Escoamento Externo:

Capítulo 7

Escoamento Externo

$$TC\begin{cases} Nu_x = f\left(x, \mathrm{Re}_x, \mathrm{Pr}\right) \\ \overline{Nu_x} = f\left(\mathrm{Re}_x, \mathrm{Pr}\right) \end{cases}$$

$$TM\begin{cases} Sh_x = f\left(x, \mathrm{Re}_x, Sc\right) \\ \overline{Sh_x} = f\left(\mathrm{Re}_x, Sc\right) \end{cases}$$

Wilhelm Nusselt 1882 - 1957

Wilhelm Nusselt, a German engineer, was born November 25, 1882, at Nurnberg, Germany. He studied machinery at the Technical Universities of Berlin-Charlottenburg and Munchen and graduated in 1904. He conducted advanced studies in mathematics and physics and became an assistant to O. Knoblauch at the Laboratory for Technical Physics in Munchen. He completed his doctoral thesis on the "Conductivity of Insulating Materials" in 1907, using the "Nusselt Sphere" for his experiments. From 1907 to 1909 he worked as an assistant to Millier in Dresden, and qualified for a Professorship with his work on "Heat and Momentum Transfer in Tubes."

In 1915, Nusselt published his pioneering paper: *The Basic Laws of Heat Transfer*, in which he first proposed the dimensionless groups now known as the principal parameters in the similarity theory of heat transfer. Other famous works were concerned with the film condensation of steam on vertical surfaces, the combustion of pulverized coal and the analogy between heat and mass transfer in evaporation. Found among the primarily mathematical works of Nusselt are the well known solutions for laminar heat transfer in the entrance region of tubes, for heat exchange in cross-flow and the basic theory of regenerators.

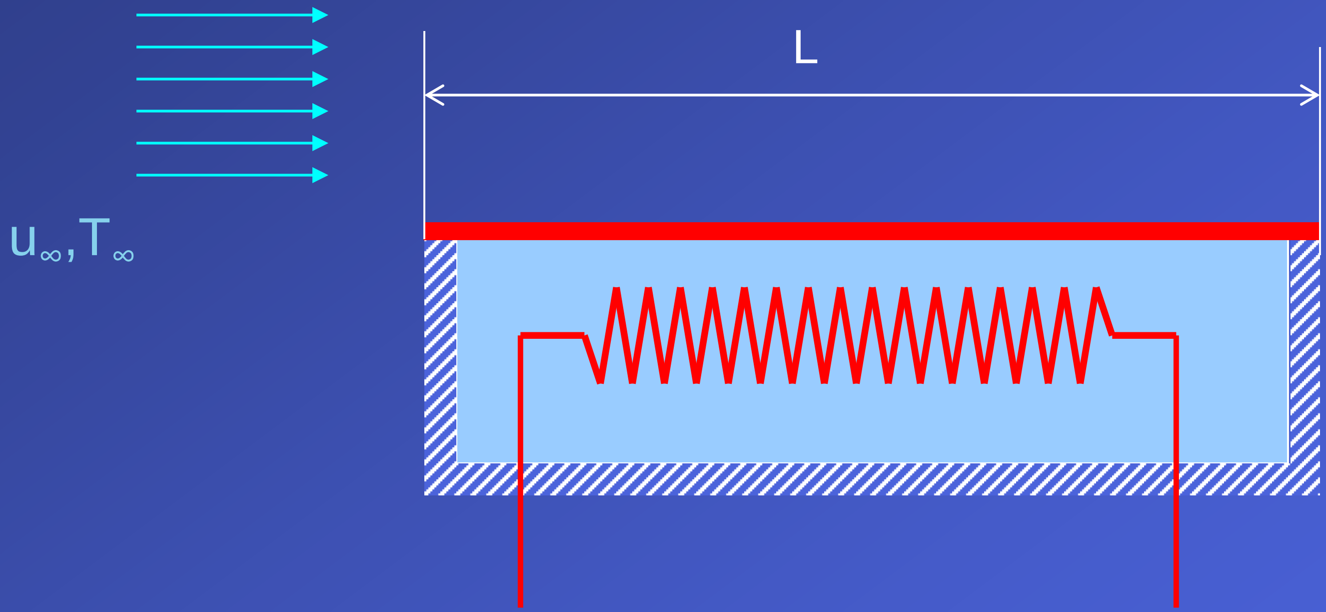
Nusselt was a professor at the Technical Universities of Karlsruhe from 1920-1925 and at Munchen from 1925 until his retirement in 1952. He was awarded the Gauss-Medal and the Grashof Commemorative Medal. Nusselt died in Munchen on September 1, 1957.

Ecoamento Externo

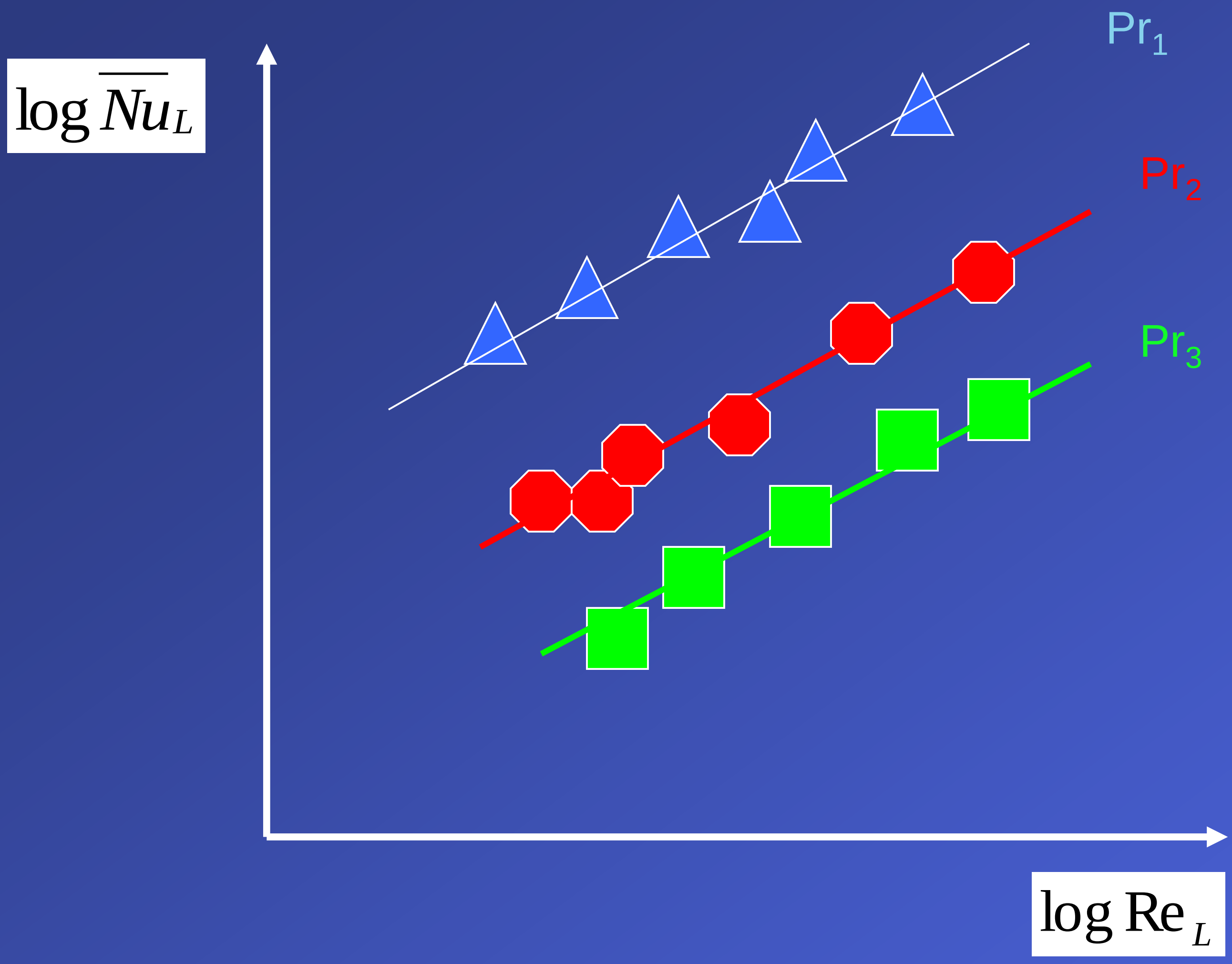
- Obtenção das Relações
  - Métodos Experimentais (Empíricos)
  - Métodos Teóricos ou Numéricos

