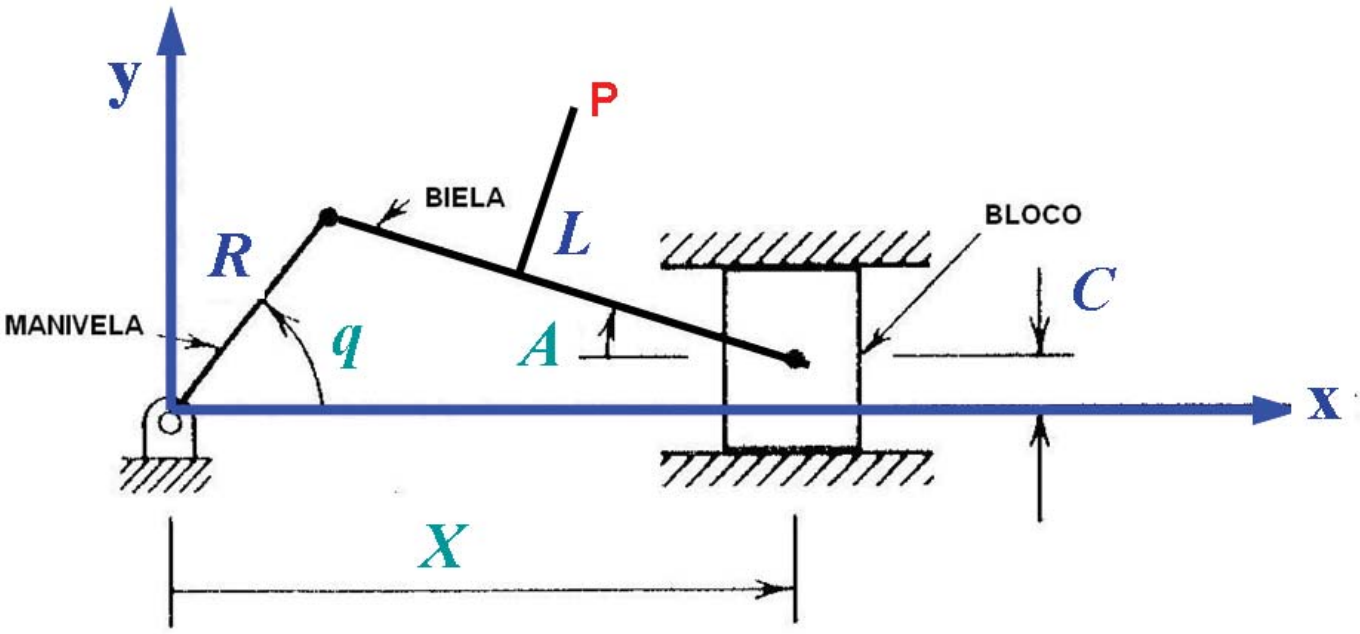
Análise de ponto de interesse



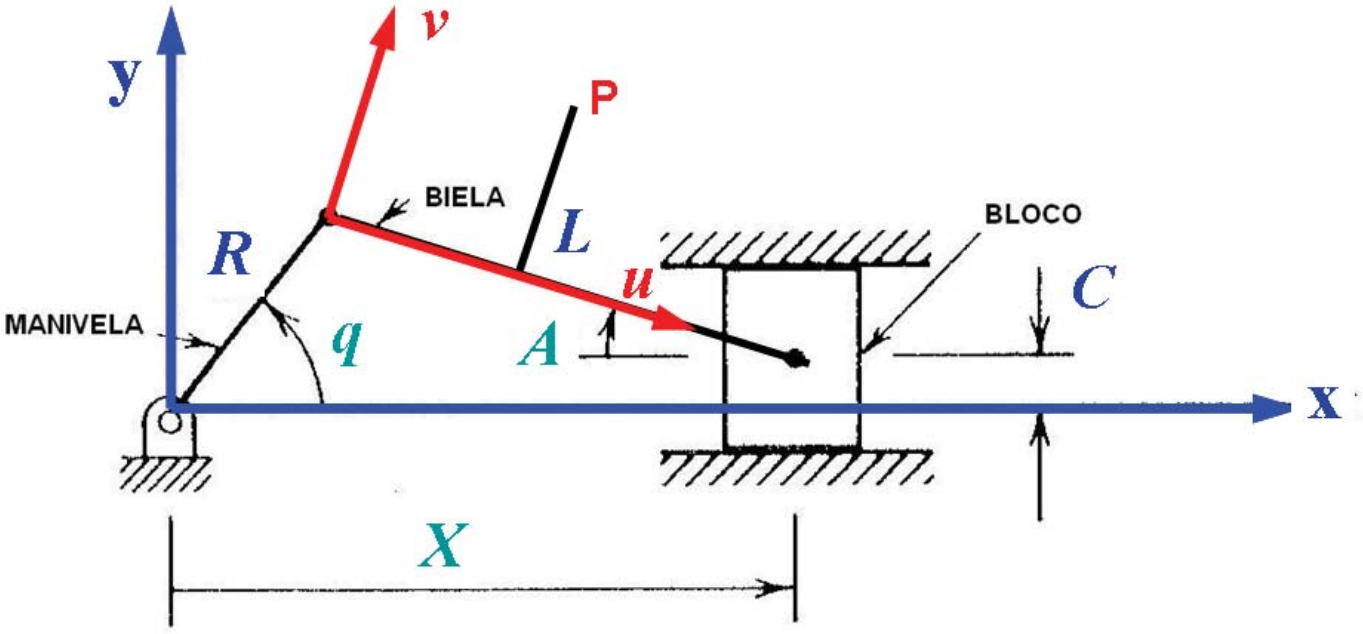
