$\boxed{1}$

$\boxed{1}$ $V_1, V_2 = ?$ $\rho = 1000\ kg/m^3$ $A_1 = 0{,}1\ m^2$

$\dot{M} = 50\ kg/s$ REGIME PERMANENTE $A_2 = 2A_1$

$\dot{M} = \rho V A \rightarrow V_1 = \dfrac{\dot{M}}{\rho A_1} = 0{,}5\ m/s$ // $V_2 A_2 = V_1 A_1 \Rightarrow V_2 = V_1 \dfrac{A_1}{A_2} = 0{,}25\ m/s$ //

$$\dot{M} = \int_A \rho \vec{V} \cdot \hat{n}\, dA$$

$\boxed{2}$ $V_2 = 2\bar{V}_2 \left[1 - \left(\dfrac{r}{R}\right)^2\right]$ $\bar{V}_2 = 3\ m/s$ $R_1 = 0{,}01\ m$ $\bar{V}_1 = ?$

$\rho = 10^3\ kg/m^3$ $R_2 = 0{,}02\ m$ $\dot{M}_1, \dot{M}_2 = ?$

REGIME PERMANENTE

PROCEDIMENTO 1

$$\dot{M} = \int_A \rho V\, dA \longrightarrow \dot{M}_2 = \rho \int_{A_2} V_2\, dA_2 \qquad a = \pi r^2 \rightarrow da = 2\pi r\, dr$$

$$\dot{M}_2 = \rho \int_0^{R_2} 2\bar{V}_2 \left[1 - \left(\frac{r}{R_2}\right)^2\right] 2\pi r\, dr = 4\pi\rho\bar{V}_2 \int_0^{R_2} \left(r - \frac{r^3}{R_2^2}\right) dr = 4\pi\rho\bar{V}_2 \left(\frac{R_2^2}{2} - \frac{R_2^4}{4R_2^2}\right)$$

$$\dot{M}_2 = \pi\rho\bar{V}_2\left(2R_2^2 - R_2^2\right) = \underline{\pi R_2^2 \rho \bar{V}_2} = \rho \bar{V}_2 A_2$$

PROCEDIMENTO 2

$\bar{V}_1 A_1 = \bar{V}_2 A_2 \rightarrow \bar{V}_1 = \bar{V}_2 \dfrac{A_2}{A_1} = \bar{V}_2 \dfrac{\pi R_2^2}{\pi R_1^2} = \bar{V}_2 \left(\dfrac{R_2}{R_1}\right)^2 \longrightarrow \bar{V}_1 = 12\ m/s$ //

$\rho \bar{V}_1 A_1 = \rho \bar{V}_2 A_2$ ou $\dot{M}_1 = \dot{M}_2 = \rho \pi R_1^2 \bar{V}_1 = 3{,}8\ kg/s$ //

$\boxed{3}$ $\bar{V}_2 \Big|_{\text{duto}}^{\text{em r. ao}} = \bar{V}_2 - V_{duto} = 3 - 1 = 2\ m/s$

$\bar{V}_1 \Big|_{\text{duto}}^{\text{em r. ao}} = \bar{V}_2\Big|_{duto} \left(\dfrac{R_2}{R_1}\right)^2 = 8\ m/s$

$\dot{M}_1 = \dot{M}_2 = \rho A_1 \bar{V}_1 = \rho \pi R_1^2 \bar{V}_1 = 2{,}5\ kg/s$ //

CAP. 3 - EXERCÍCIOS ②

17 Mai 95

$\boxed{4}$ $V_3 = ?$ → PARA → $\theta = 30°$ e $60°$

$\dot{M}_1 = 20$ kg/s ent.

$V_4 = 2$ m/s ent.

$Q_2 = 0{,}05\ m^3/s$ sai

$A_1 = A_2 = 0{,}02\ m^2$

$A_3 = A_4 = 0{,}01\ m^2$

$\rho = 1000\ kg/m^3$

$$\rho \int_{SC} V\,dA = 0 = \Sigma \dot{M} = \Sigma Q$$

$$\dot{M}_1 = \rho V_1 A_1 \rightarrow V_1 = \frac{\dot{M}_1}{\rho A_1} = 1\ m/s$$

$$Q_2 = V_2 A_2 \rightarrow V_2 = \frac{Q_2}{A_2} = 2{,}5\ m/s$$

$$-V_1 A_1 + V_2 A_2 \underbrace{- V_3 A_3}_{\text{entrando}} - V_4 A_4 = 0 \rightarrow V_3 = \frac{-V_1 A_1 - V_4 A_4 + V_2 A_2}{A_3} = 1\ m/s$$