Escoamento Externo

- Método Empírico

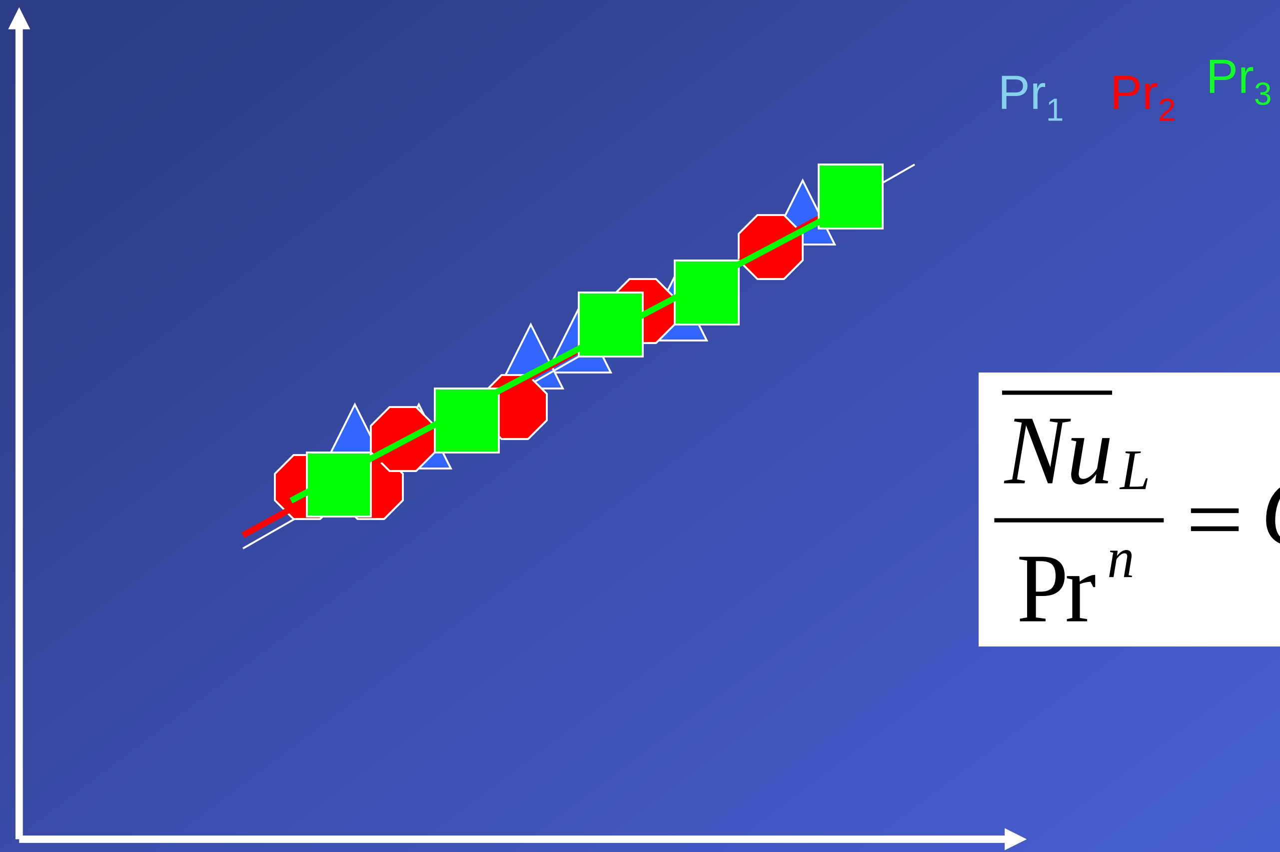


$$i.V = q = \overline{h_L} A_S \left(T_S - T_\infty \right)$$



$$\overline{Nu}_L = C\,\mathrm{Re}_L^m\,\mathrm{Pr}^n$$

$$\log\left(\frac{\overline{Nu}_L}{\mathrm{Pr}^n}\right)$$



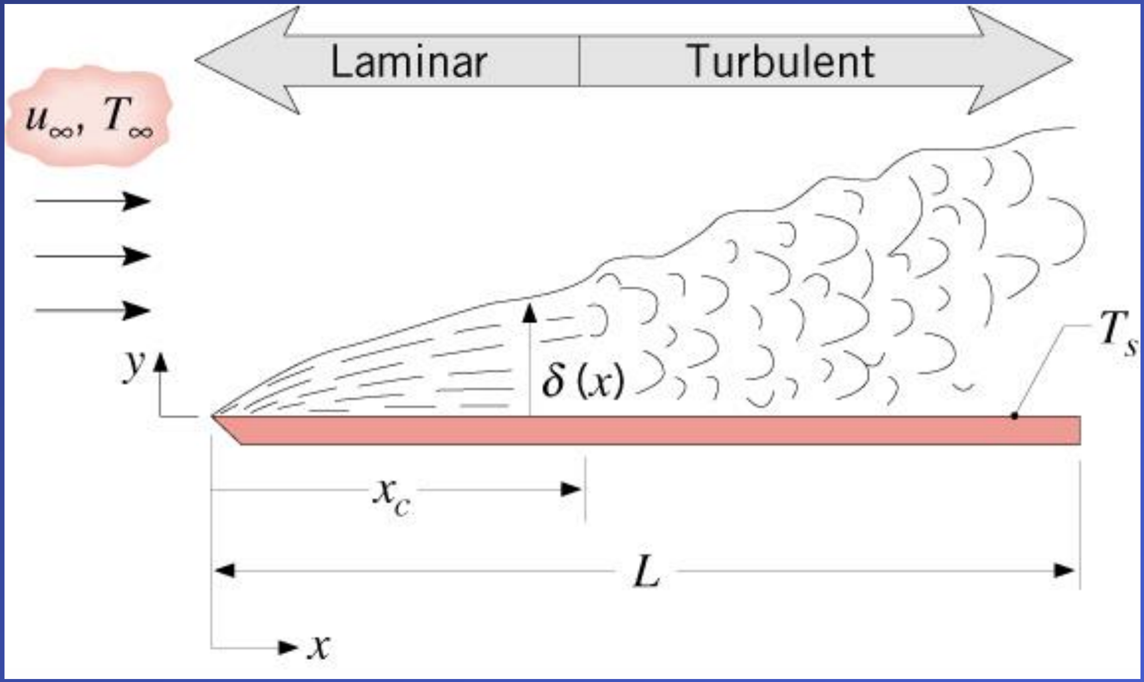
$$\log \mathrm{Re}_L$$

$$\frac{\overline{Nu}_L}{\mathrm{Pr}^n} = C\,\mathrm{Re}_L^m$$

Para TM

$$\overline{Sh}_L = C\,\mathrm{Re}_L^m\,Sc^n$$

Escoamento externo



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$$Re_L = \frac{\rho u_\infty L}{\mu} = \frac{u_\infty L}{\nu}$$

Reynolds crítico, $Re_{x,c}$.

$Re_L < Re_{x,c} \rightarrow$ laminar flow throughout

$$Re_{x,c} \approx 5 \times 10^5.$$

$Re_L > Re_{x,c} \rightarrow$ transition to turbulent flow at $x_c / L \equiv Re_{x,c} / Re_L$

Solução por Similaridade

- Escoamento laminar
- Incompressível
- Regime Estacionário
- Propriedades constantes
- Dissipação viscosa desprezível
- Gradiente de pressão desprezível

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(\frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(+\frac{\partial^2 T}{\partial y^2}\right)$$

$$u\frac{\partial \rho_A}{\partial x}+v\frac{\partial \rho_A}{\partial y}=D_{AB}\left(\frac{\partial^2 \rho_A}{\partial y^2}\right)$$

u e v não dependem de T e ρ

Solução por Similaridade: Solução de Blasius

Blasius, H. (1908), *Grenzschichten in Flussigkeiten mit kleiner Reibung*, Z. Math. Phys. vol 56, pp. 1-37.

Similarity Solution

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial y^2}\right)$$

$$\delta \approx \sqrt{\frac{\nu x}{u_\infty}}$$

$$\eta \approx \frac{y}{\delta(x)} = y\sqrt{\frac{u_\infty}{\nu x}}$$

$$\frac{u}{u_\infty} = \phi\left(\frac{y}{\delta}\right) = \phi(\eta)$$

Substituindo em linhas de corrente

$$u = \frac{\partial \psi}{\partial y}; \quad v = -\frac{\partial \psi}{\partial x}$$

$$f(\eta) = \frac{\psi}{u_\infty \sqrt{\frac{\nu x}{u_\infty}}}$$

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = u_\infty \sqrt{\frac{\nu x}{u_\infty}}\frac{df}{d\eta}\sqrt{\frac{u_\infty}{\nu x}} = u_\infty \frac{df}{d\eta}$$

portanto

$$\frac{u}{u_\infty} = \frac{df}{d\eta}$$