1- Sistema global (x,y)



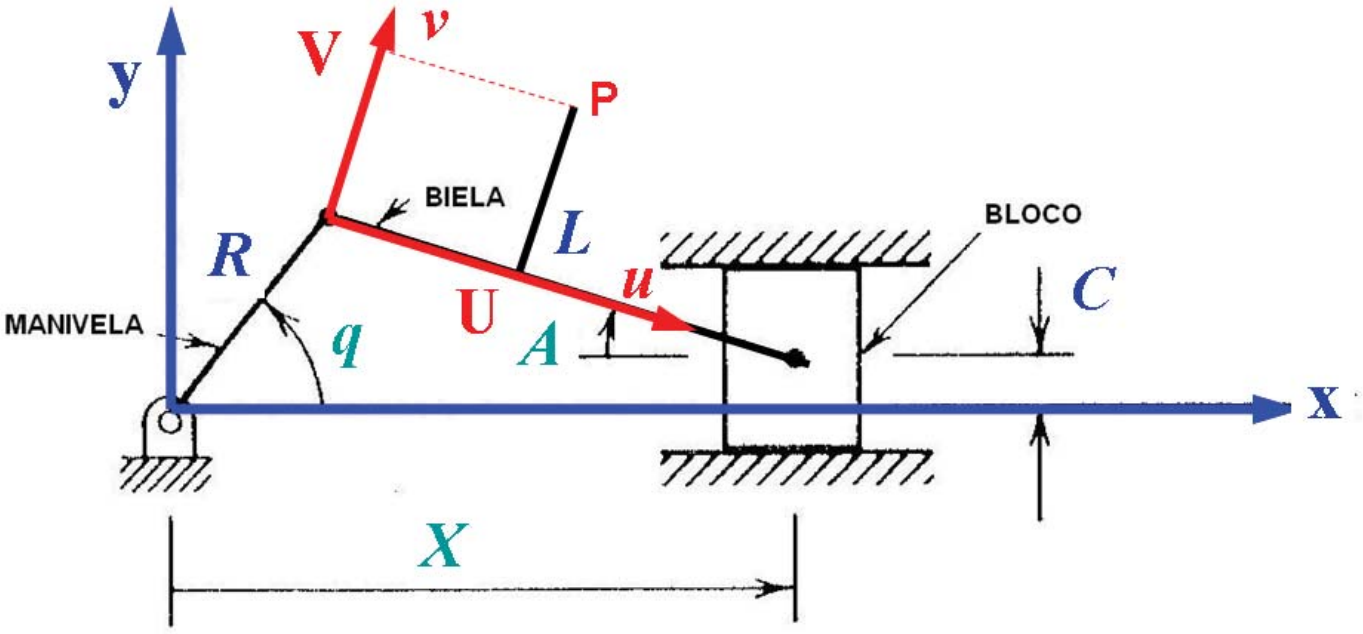
2- Seleção do ponto de interesse (P)