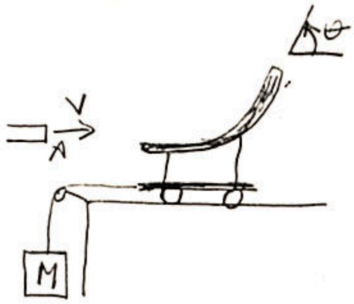
entrando no V. C

5 ~~4.56~~ OK

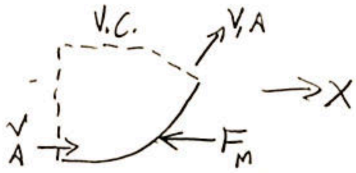
$V = 15\ m/s$

$\theta = 50^\circ$

$A = 0{,}05\ m^2$



Qual M para carrinho ficar parado?



F_M = força devido a massa M sobre o V.C. p/ mantê-lo parado

$$\frac{\partial}{\partial t}\int_{VC} \vec{V}\rho\, dV + \int_{SC} \vec{V}\rho\, \vec{V}\cdot d\vec{A} = \vec{F}$$

regime permanente $\rightarrow$ x: $\int_{SC} u\rho\, \vec{V}\cdot d\vec{A} = F_x$

$$|V| \times \{-|\rho V A|\} + |V|\cos\theta \times \{|\rho V A|\} = -F_M$$

$$F_M = |\rho V^2 A| - |\rho V^2 A|\cos\theta = |\rho V^2 A|(1-\cos\theta)$$

$P = Mg$

$P = F_M$

$$M = \frac{F_M}{g}$$

$$\boxed{M = \frac{|\rho V^2 A|(1-\cos\theta)}{g}}$$

$g = 9{,}8 \quad \rho = 1000$

$M = 410\ kg$ //

6 ~~4.60~~ OK

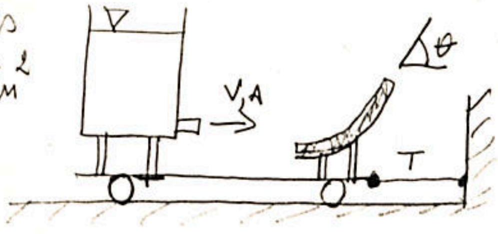
Qual a tensão T no fio?

$V = 10\ m/s$

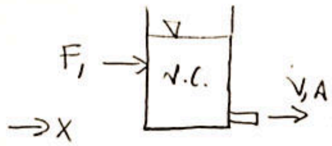
$A = 6 \times 10^{-4}\ m^2$

ÁGUA

$\theta = 90^\circ$



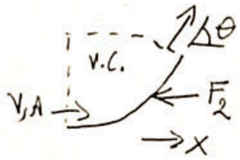
$$\frac{\partial}{\partial t}\int_{VC} \vec{V}\rho\, d\forall + \int_{SC} \vec{V}\rho\, \vec{V}\cdot d\vec{A} = \vec{F}$$



F_1 = força p/ manter tanque parado (V.C.)

regime permanente: x: $\int_{SC} u\rho\, \vec{V}\cdot d\vec{A} = \vec{F}$

$|V| \times \{|\rho V A|\} = F_1 \rightarrow F_1 = |\rho V^2 A| \qquad \vec{F_1} = |\rho V^2 A|\,\hat{i}$



F_2 = força p/ manter a aleta (V.C.) parada

$|V| \times \{-|\rho V A|\} + |V|\cos\theta \times \{|\rho V A|\} = -F_2$

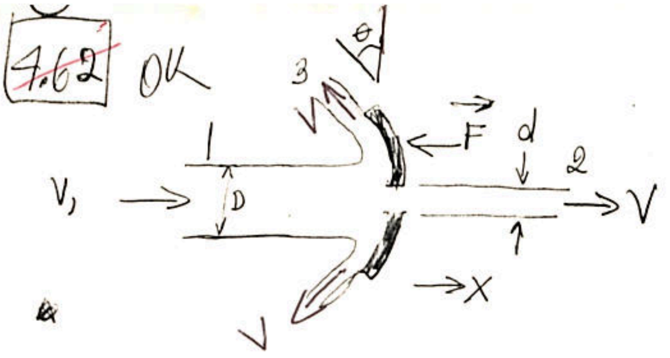
$F_2 = |\rho V^2 A|(1 - \cos\theta)$

para $\theta = 90^\circ \qquad F_2 = |\rho V^2 A| \qquad \vec{F_2} = -|\rho V^2 A|\,\hat{i}$