Substituindo na eq. Da QM

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0$$

Condições de contorno: $u(x,0) = v(x,0) = 0 \quad e \quad u(x,\infty) = u_\infty$

$$\left.\frac{df}{d\eta}\right|_{\eta=0} = f'(0) = 0 \qquad \left.\frac{df}{d\eta}\right|_{\eta=\infty} = 1$$

A solução pode ser feita em série ou integração numérica

TABLE 5-1 Function $F(\eta)$ for Boundary Layer Along a Flat Plate at Zero Incidence [3]

$\eta = y\sqrt{U_\infty/\nu x}$	F	$F' = u/U_\infty$	F''
0	0	0	0.33206
0.2	0.00664	0.06641	0.33199
0.4	0.02656	0.13277	0.33147
0.6	0.05974	0.19894	0.33008
0.8	0.10611	0.26471	0.32739
1.0	0.16557	0.32979	0.32301
1.2	0.23795	0.39378	0.31659
1.4	0.32298	0.45627	0.30787
1.6	0.42032	0.51676	0.29667
1.8	0.52952	0.57477	0.28293
2.0	0.65003	0.62977	0.26675
2.2	0.78120	0.68132	0.24835
2.4	0.92230	0.72899	0.22809
2.6	1.07252	0.77246	0.20646
2.8	1.23099	0.81152	0.18401
3.0	1.39682	0.84605	0.16136
3.2	1.56911	0.87609	0.13913
3.4	1.74696	0.90177	0.11788
3.6	1.92954	0.92333	0.09809
3.8	2.11605	0.94112	0.08013
4.0	2.30576	0.95552	0.06424
4.2	2.49806	0.96696	0.05052
4.4	2.69238	0.97587	0.03897
4.6	2.88826	0.98269	0.02948
4.8	3.08534	0.98779	0.02187
5.0	3.28329	0.99155	0.01591
5.2	3.48189	0.99425	0.01134
5.4	3.68094	0.99616	0.00793
5.6	3.88031	0.99748	0.00543
5.8	4.07990	0.99838	0.00365
6.0	4.27964	0.99898	0.00240
6.2	4.47948	0.99937	0.00155
6.4	4.67938	0.99961	0.00098
6.6	4.87931	0.99977	0.00061
6.8	5.07928	0.99987	0.00037
7.0	5.27926	0.99992	0.00022
7.2	5.47925	0.99996	0.00013
7.4	5.67924	0.99998	0.00007
7.6	5.87924	0.99999	0.00004
7.8	6.07923	1.00000	0.00002
8.0	6.27923	1.00000	0.00001
8.2	6.47923	1.00000	0.00001
8.4	6.67923	1.00000	0.00000
8.6	6.87923	1.00000	0.00000
8.8	7.07923	1.00000	0.00000

- with $u/u_\infty = 0.99$ at $\eta = 5.0$,

$$\delta = \frac{5.0}{\left(u_\infty / \nu x\right)^{1/2}} = \frac{5x}{\left(\mathrm{Re}_x\right)^{1/2}}$$

- with $\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu u_\infty \sqrt{u_\infty / \nu x} \left.\frac{d^2 f}{d\eta^2}\right|_{\eta=0}$

and $\left. d^2 f / d\eta^2 \right|_{\eta=0} = 0.332$,

$$C_{f,x} \equiv \frac{\tau_{s,x}}{\rho u_\infty^2 / 2} = 0.664\,\mathrm{Re}_x^{-1/2}$$