3-Sistema LOCAL (u,v)

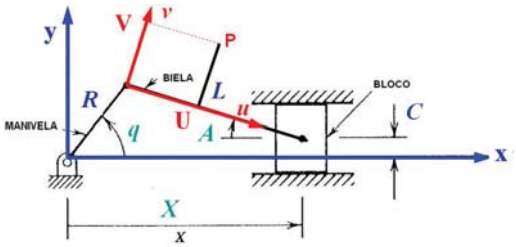


4-Coordenadas locais (U,V)



5-Coordenadas globais (X,Y)

$$\begin{pmatrix} X_P \\ Y_P \end{pmatrix} = \begin{pmatrix} O_x \\ O_y \end{pmatrix} + \begin{pmatrix} u_x & v_x \\ u_y & v_y \end{pmatrix} \cdot \begin{pmatrix} U_P \\ V_P \end{pmatrix}$$



$$\begin{pmatrix} X_P \\ Y_P \end{pmatrix} = \begin{pmatrix} R\cdot\cos(q) \\ R\cdot\sin(q) \end{pmatrix} + \begin{pmatrix} \cos(A) & \sin(A) \\ -\sin(A) & \cos(A) \end{pmatrix} \cdot \begin{pmatrix} U_P \\ V_P \end{pmatrix}$$

$$\begin{pmatrix} X_P \\ Y_P \end{pmatrix} = \begin{pmatrix} U_P\cdot\cos(A) + V_P\cdot\sin(A) + R\cdot\cos(q) \\ V_P\cdot\cos(A) - U_P\cdot\sin(A) + R\cdot\sin(q) \end{pmatrix}$$

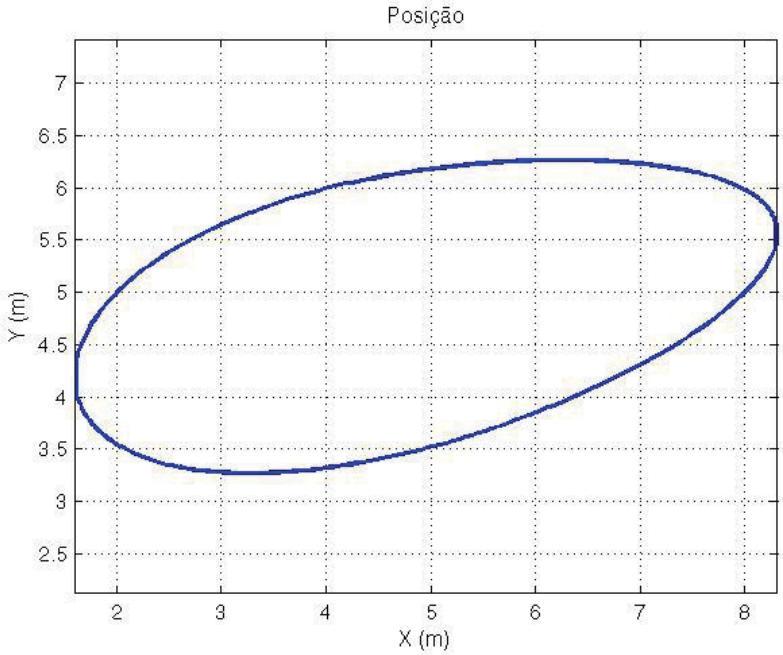
$$X_P(q) := U_P\cdot\cos(A(q)) + V_P\cdot\sin(A(q)) + R\cdot\cos(q)$$

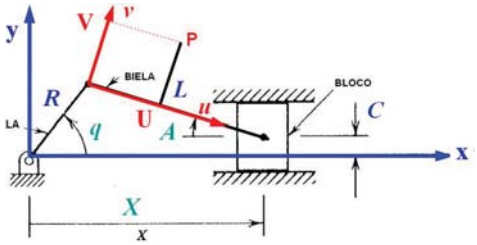
$$Y_P(q) := V_P\cdot\cos(A(q)) - U_P\cdot\sin(A(q)) + R\cdot\sin(q)$$