$\vec{T} = \vec{F_1} + \vec{F_2} = |\rho V^2 A|\,\hat{i} - |\rho V^2 A|\,\hat{i}$

$\boxed{\vec{T} = 0}$

7 ~~4.62~~ OK



Qual a força $\vec{F}$ necessária para manter o disco parado?

$$\int_{SC} u \rho \vec{V} \cdot d\vec{A} = F_x$$

$$|V| \times \{-|\rho V A_1|\} + |V| \times \{|\rho V A_2|\} - |V \operatorname{sen}\theta| \times \{|\rho V A_3|\} = -F$$

$$F = |\rho V^2| \{|A_1| - |A_2| + |A_3 \operatorname{sen}\theta|\}$$

$$A_1 = \frac{\pi D^2}{4} \qquad A_2 = \frac{\pi d^2}{4}$$

$$\int_{SC} \vec{V} \cdot d\vec{A} = 0 \longrightarrow -|V A_1| + |V A_2| + |V A_3| = 0 \qquad A_3 = A_1 - A_2$$

$$F = |\rho V^2| [A_1 - A_2 + (A_1 - A_2) \operatorname{sen}\theta] = |\rho V^2| [A_1 (1 + \operatorname{sen}\theta) - A_2 (1 + \operatorname{sen}$$

$$F = (1 + \operatorname{sen}\theta) \rho V^2 (A_1 - A_2) = (1 + \operatorname{sen}\theta) \rho V^2 \left(\frac{\pi D^2}{4} - \frac{\pi d^2}{4} \right)$$

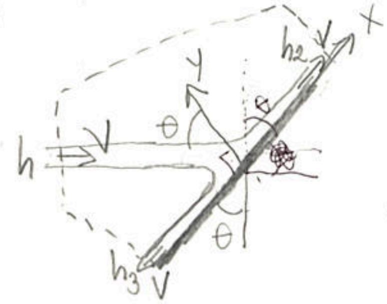
$$\boxed{F = \frac{\pi}{4} (1 + \operatorname{sen}\theta) \rho V^2 (D^2 - d^2)}$$

$\rho = 1000\ kg/m^3$ $\quad D = 0,1\ m$ $\quad d = 0,025\ m$ $\quad \theta = 45^\circ$ $\quad V = 5\ m/s$

$F = 314\ N$ $\qquad \boxed{\vec{F} = -314\ N\ \hat{\imath}}$

$$F = 0,785 \times (1 + 0,707) \times 1000 \times 25 \times (9,38 \times 10^{-3}) =$$

8



2ª Lei N. $\int_{SC} \vec{V} \rho \vec{V} \cdot \hat{n} \, dA = \vec{F}$

DIREÇÃO X: $V \operatorname{sen}\theta \, \rho (-V) h + V \rho V h_2 - V \rho V h_3 = 0$

$$h_2 = h \operatorname{sen}\theta + h_3 \quad (*)$$

MASSA: $\int_{SC} \rho \vec{V} \cdot \hat{n} \, dA = 0 \rightarrow -Vh + Vh_2 + Vh_3 = 0 \rightarrow h_3 = h - h_2 \quad (**)$

COM (**) EM (*): $h_2 = h \operatorname{sen}\theta + h - h_2 \quad \therefore \quad 2h_2 = h(1 + \operatorname{sen}\theta)$

$$\boxed{\frac{h_2}{h} = \frac{1 + \operatorname{sen}\theta}{2}}$$

$\theta = 0°$ (placa vertical) $\rightarrow \frac{h_2}{h} = \frac{1}{2}$

$\theta = 90°$ (placa horizontal) $\rightarrow \frac{h_2}{h} = 1$

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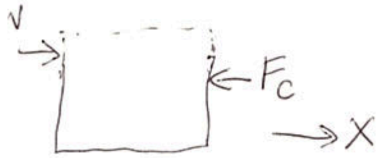