Da mesma forma para a equação da energia

$$T^* \equiv \left[(T - T_s) / (T_\infty - T_s) \right] \qquad \frac{d^2T^*}{d\eta^2} + \frac{\Pr}{2} f \frac{dT^*}{d\eta} = 0$$

and $dT^* / d\eta \big|_{\eta=0} = 0.332 \ \Pr^{1/3}$ for $\Pr > 0.6$,

- with $h_x = q_s'' / (T_s - T_\infty) = k \partial T^* / \partial y \big|_{y=0} = k (u_\infty / vx)^{1/2} dT^* / d\eta \big|_{\eta=0}$

$$Nu_x = \frac{h_x x}{k} = 0.332 \ \mathrm{Re}_x^{1/2} \Pr^{1/3}$$

← Pr>0,6

and

$$\frac{\delta}{\delta_t} = \Pr^{1/3}$$

laminar

Da mesma forma para a equação de TM

$$\rho_A^* = \frac{\rho_A - \rho_{A,s}}{\rho_{A,\infty} - \rho_{A,s}}$$

$$\frac{d^2 \rho^*{}_A}{d\eta^2} + \frac{Sc}{2} f \frac{d\rho^*{}_A}{d\eta^2} = 0$$

$$com \quad \left.\frac{d\rho_A^*}{d\eta}\right|_{\eta=0} = 0{,}332\, Sc^{1/3} \quad para \quad Sc \geq 0{,}6$$

$$sendo \quad h_{m,x} = \frac{n_A''}{\rho_{A,S} - \rho_{A,\infty}} = D_{AB} \left.\frac{\partial \rho_A^*}{\partial y}\right|_{y=0} = D_{AB} \sqrt{\frac{u_\infty}{\upsilon x}} \left.\frac{\partial \rho_A^*}{\partial \eta}\right|_{\eta=0}$$

$$Sh_x = \frac{h_{m,x} x}{D_{AB}} = 0{,}332\, \mathrm{Re}_x^{1/2}\, Sc^{1/3} \quad Sc \geq 0{,}6$$ ←

$$\frac{\delta}{\delta_c} = Sc^{1/3}$$

laminar

- Cálculo dos valores médios:

$$\bar{\tau}_{s,x} \equiv \frac{1}{x}\int_0^x \tau_s dx$$

$$\overline{C_{f,x}} = 1.328\ \mathrm{Re}_x^{-1/2}$$

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx = 0{,}332\left(\frac{k_f}{x}\right)\mathrm{Pr}^{1/3}\left(\frac{u_\infty}{\upsilon}\right)^{1/2}\int_0^x \frac{1}{x^{1/2}}dx$$

$$\bar{h}_x = 0{,}664\left(\frac{k_f}{x}\right)\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}\left(\frac{u_\infty}{\upsilon}\right)^{1/2} \qquad \mathrm{Pr} \geq 0{,}6$$

$$\bar{h}_x = 2h_x$$

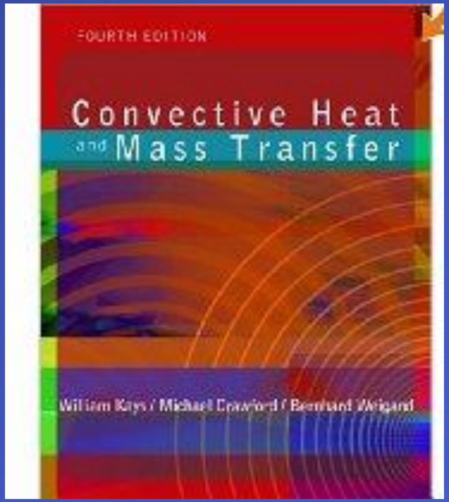
$$ou \qquad \overline{Nu}_x = \frac{\bar{h}_x x}{k_f} = 0{,}664\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} \geq 0{,}6$$

De maneira similar

$$\overline{Sh}_x = \frac{\bar{h}_{m,x} x}{D_{AB}} = 0{,}664\,\mathrm{Re}_x^{1/2}\,Sc^{1/3} \qquad Sc \geq 0{,}6$$

- Temperatura de filme $T_f = \dfrac{T_s + T_\infty}{2}$

Outras correlações



Para Pr<<1 (metais líquidos) , $\delta_t >> \delta$, pode-se considerar velocidade uniforme $u=u_\infty$ (solução de Kays e Crawford, 1980)

$$Nu_x = 0{,}565 Pe_x^{1/2} \quad \Pr \le 0{,}05 \quad e \quad Pe \ge 100$$
$$onde \quad Pe = \mathrm{Re}_x \Pr \quad (número \quad de \quad Peclet)$$

Uma correlação geral para placa isotérmica (Churchill e Ozoe, 1973)

$$Nu_x = \frac{0{,}3387 \mathrm{Re}_x^{1/2} \Pr^{1/3}}{\left[1 + (0{,}0468/\Pr)^{2/3}\right]^{1/4}} \qquad Pe_x > 100$$

$$sendo \quad \overline{Nu}_x = Nu_x$$

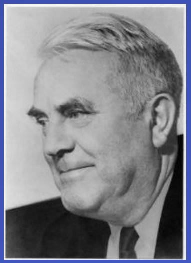
Escoamento Turbulento

- Parametros locais, a partir de dados experimentais (Schlichting, 1968):

$$C_{f,x} = 0{,}0592\,\mathrm{Re}_x^{-1/5}, \quad \mathrm{Re}_x \le 10^7 \quad (15\% \ de \ erro \ para \ \ 10^7 \le \mathrm{Re}_x \le 10^8)$$

$$\delta_{turb} = 0{,}37\,x\,\mathrm{Re}_x^{-1/5} \qquad \delta_{turb} \ne f(\mathrm{Pr} \ \ ou \ \ Sc)$$

$$\delta \approx \delta_t \approx \delta_c$$



Prof. Dr. phil. Hermann Schlichting

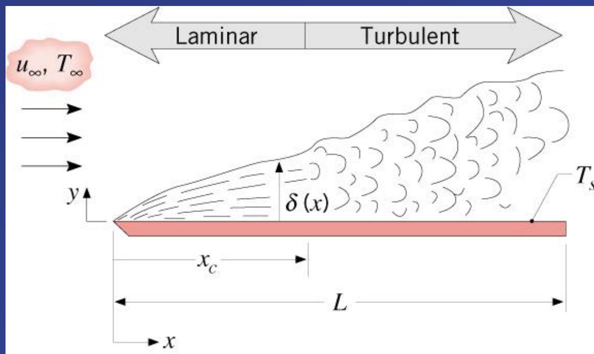
Utilizando a analogia de Reynolds modificada, Cap. 6

$$\frac{C_f}{2} = St\,\mathrm{Pr}^{2/3} \quad (cap. \ \ 6)$$

$$Nu_x = St\,\mathrm{Re}_x\,\mathrm{Pr} = 0{,}0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3} \qquad 0{,}6 < \mathrm{Pr} < 60$$

$$Sh_x = St_m\,\mathrm{Re}_x\,Sc = 0{,}0296\,\mathrm{Re}_x^{4/5}\,Sc^{1/3} \qquad 0{,}6 < Sc < 3000$$