5-Coordenadas globais (X,Y)

$$X_P = R.\cos(q) + U_P.\cos(A(q)) + V_P.\sin(A(q))$$

$$Y_P = R.\text{sen}(q) + V_P.\cos(A(q)) - U_P.\sin(A(q))$$





R=3
L=10
C=0
U_P=L/2
V_P=L/2

Equações das velocidades

$$\dot{X}_P(q) = -U_P \cdot \sin(A(q)) \cdot Ka(q) \cdot \dot{q} + V_P \cdot \cos(A(q)) \cdot Ka(q) \cdot \dot{q} - R \cdot \sin(q) \cdot \dot{q}$$

$$\dot{Y}_P(q) = -V_P \cdot \sin(A(q)) \cdot Ka(q) \cdot \dot{q} - U_P \cdot \cos(A(q)) \cdot Ka(q) \cdot \dot{q} + R \cdot \cos(q) \cdot \dot{q}$$

$$K_{PX}(q) := \left(-U_P \cdot \sin(A(q)) + V_P \cdot \cos(A(q))\right) \cdot Ka(q) - R \cdot \sin(q)$$

$$K_{Py}(q) := \left(-V_P \cdot \sin(A(q)) - U_P \cdot \cos(A(q))\right) \cdot Ka(q) + R \cdot \cos(q)$$

$$\dot{X}_P(q) := K_{PX}(q) \cdot \dot{q}$$

$$\dot{Y}_P(q) := K_{Py}(q) \cdot \dot{q}$$

$$Vel(q) := \sqrt{\dot{X}_P(q)^2 + \dot{Y}_P(q)^2}$$

Equações das acelerações

$$L_{PX} = \frac{d}{dq} K_{PX} \qquad L_{Py} = \frac{d}{dq} K_{Py}$$

$$L_{PX}(q) = \left(V_P \cdot \cos(A(q)) - U_P \cdot \sin(A(q))\right) \cdot La(q) + \left(-U_P \cdot \cos(A(q)) - V_P \cdot \sin(A(q))\right) \cdot Ka(q)^2 - R \cdot \cos(q)$$

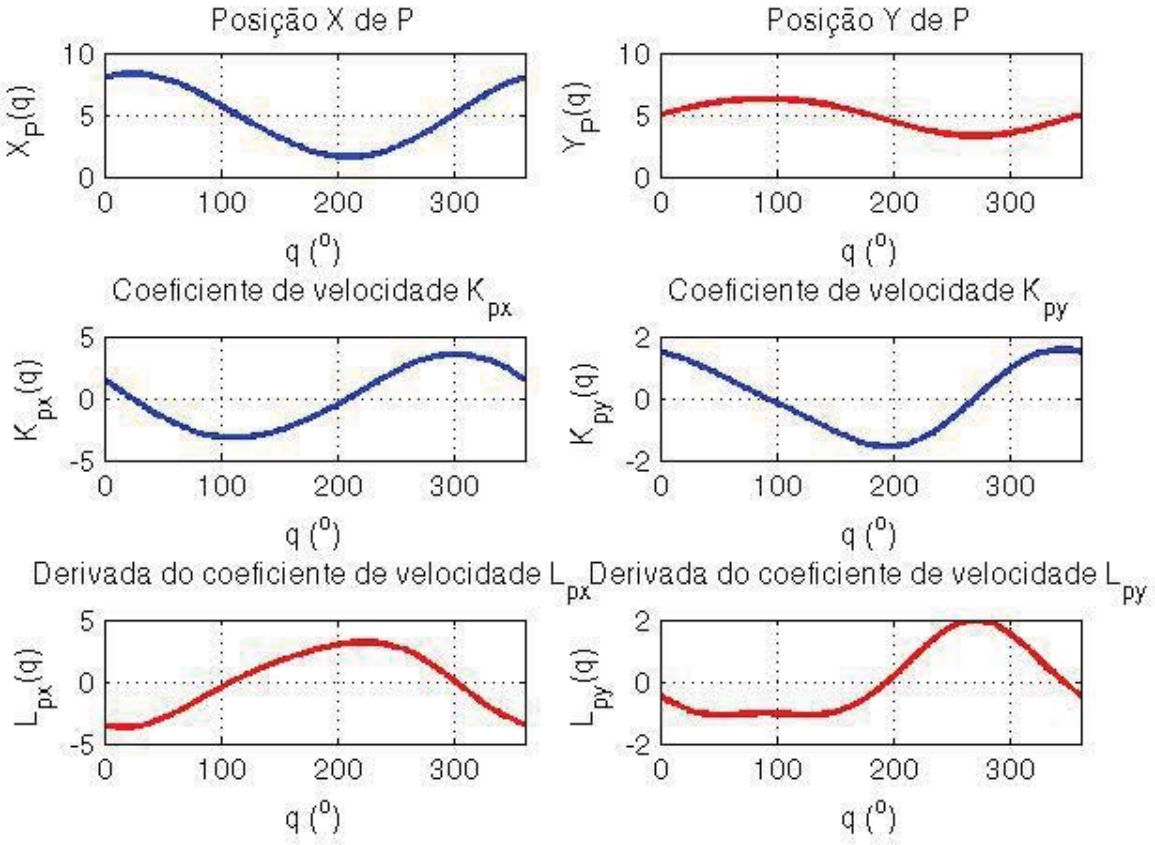
$$L_{Py}(q) = \left(-U_P \cdot \cos(A(q)) - V_P \cdot \sin(A(q))\right) \cdot La(q) + \left(U_P \cdot \sin(A(q)) - V_P \cdot \cos(A(q))\right) \cdot Ka(q)^2 - R \cdot \sin(q)$$