9 MASSA: $\frac{\partial}{\partial t}\int_{VC} \rho \, dV + \int_{SC} \rho \, \vec{V} \cdot \hat{n} \, dA = 0$

$\frac{\partial M}{\partial t} - \rho(V-U)A = 0 \quad \therefore \quad \int_{M_0}^{M} dM = \int_0^t \rho(V-U)A \, dt$ onde $U = U(t)$

$\frac{dM}{dt} = \rho(V-U)A \quad (*)$

$\cancel{\frac{\partial}{\partial t}\int_{VC} \vec{V} \rho \, dV}^{\simeq 0} + \int_{SC} \vec{V} \rho \, \vec{V} \cdot \hat{n} \, dA = \vec{F}$ $\qquad (V-U)\rho\{-|V-U|\}A = -F_C$ (força p/ manter carrinho imóvel $\to U=0$)

$F_C = \rho(V-U)^2 A \to \vec{F}_j = \rho(V-U)^2 A \, \hat{\imath}$ (força do jato sobre o carrinho) $\qquad F_j = M\frac{dU}{dt}$

$\cancel{F_j = \frac{d}{dt}(MU) =} \qquad M\frac{dU}{dt} = \rho(V-U)^2 A \quad \to \quad \rho(V-U)A = \frac{M}{(V-U)}\frac{dU}{dt} \quad (**)$

Com (**) em (*)

$\frac{dM}{dt} = \rho(V-U)A = \frac{M}{(V-U)}\frac{dU}{dt} \qquad \frac{dM}{M} = \frac{dU}{(V-U)}$

$\int_{M_0}^{M} \frac{dM}{M} = \int_0^U \frac{dU}{(V-U)} = -\int_V^{V-U} \frac{dZ}{Z} \qquad Z = V-U, \; dZ = -dU$

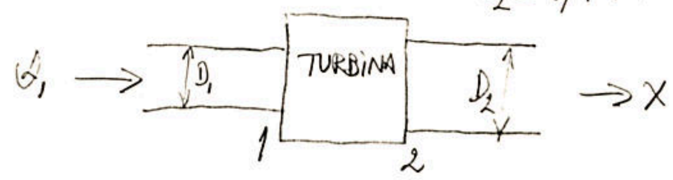
$\ln M \Big|_{M_0}^{M} = -\ln Z \Big|_V^{V-U} \qquad \ln\left(\frac{M}{M_0}\right) = -\left[\ln(V-U) - \ln(V)\right] = -\ln\left(\frac{V-U}{V}\right) = \ln\left[\frac{V}{(V-U)}\right]$

$$\boxed{M = \frac{M_0 V}{(V-U)}}$$

⑩ OK MEC-FLU LISTA 3 [7] NSet 94

10 ~~4.204~~ $\mathcal{Q}_1 = 0{,}6\,m^3/s$ ÁGUA $D_1 = 0{,}3\,m$ $D_2 = 0{,}4\,m$ $\dot{W}_e = 60\,kW$ $\rho = 1000\,kg/m^3$

$p_2 - p_1 = ?$



$\dot{W}_e = -\dot{W}$

Regime permanente:
$$\int_{SC}\left(h + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A} = \dot{Q} + \dot{W}_e \qquad h = u + \frac{p}{\rho}$$

Considerando: $\dot{Q} = 0,\ u_1 = u_2,\ z_1 = z_2$

$$\int_{SC}\left(\frac{p}{\rho} + \frac{V^2}{2}\right)\rho\vec{V}\cdot d\vec{A} = \dot{W}_e \qquad |\rho V A| = \dot{m} = \rho\mathcal{Q}$$

$$\mathcal{Q} = V_1 A_1 = V_1 \frac{\pi D_1^2}{4} \qquad V_1 = \frac{4\mathcal{Q}}{\pi D_1^2} \cong 8{,}49\,m/s$$

$$\int_{SC}\vec{V}\cdot d\vec{A} = 0 \qquad -V_1A_1 + V_2A_2 = 0 \qquad V_2 = V_1\frac{A_1}{A_2} = V_1\frac{\frac{\pi D_1^2}{4}}{\frac{\pi D_2^2}{4}} = V_1\left(\frac{D_1}{D_2}\right)^2 \cong 4{,}78\ m/s$$

$$\dot{m}\left(\frac{p_2}{\rho} + \frac{V_2^2}{2} - \frac{p_1}{\rho} - \frac{V_1^2}{2}\right) = -\dot{W} \qquad (p_2 - p_1) = \left[-\frac{\dot{W}}{\dot{m}} + \frac{(V_1^2 - V_2^2)}{2}\right]\rho$$