Camada limite mista



$$\bar{h}_L = \frac{1}{L}\left(\int_0^{x_c} h_{1am}\,dx + \int_{x_c}^{L} h_{turb}\,dx\right)$$

Substituindo e assumindo $\mathrm{Re}_{x,c} = 5\times10^{5},$

$$\overline{Nu}_L = \left(0.037\,\mathrm{Re}_L^{4/5} - 871\right)\mathrm{Pr}^{1/3}$$

$$\begin{cases} 0{,}6 < \mathrm{Pr} < 60 \\ 5.10^5 < \mathrm{Re}_L < 10^8 \\ \mathrm{Re}_{x_c} = 5.10^5 \end{cases}$$

Se $x_c \ll L$

$$\overline{Nu}_L = \left(0.037\,\mathrm{Re}_L^{4/5} - 8\ldots\right)\mathrm{Pr}^{1/3}$$

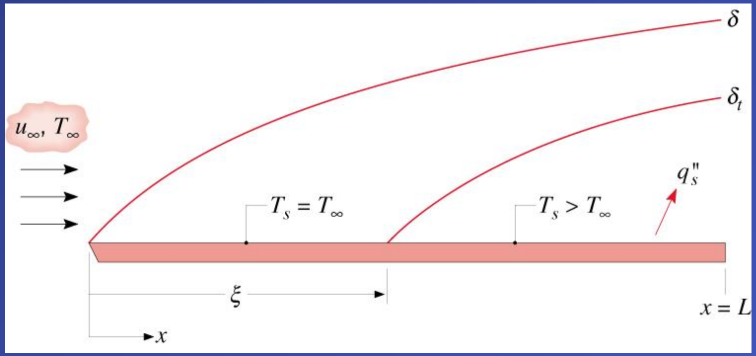
Da mesma forma, para TM

$$\overline{Sh_L} = \left(0.037\ \mathrm{Re}_L^{4/5} - 871\right) Sc^{1/3} \qquad \begin{cases} 0,6 < Sc < 3000 \\ 5.10^5 < \mathrm{Re}_L < 10^8 \\ \mathrm{Re}_{x_c} = 5.10^5 \end{cases}$$

Se $x_c << L$

$$\overline{Sh_L} = \left(0.037\ \mathrm{Re}_L^{4/5} - 8\ldots\right) Sc^{1/3}$$

Special Cases: Unheated Starting Length (USL) and/or Uniform Heat Flux



For both uniform surface temperature (UST) and uniform surface heat flux (USF), the effect of the USL on the local Nusselt number may be represented as follows:

$$Nu_x = \frac{Nu_x\big|_{\xi=0}}{\left[1-(\xi/x)^a\right]^b}$$

$$Nu_x\big|_{\xi=0} = C\,\mathrm{Re}_x^m\,\mathrm{Pr}^{1/3}$$

	Laminar		Turbulent	
	UST	USF	UST	USF
a	3/4	3/4	9/10	9/10
b	1/3	1/3	1/9	1/9
C	0.332	0.453	0.0296	0.0308
m	1/2	1/2	4/5	4/5

- Valores médios para UST:

$$q''_s = h_x (T_s - T_\infty) \qquad q = \bar{h}_L A_s (T_s - T_\infty)$$

$$\overline{Nu}_L = \overline{Nu}_L \Big|_{\xi=0} \frac{L}{(L-\xi)} \left[1 - (\xi/L)^{(p+1)/(p+2)}\right]^{p/(p+1)}$$

$p = 2$ for laminar flow throughout

$p = 8$ for turbulent flow throughout

$\bar{h}_L \rightarrow$ numerical integration for laminar/turbulent flow

$$\bar{h}_L = \frac{1}{L}\left[\int_\xi^{x_c} h_{1am}\, dx + \int_{x_c}^{L} h_{turb}\, dx\right]$$

- USF:

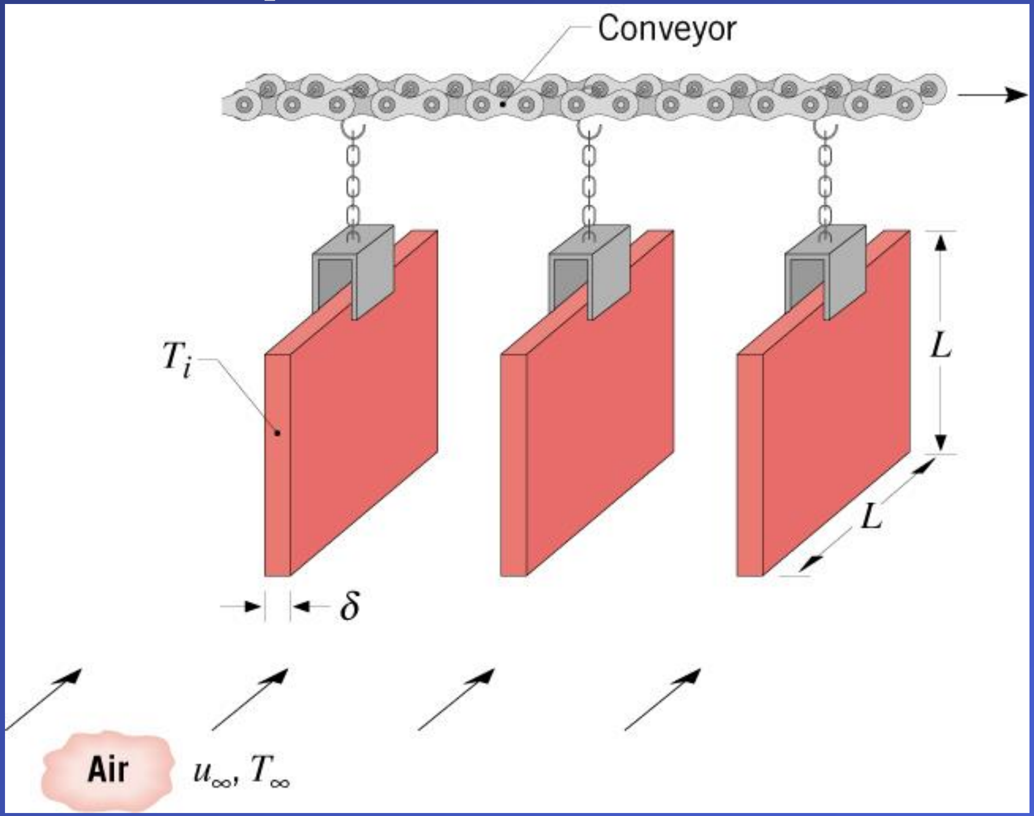
$$T_s = T_\infty + \frac{q''_s}{h_x} \qquad q = q''_s A_s$$

- Treatment of Non-Constant Property Effects:

Evaluate properties at the film temperature.

$$T_f = \frac{T_s + T_\infty}{2}$$

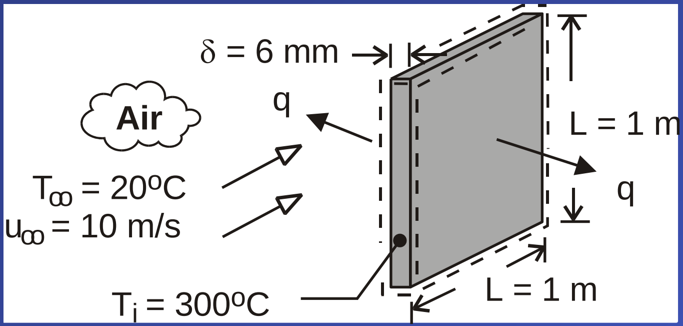
Problem 7.24: Convection cooling of steel plates on a conveyor by air in parallel flow.