$$\ddot{X}_P(q) = K_{PX}(q) \cdot \ddot{q} + L_{PX}(q) \cdot \dot{q}^2$$

$$\ddot{Y}_P(q) = K_{Py}(q) \cdot \ddot{q} + L_{Py}(q) \cdot \dot{q}^2$$

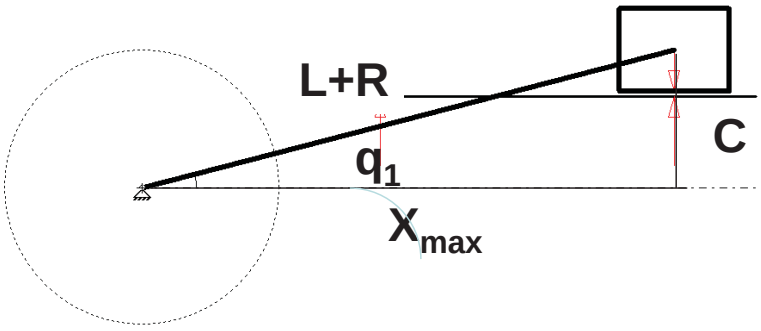
$$A_P(q) = \sqrt{\ddot{X}_P(q)^2 + \ddot{Y}_P(q)^2}$$

Análise de Pontos de Interesse - Resultados



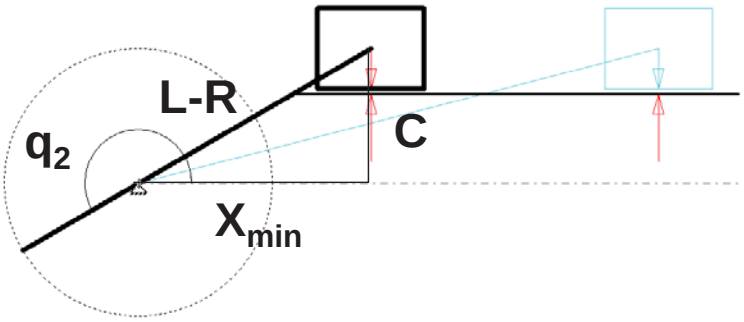
R=3
L=10
C=0
U_P=L/2
V_P=L/2

Limites do mecanismo biela-manivela



$$q_1 = \arcsin\left(\frac{C}{L+R}\right)$$

$$X_{\max} = \sqrt{(L+R)^2 - C^2}$$



$$q_2 = \arcsin\left(\frac{C}{L-R}\right) + 180^0$$

$$X_{\min} = \sqrt{(L-R)^2 - C^2}$$

E se o acionamento for outro?

variável primária – q

Variáveis secundárias – A, B