Solution

Part (a) Use absolute temperature in the exact formula, Eq. (2.20):

$$p = p_a\left[1 - \frac{(0.00650\ \text{K/m})(5000\ \text{m})}{288.16\ \text{K}}\right]^{5.26} = (101{,}350\ \text{Pa})(0.8872)^{5.26}$$

$$= 101{,}350(0.5328) = 54{,}000\ \text{Pa} \qquad \textit{Ans. (a)}$$

This is the standard-pressure result given at $z = 5000$ m in Table A.6.

Part (b) If the atmosphere were isothermal at 288.16 K, Eq. (2.18) would apply:

$$p \approx p_a \exp\left(-\frac{gz}{RT}\right) = (101{,}350\ \text{Pa}) \exp\left\{-\frac{(9.807\ \text{m/s}^2)(5000\ \text{m})}{[287\ \text{m}^2/(\text{s}^2 \cdot \text{K})](288.16\ \text{K})}\right\}$$

$$= (101{,}350\ \text{Pa}) \exp(-0.5929) \approx 56{,}000\ \text{Pa} \qquad \textit{Ans. (b)}$$

This is 4 percent higher than the exact result. The isothermal formula is inaccurate in the troposphere.

Is the Linear Formula Adequate for Gases?

The linear approximation from Eq. (2.14), $\delta p \approx -\rho g\, \delta z$, is satisfactory for liquids, which are nearly incompressible. For gases, it is inaccurate unless δz is rather small. Problem P2.26 asks you to show, by binomial expansion of Eq. (2.20), that the error in using constant gas density to estimate δp from Eq. (2.14) is small if

$$\delta z \ll \frac{2T_0}{(n - 1)B} \tag{2.21}$$

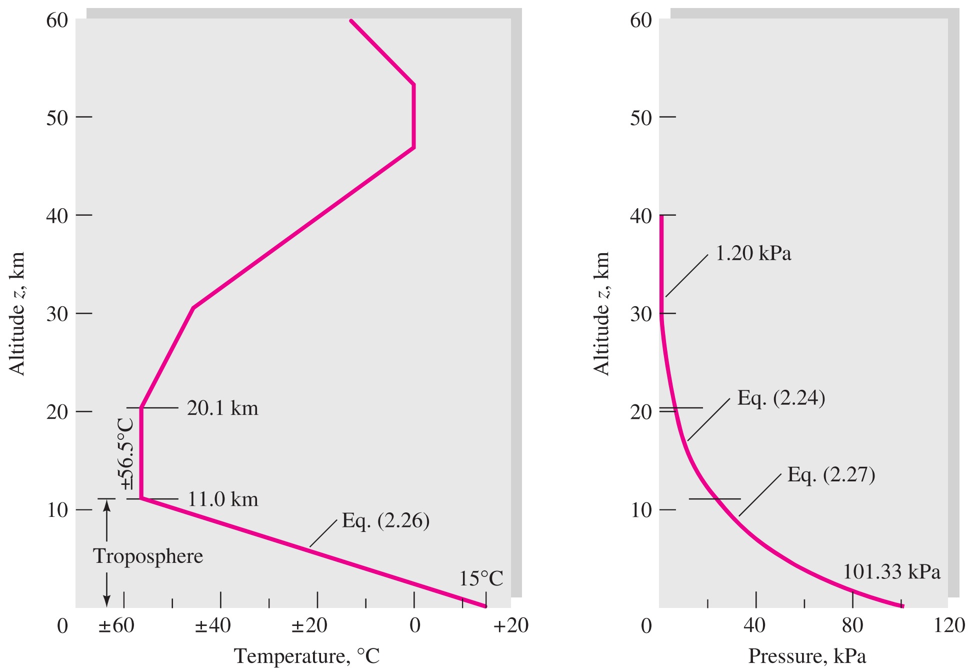

Fig. 2.7 Temperature and pressure distribution in the U.S. standard atmosphere. *(From Ref. 1.)*