KNOWN: Plate dimensions and initial temperature. Velocity and temperature of air in parallel flow over plates.

FIND: Initial rate of heat transfer from plate. Rate of change of plate temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible radiation, (2) Negligible effect of conveyor velocity on boundary layer development, (3) Plates are isothermal, (4) Negligible heat transfer from edges of plate, (5) $Re_{x,c} = 5\times10^5$, (6) Constant properties.

PROPERTIES: *Table A-1*, AISI 1010 steel (573K): k_p = 49.2 W/m·K, c = 549 J/kg·K, ρ = 7832 kg/m^3. *Table A-4*, Air (p = 1 atm, T_f = 433K): ν = 30.4 × 10^{-6} m^2/s, k = 0.0361 W/m·K, Pr = 0.688.

ANALYSIS: The initial rate of heat transfer from a plate is

$$q = 2\bar{h}A_s\left(T_i - T_\infty\right) = 2\bar{h}L^2\left(T_i - T_\infty\right)$$

With $Re_L = u_\infty L/\nu = 10\,m/s \times 1m/30.4\times10^{-6}m^2/s = 3.29\times10^5$, flow is laminar over the entire surface.

Hence,

$$\overline{Nu}_L = 0.664\,Re_L^{1/2}\,Pr^{1/3} = 0.664\left(3.29\times10^5\right)^{1/2}\left(0.688\right)^{1/3} = 336$$

$$\bar{h} = \left(k/L\right)\overline{Nu}_L = \left(0.0361\,W/m\cdot K/1m\right)336 = 12.1\,W/m^2\cdot K$$

$$q = 2\times12.1\,W/m^2\cdot K\left(1m\right)^2\left(300-20\right)°C = 6780\,W$$

Performing an energy balance at an instant of time for a control surface about the plate, $-\dot{E}_{out} = \dot{E}_{st}$,

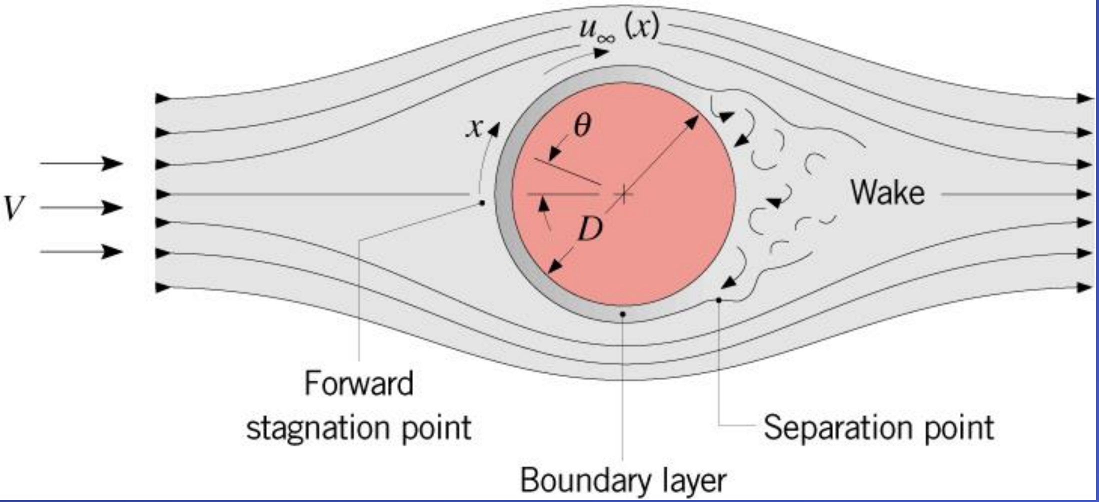
$$\rho \delta L^2 c \left.\frac{dT}{dt}\right|_i = -\bar{h}\, 2L^2 \left(T_i - T_\infty\right)$$

$$\left.\frac{dT}{dt}\right|_i = -\frac{2\left(12.1\,W/m^2\cdot K\right)(300-20)^\circ C}{7832\ kg/m^3 \times 0.006m \times 549\,J/kg\cdot K} = -0.26^\circ C/s$$

COMMENTS: (1) With $Bi = \bar{h}(\delta/2)/k_p = 7.4 \times 10^{-4}$, use of the lumped capacitance method is appropriate.

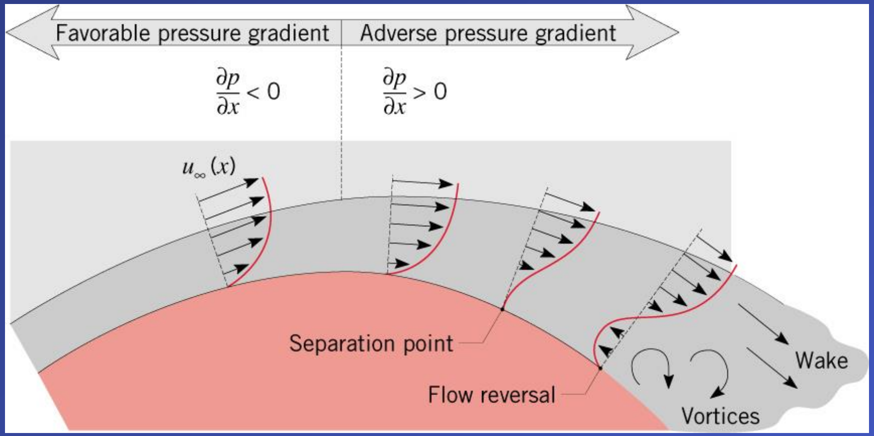
(2) Despite the large plate temperature and the small convection coefficient, if adjoining plates are in close proximity, radiation exchange with the surroundings will be small and the assumption of negligible radiation is justifiable.