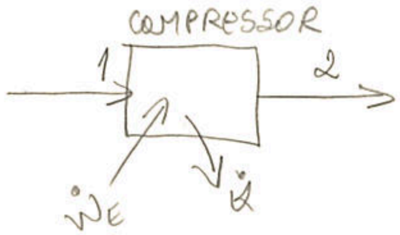
$$(p_2 - p_1) = -\frac{\dot{W}}{\mathcal{Q}} + \left[\frac{V_1^2}{2} - \frac{V_1^2}{2}\left(\frac{D_1}{D_2}\right)^4\right]\rho = -\frac{\dot{W}}{\mathcal{Q}} + \rho\frac{V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right]$$

$$(p_2 - p_1) = -\frac{\dot{W}}{\mathcal{Q}} + \frac{\rho}{2}\left(\frac{4\mathcal{Q}}{\pi D_1^2}\right)^2\left[1 - \left(\frac{D_1}{D_2}\right)^4\right] = -\frac{\dot{W}}{\mathcal{Q}} + \frac{8\rho\mathcal{Q}^2}{\pi^2}\left(\frac{1}{D_1^4} - \frac{1}{D_2^4}\right)$$

$$\boxed{(p_2 - p_1) = \frac{8\rho\mathcal{Q}^2}{\pi^2}\left(\frac{1}{D_1^4} - \frac{1}{D_2^4}\right) - \frac{\dot{W}}{\mathcal{Q}}}$$

$(p_2 - p_1) = -100\,000 + 24627$ $\qquad (p_2 - p_1) \cong -75{,}4\ kPa$ //

$\boxed{11}$ AR $\qquad \dot{M} = 1\ kg/s$



$\dot{W}_E = ?$

$V_1 = 75\ m/s \qquad p_1 = 101\ kPa$

$V_2 = 125\ m/s \qquad T_1 = 300\ K$

$T_2 = 345\ K \qquad p_2 = 200\ kPa$

$q = 18\ kJ/kg \qquad \dot{Q} = q \cdot \dot{M} = 18\ kW$

$Z =$ constante $\qquad c_p = 1004\ J/kg.K$

$$\cancel{\frac{\partial}{\partial t}\int_{VC}\left(u + \frac{V^2}{2} + gz\right)\rho\, dV}^{\simeq 0} + \int_{SC}\left(\overset{u+p\vartheta}{h} + \frac{V^2}{2} + gz\right)\rho\, \vec{V}\cdot\hat{n}\, dA = \dot{Q} + \dot{W}_E$$

$$\int_{SC}\left(h + \frac{V^2}{2}\right)\rho\, \vec{V}\cdot\hat{n}\, dA = \dot{Q} + \dot{W}_E \qquad h = c_p T$$

$$\boxed{\dot{W}_E = \dot{M}\left[c_p(T_2 - T_1) + \frac{(V_2^2 - V_1^2)}{2}\right] - \dot{Q}}$$

$p = \rho R T \qquad p\vartheta = RT$

$\vartheta = \dfrac{RT}{p}$

$$\dot{W}_E = 1\left[1004(345-300) + \frac{(125^2 - 75^2)}{2}\right] - 18000$$

$$= 45180 + 5000 - 18000 = 32180\ W$$

$\vartheta_1 = 0{,}852$

$\vartheta_2 = 0{,}495\ m^3/kg$

$c_\vartheta = 717\ J/kg.K$

$u = c_\vartheta T$

OK

$$\dot{W}_E = \dot{M}\left[c_\vartheta(T_2 - T_1) + p_2\vartheta_2 - p_1\vartheta_1 + \frac{(V_2^2 - V_1^2)}{2}\right] - \dot{Q}$$

$$32265 + 12948 \quad + 5000 \quad -18000 = 32213\ W$$

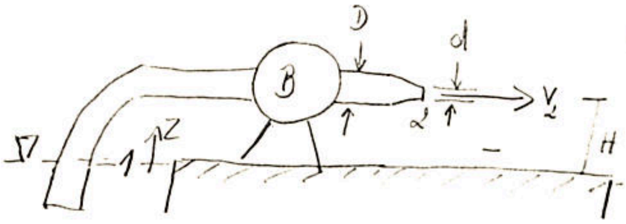
$\dot{W}_E \cong 32{,}2\ kW$

12 ~~4.212~~ OK

$D = 0,075\,m$ $d = 0,025\,m$ $g = 9,8\,m/s^2$

ÁGUA $\dot{W}_B = 10\,kW$ $H = 3\,m$ $Q = ?$

$\rho = 1000\,kg/m^3$ $H_{max} = ?$ (altura máxima a q a água pode ser elevada)



