EXAMPLE 1.1

Consider the low-speed flow of air over an airplane wing at standard sea level conditions; the free-stream velocity far ahead of the wing is 100 mi/h. The flow accelerates over the wing, reaching a maximum velocity of 150 mi/h at some point on the wing. What is the percentage pressure change between this point and the free stream?

■ Solution

Since the airspeeds are relatively low, let us (for the first and *only* time in this book) assume incompressible flow, and use Bernoulli's equation for this problem. (See Ref. 1 for an elementary discussion of Bernoulli's equation, as well as Ref. 104 for a more detailed presentation of the role of this equation in the solution of incompressible flow. Here, we assume that the reader is familiar with Bernoulli's equation—its use and its limitations. If not, examine carefully the appropriate discussions in Refs. 1 and 104.) Let points 1 and 2 denote the free stream and wing points, respectively. Then, from Bernoulli's equation,

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$

or

$$p_1 - p_2 = \tfrac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

At standard sea level, $\rho = 0.002377$ slug/ft^3. Also, using the handy conversion that 60 mi/h = 88 ft/s, we have $V_1 = 100$ mi/h $= 147$ ft/s and $V_2 = 150$ mi/h $= 220$ ft/s. (Note that, as always in this book, we will use *consistent* units; for example, we will use either the English Engineering System, as in this problem, or the International System. See the footnote in Sec. 1.4 of this book, as well as Chap. 2 of Ref. 1. By using consistent units, *none* of our basic equations will ever contain conversion factors, such as g_c and J, as is found in some references.) With this information, we have

$$\begin{aligned} p_1 - p_2 &= \tfrac{1}{2}\rho\left(V_2^2 - V_1^2\right) \\ &= \tfrac{1}{2}(0.002377)[(220)^2 - (147)^2] = 31.8 \text{ lb/ft}^2 \end{aligned}$$

The fractional change in pressure referenced to the free-stream pressure, which at standard sea level is $p_1 = 2116$ lb/ft^2, is obtained as

$$\frac{p_1 - p_2}{p_1} = \frac{31.8}{2116} = 0.015$$

Therefore, the *percentage* change in pressure is 1.5 percent. In expanding over the wing surface, the pressure changes by *only* 1.5 percent. This is a case where, in Eq. (1.6), dp is small, and hence $d\rho$ is small. The purpose of this example is to demonstrate that, in low-speed flow problems, the *percentage* change in pressure is always small, and this, through Eq. (1.6), justifies the *assumption* of incompressible flow ($d\rho = 0$) for such flows. However, at high flow velocities, the change in pressure is not small, and the density must be treated as variable. This is the regime of compressible flow—the subject of this book. *Note:* Bernoulli's equation used in this example is good *only* for incompressible flow, therefore it will not appear again in any of our subsequent discussions. Experience has shown that, because it is one of the first equations usually encountered by students in the study of fluid dynamics, there is a tendency to use Bernoulli's equation for situations where it is not valid. Compressible flow is one such situation. Therefore, for our subsequent discussions in this book, remember *never* to invoke Bernoulli's equation.

EXAMPLE 1.2

A pressure vessel that has a volume of 10 m^3 is used to store high-pressure air for operating a supersonic wind tunnel. If the air pressure and temperature inside the vessel are 20 atm and 300 K, respectively, what is the mass of air stored in the vessel?

■ Solution

Recall that 1 atm = 1.01×10^5 N/m^2. From Eq. (1.9)

$$\rho = \frac{p}{RT} = \frac{(20)(1.01 \times 10^5)}{(287)(300)} = 23.46 \text{ kg/m}^3$$

The total mass stored is then

$$M = \mathscr{V}\rho = (10)(23.46) = \boxed{234.6 \text{ kg}}$$

EXAMPLE 1.3

Calculate the isothermal compressibility for air at a pressure of 0.5 atm.

■ Solution

From Eq. (1.3)

$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

From Eq. (1.8)

$$v = \frac{RT}{p}$$

Thus

$$\left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2}$$

Hence

$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T = -\left(\frac{p}{RT}\right)\left(-\frac{RT}{p^2}\right) = \frac{1}{p}$$

We see that the isothermal compressibility for a perfect gas is simply the reciprocal of the pressure:

$$\tau_T = \frac{1}{p} = \frac{1}{0.5} = \boxed{2 \text{ atm}^{-1}}$$

In terms of the International System of units, where $p = (0.5)(1.01 \times 10^5) = 5.05 \times 10^4$ N/m^2,

$$\tau_T = \boxed{1.98 \times 10^{-5} \text{m}^2/\text{N}}$$

In terms of the English Engineering System of units, where $p = (0.5)(2116) = 1058$ lb/ft^2,

$$\tau_T = \boxed{9.45 \times 10^{-4} \text{ ft}^2/\text{lb}}$$

EXAMPLE 1.4

For the pressure vessel in Example 1.2, calculate the total internal energy of the gas stored in the vessel.