Cilindro com escoamento transversal



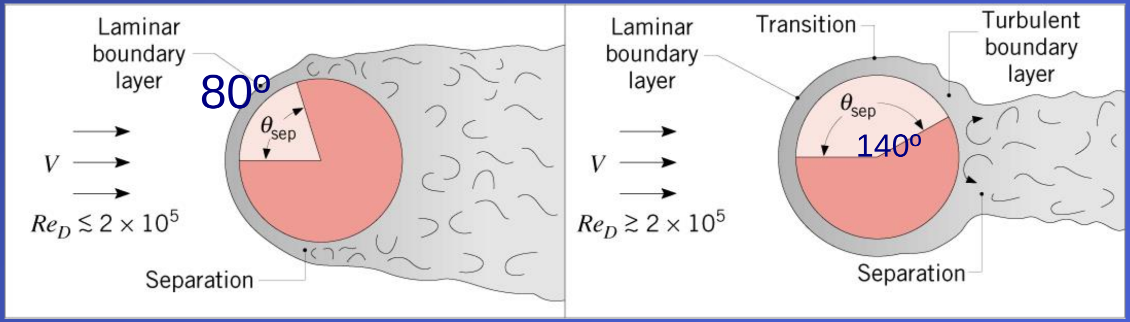
- Stagnation point: Location of zero velocity $(u_\infty = 0)$ and maximum pressure.
- Followed by boundary layer development under a favorable pressure gradient $(dp/dx < 0)$ and hence acceleration of the free stream flow $(du_\infty/dx > 0)$.
- As the rear of the cylinder is approached, the pressure must begin to increase. Hence, there is a minimum in the pressure distribution, *p(x)*, after which boundary layer development occurs under the influence of an adverse pressure gradient $(dp/dx > 0,\ du_\infty/dx < 0)$.

– Separation occurs when the velocity gradient $du/dy\big|_{y=0}$ reduces to zero



and is accompanied by flow reversal and a downstream wake.

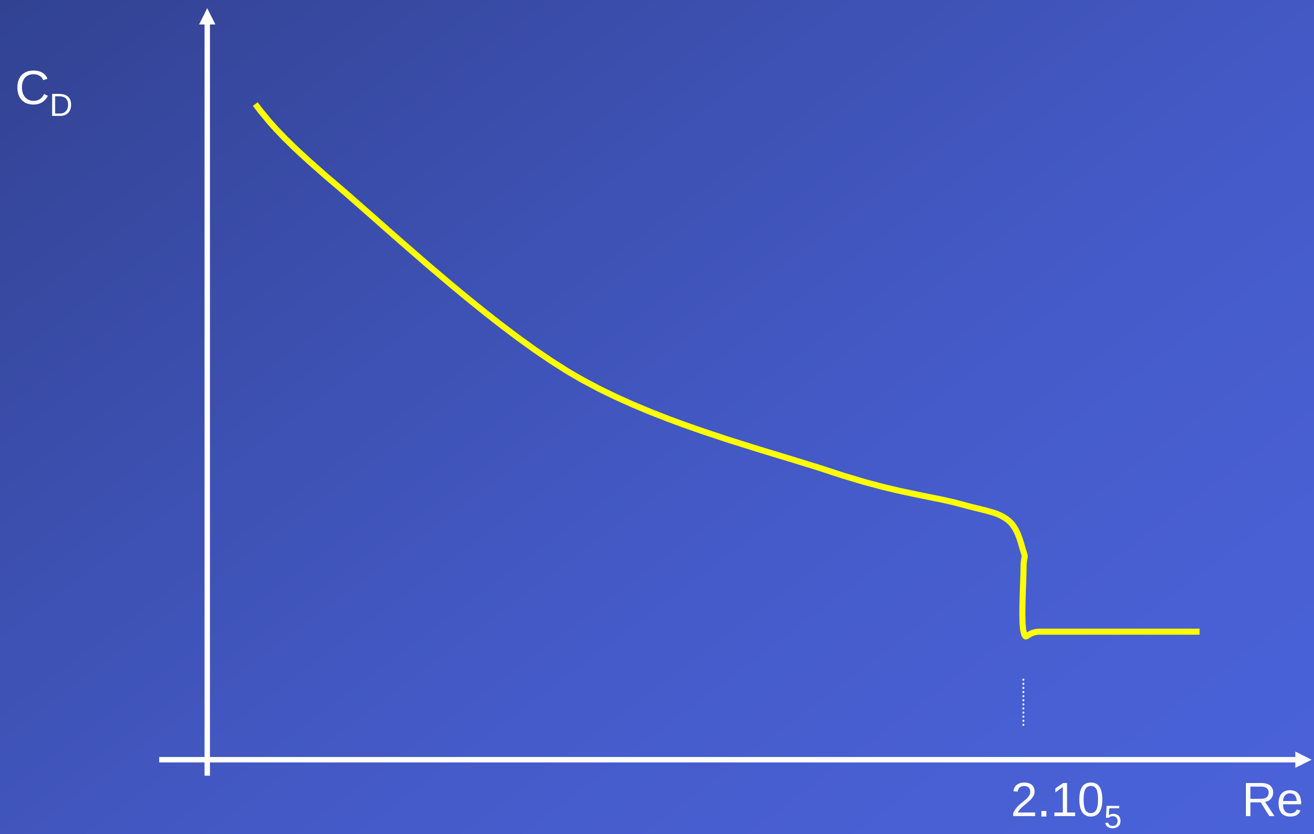
– Location of separation depends on boundary layer transition.

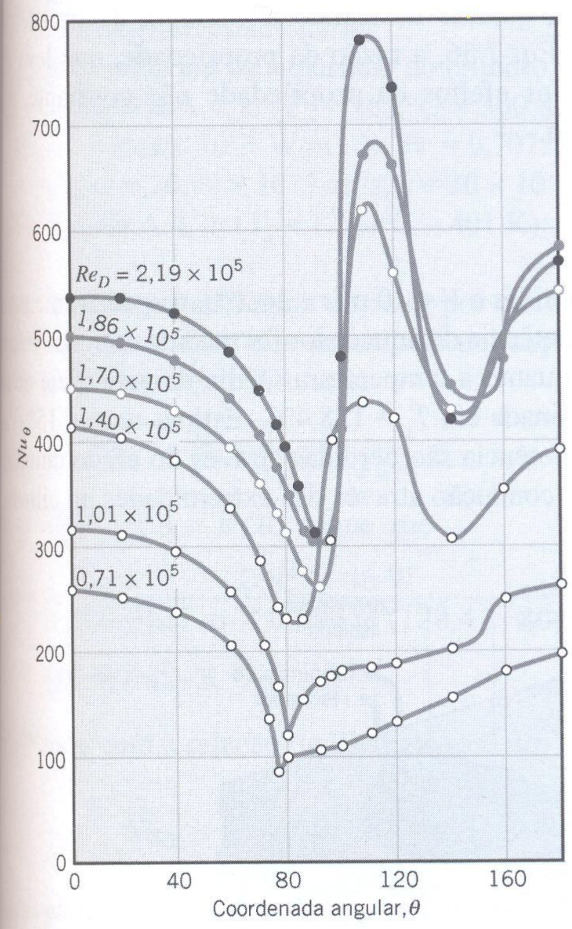


$$\mathrm{Re}_D \equiv \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

The dimensionless form of the drag force is

$$C_D = \frac{F_D}{A_f\left(\rho V^2/2\right)} \rightarrow \text{Figure 7.8}$$





7.9 Número de Nusselt local para fluxo de ar normal a um cilindro circular. Adaptado com permissão de W. H. Giedt, *Trans. ASME*, 71, 375, 1949.