$Z_1 = 0$

$Z_2 = H$ F_j = força do jato s/o banco

$$\int_{SC} \vec{V}\cdot d\vec{A} = 0 \qquad -V_1A_1 + V_2A_2 = 0 \qquad V_1 = V_2\frac{A_2}{A_1} \qquad A_2 = \frac{\pi d^2}{4} \quad A_1 = \frac{\pi D^2}{4}$$

$$V_1 = V_2\frac{\frac{\pi d^2}{4}}{\frac{\pi D^2}{4}} = V_2\left(\frac{d}{D}\right)^2 \qquad \dot{M} = \rho V_2 A_2 = \rho V_2 \frac{\pi d^2}{4}$$

$A_2 = 4,909 \times 10^{-4}\,m^2$

$A_1 = 4,418 \times 10^{-3}\,m^2$

$Q = V \cdot A$

$$\int_{SC}\left(\frac{V^2}{2} + gZ\right)\rho\vec{V}\cdot d\vec{A} = \dot{W}_B \qquad \dot{Q} = 0 \text{ e } h_1 = h_2$$

$$\left(\frac{V_2^2}{2} + gZ_2 - \frac{V_1^2}{2} - gZ_1\right)\dot{M} = \dot{W}_B \qquad \left[\frac{V_2^2}{2} + gZ_2 - \frac{V_2^2}{2}\left(\frac{d}{D}\right)^4\right]\rho V_2 \frac{\pi d^2}{4}$$

$$\left(\frac{Q^2}{2A_2^2} + gZ_2 - \frac{Q^2}{2A_1^2}\right)\rho Q = \dot{W}_B \qquad \boxed{\frac{\rho Q^3}{2}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) + gZ_2\rho Q = \dot{W}_B}$$

$ax^3 + bx^2 + cx + d = 0$ (eq. do 3º grau) p. 168 BRONSTEIN

$$a = \frac{\rho}{2}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) = 2,0492 \times 10^9 \,/\, b = 0 \,/\, c = gZ_2\rho = 29400 \,/\, d = -\dot{W}_B = -10000$$

$$q = -2,441 \times 10^{-6} \qquad p = 4,782 \times 10^{-6} \qquad r = \pm\sqrt{|p|} = \pm 2,187 \times 10^{-3}$$

$$p > 0 \rightarrow \operatorname{senh}\varphi = \frac{q}{r^3} = +233,26 \qquad \varphi = 6,145^\circ$$

$$Q = -2r\operatorname{senh}\frac{\varphi}{3} \longrightarrow Q = 0,0167\,m^3/s \;//\; \text{OK}$$

$$V_1 = \frac{Q}{A_1} \cong 3,78\,m/s \qquad V_2 = \frac{Q}{A_2} \cong 34,0\,m/s \qquad \dot{M} = \rho Q = 16,7\,kg/s$$

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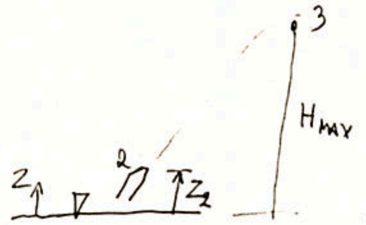
<u>H_{MAX}</u>

$$\int_{SC}\left(\frac{V^2}{2}+gz\right)\rho\vec{V}\cdot d\vec{A}=0$$



$V_3 = 0$

$$\left(\frac{V_2^2}{2}+gz_2-\cancel{\frac{V_3^2}{2}}^{\to 0}-gz_3\right)\dot{M}=0 \qquad \frac{V_2^2}{2}+gH-gH_{MAX}=0$$

$$\boxed{H_{MAX}=H+\frac{V_2^2}{2g}} \qquad H_{MAX}\cong 3+59,=62\ m\ // \quad OK$$

$$\boxed{F_j=\rho V_2^2 A_2} \qquad F_j \cong 567\,N\ //$$