■ Solution

From Eq. (1.23)

$$c_v = \frac{R}{\gamma - 1} = \frac{287}{1.4 - 1} = 717.5 \text{ J/kg} \cdot \text{K}$$

From Eq. (1.19)

$$e = c_v T = (717.5)(300) = 2.153 \times 10^5 \text{ J/kg}$$

From Example 1.2, we calculated the mass of air in the vessel to be 234.6 kg. Thus, the total internal energy is

$$E = Me = (234.6)(2.153 \times 10^5) = \boxed{5.05 \times 10^7 \text{ J}}$$

EXAMPLE 1.5

Consider the air in the pressure vessel in Example 1.2. Let us now heat the gas in the vessel. Enough heat is added to increase the temperature to 600 K. Calculate the change in entropy of the air inside the vessel.

■ Solution

The vessel has a constant volume; hence as the air temperature is increased, the pressure also increases. Let the subscripts 1 and 2 denote the conditions before and after heating, respectively. Then, from Eq. (1.8),

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} = \frac{600}{300} = 2$$

In Example 1.4, we found that $c_v = 717.5$ J/kg · K. Thus, from Eq. (1.20)

$$c_p = c_v + R = 717.5 + 287 = 1004.5 \text{ J/kg} \cdot \text{K}$$

From Eq. (1.36)

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$
$$= 1004.5 \ln 2 - 287 \ln 2 = 497.3 \text{ J/kg} \cdot \text{K}$$

From Example 1.2, the mass of air inside the vessel is 234.6 kg. Thus, the total entropy change is

$$S_2 - S_1 = M(s_2 - s_1) = (234.6)(497.3) = \boxed{1.167 \times 10^5 \text{ J/K}}$$

EXAMPLE 1.6

Consider the flow through a rocket engine nozzle. Assume that the gas flow through the nozzle is an isentropic expansion of a calorically perfect gas. In the combustion chamber, the gas which results from the combustion of the rocket fuel and oxidizer is at a pressure and temperature of 15 atm and 2500 K, respectively; the molecular weight and specific heat at constant pressure of the combustion gas are 12 and 4157 J/kg · K, respectively. The gas expands to supersonic speed through the nozzle, with a temperature of 1350 K at the nozzle exit. Calculate the pressure at the exit.

■ Solution

From our earlier discussion on the equation of state,

$$R = \frac{\mathscr{R}}{\mathscr{M}} = \frac{8314}{12} = 692.8 \text{ J/kg} \cdot \text{K}$$

From Eq. (1.20)

$$c_v = c_p - R = 4157 - 692.8 = 3464 \text{ J/kg} \cdot \text{K}$$

Thus

$$\gamma = \frac{c_p}{c_v} = \frac{4157}{3464} = 1.2$$

From Eq. (1.43), we have

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} = \left(\frac{1350}{2500}\right)^{1.2/(1.2-1)} = 0.0248$$

$$p_2 = 0.025 p_1 = (0.0248)(15 \text{ atm}) = \boxed{0.372 \text{ atm}}$$

EXAMPLE 1.7

Calculate the isentropic compressibility for air at a pressure of 0.5 atm. Compare the result with that for the isothermal compressibility obtained in Example 1.3.

■ Solution

From Eq. (1.4), the isentropic compressibility is defined as

$$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s$$

Since $v = 1/\rho$, we can write Eq. (1.4) as

$$\tau_s = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_s \tag{E.1}$$

The variation between p and ρ for an isentropic process is given by Eq. (1.43)

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

which is the same as writing

$$p = c\rho^{\gamma} \tag{E.2}$$

where c is a constant. From Eq. (E.2)

$$\left(\frac{\partial p}{\partial \rho}\right)_s = c\gamma\rho^{\gamma-1} = \frac{p}{\rho^{\gamma}}(\gamma\rho^{\gamma-1}) = \frac{\gamma p}{\rho} \tag{E.3}$$

From Eqs. (E.1) and (E.3),

$$\tau_s = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_s = \frac{1}{\rho}\left(\frac{\partial p}{\partial \rho}\right)_s^{-1} = \frac{1}{\rho}\left(\frac{\gamma p}{\rho}\right)^{-1}$$

Hence,

$$\tau_s = \frac{1}{\gamma p} \tag{E.4}$$

Recall from Example 1.3 that $\tau_T = 1/p$. Hence,

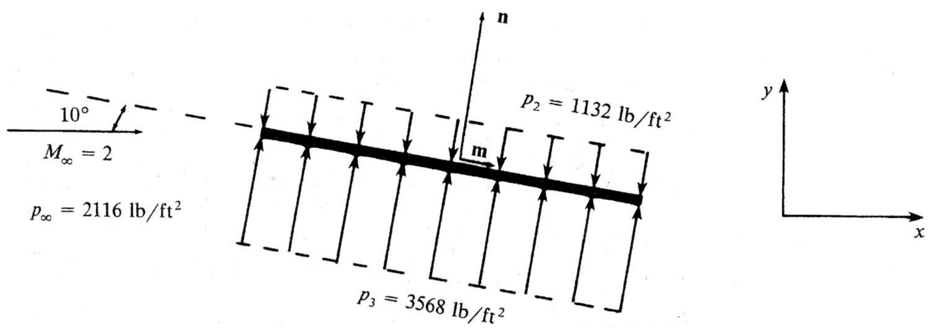
$$\tau_s = \frac{\tau_T}{\gamma} \tag{E.5}$$

Note that τ_s is smaller than τ_T by the factor γ. From Example 1.3, we found that for $p = 0.5$ atm, $\tau_T = 1.98 \times 10^{-5}$ m²/N. Hence, from Eq. (E.5)

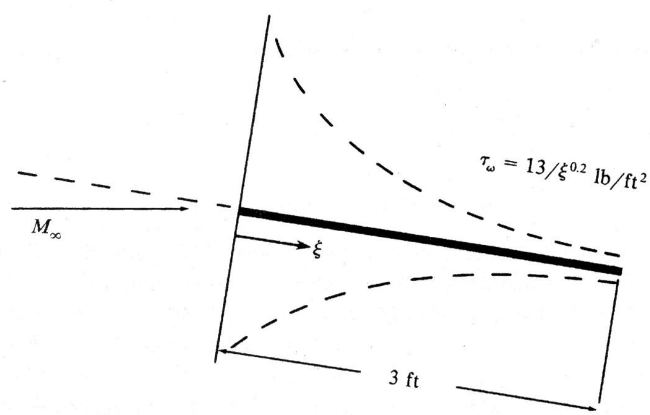
$$\tau_s = \frac{1.98 \times 10^{-5}}{1.4} = \boxed{1.41 \times 10^{-5}\ \text{m}^2/\text{N}}$$

EXAMPLE 1.8

A flat plate with a chord length of 3 ft and an infinite span (perpendicular to the page in Fig. 1.12) is immersed in a Mach 2 flow at standard sea level conditions at an angle of attack of 10°. The pressure distribution over the plate is as follows: upper surface, $p_2 = \text{const} = 1132$ lb/ft²; lower surface, $p_3 = \text{const} = 3568$ lb/ft². The local shear stress is given by



Pressure distribution



Shear stress distribution

Figure 1.12 | Geometry for Example 1.8.

$\tau_w = 13/\xi^{0.2}$, where τ_w is in pounds per square feet and ξ is the distance in feet along the plate from the leading edge. Assume that the distribution of τ_w over the top and bottom surfaces is the same. (We make this assumption for simplicity in this example. In reality, the shear stress distributions over the top and bottom surfaces will be different because the flow properties over these two surfaces are different.) Both the pressure and shear stress distributions are sketched qualitatively in Fig. 1.12. Calculate the lift and drag per unit span on the plate.

■ Solution

Considering a unit span,

$$-\oint\!\!\!\oint p\,d\mathbf{S} = \left[-\int_0^3 p_2\,d\xi + \int_0^3 p_3\,d\xi\right]\mathbf{n} = [-(1132)(3) + (3568)(3)]\mathbf{n} = 7308\mathbf{n}$$

From Eq. (1.46)

$$L = y \text{ component of} \left[-\oint\!\!\!\oint p\,d\mathbf{S}\right] = 7308\cos 10^\circ = \boxed{7197 \text{ lb}} \text{ per unit span}$$

From Eq. (1.47)

$$\text{Pressure drag} = \text{wave drag} \equiv D_w = x \text{ component of} \left[-\oint\!\!\!\oint p\,d\mathbf{S}\right]$$

Hence

$$D_w = 7308\sin 10^\circ = \boxed{1269 \text{ lb}} \text{ per unit span}$$

Also from Eq. (1.46)

$$\text{Skin-friction drag} \equiv D_f = x \text{ component of} \left[\oint\!\!\!\oint \tau\mathbf{m}\,dS\right]$$

$$\oint\!\!\!\oint \tau\mathbf{m}\,dS = \left[13\int_0^3 \xi^{-0.2}d\xi\right]\mathbf{m} = 16.25\xi^{4/5}\Big|_0^3\mathbf{m} = 39.13\mathbf{m}$$

Hence, recalling that shear stress acts on both sides,

$$D_f = 2(39.13)\cos 10^\circ = \boxed{77.1 \text{ lb}} \text{ per unit span}$$

The total drag is

$$D = D_w + D_f$$

$$D = 1269 \text{ lb} + 77.1 \text{ lb} = \boxed{1346 \text{ lb}}$$

Note: For this example, the drag is mainly wave drag; skin-friction drag accounts for only 5.7 percent of the total drag. This illustrates an important point. For supersonic flow over slender bodies at a reasonable angle of attack, the wave drag is the primary drag contributor at sea level, far exceeding the skin-friction drag. For such applications, the inviscid methods discussed in this book suffice, because the wave drag (pressure drag) can be obtained from such methods. We see here also why so much attention is focused on the reduction of wave drag—because it is frequently the primary drag component. At smaller angles of attack, the relative proportion of D_f to D increases. Also, at higher altitudes, where viscous effects become stronger (the Reynolds number is lower), the relative proportion of D_f to D increases.