– The Average Nusselt Number $\left(\overline{Nu}_D \equiv \bar{h}D/k\right)$:

$$\overline{Nu}_D = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3}$$

Tabela 7.2 Constantes da Eq. 7.55b para o cilindro circular em corrente cruzada [14,15]

Re_D	C	m
0,4–4	0,989	0,330
4–40	0,911	0,385
40–4000	0,683	0,466
4000–40.000	0,193	0,618
40.000–400.000	0,027	0,805

– Churchill and Bernstein Correlation:

$$\overline{Nu}_D = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$\mathrm{Re}_D\mathrm{Pr}>0{,}2$

– Cylinders of Noncircular Cross Section:

$$\overline{Nu}_D = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3}$$

$$C, m \rightarrow \text{Table 7.3}$$

Tabela 7.3 Constantes da Eq. 7.55b para cilindros não-circulares em escoamento cruzado de um gás [16]

Geometria	Re_D	C	m
Quadrado			
$V \rightarrow$ ◇ D	5×10^3–10^5	0,246	0,588
$V \rightarrow$ □ D	5×10^3–10^5	0,102	0,675
Hexágono			
$V \rightarrow$ ⬡ D	5×10^3–$1{,}95 \times 10^4$	0,160	0,638
	$1{,}95 \times 10^4$–10^5	0,0385	0,782
$V \rightarrow$ ⬡ D	5×10^3–10^5	0,153	0,638
Placa vertical			
$V \rightarrow$ ▯ D	4×10^3–$1{,}5 \times 10^4$	0,228	0,731

- Correlação de Zhukauskas

$$Nu_D = C\,\mathrm{Re}^m\,\mathrm{Pr}^n\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_S}\right)^{1/4}$$

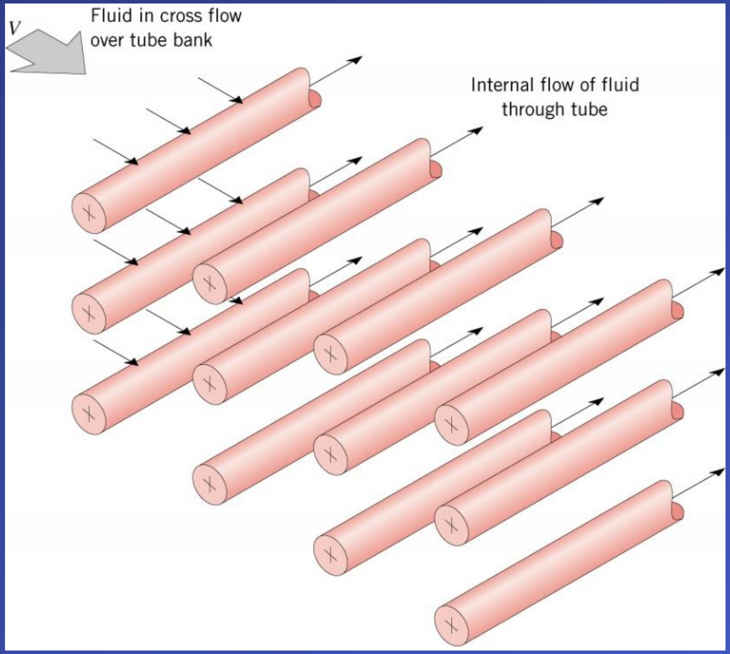
$$\begin{bmatrix} 0,7 < \mathrm{Pr} < 500 \\ 1 < \mathrm{Re}_D < 10^6 \end{bmatrix} \qquad \begin{cases} \mathrm{Pr} \leq 10 \rightarrow n = 0,37 \\ \mathrm{Pr} > 10 \rightarrow n = 0,36 \end{cases}$$

Todas as propriedades avaliadas em T_∞, exceto Pr_s avaliada em T_s

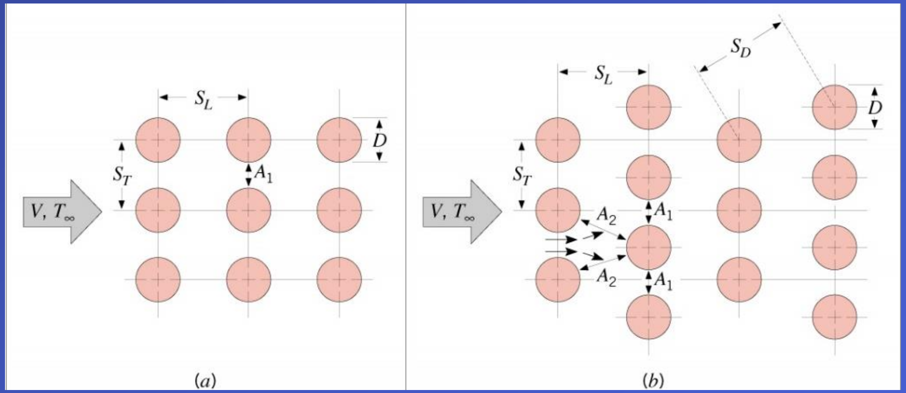
Tabela 7.4 Constantes da Eq. 7.56 para cilindro circular em escoamento cruzado [17]

Re_D	C	m
1–40	0,75	0,4
40–1000	0,51	0,5
10^3–2×10^5	0,26	0,6
2×10^5–10^6	0,076	0,7

Feixe de tubos com escoamento transversal



- Alinhado e Quincôncio:



Alinhado:

$$V_{max} = \frac{S_T}{S_T - D} V$$

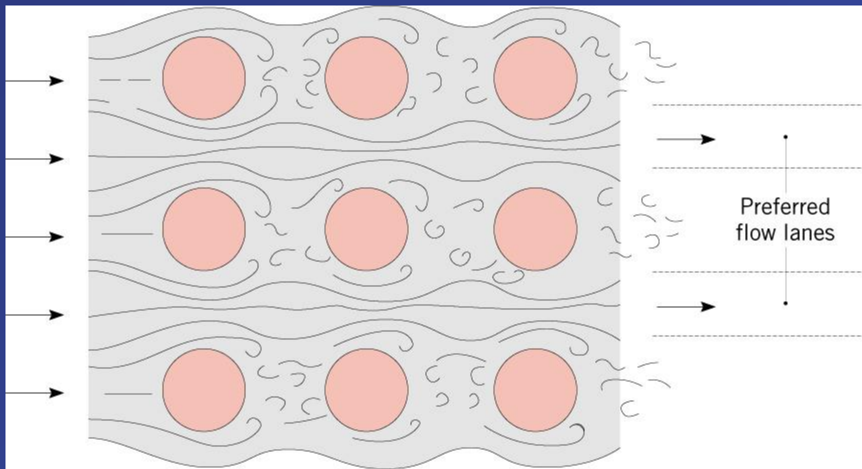
Quincôncio:

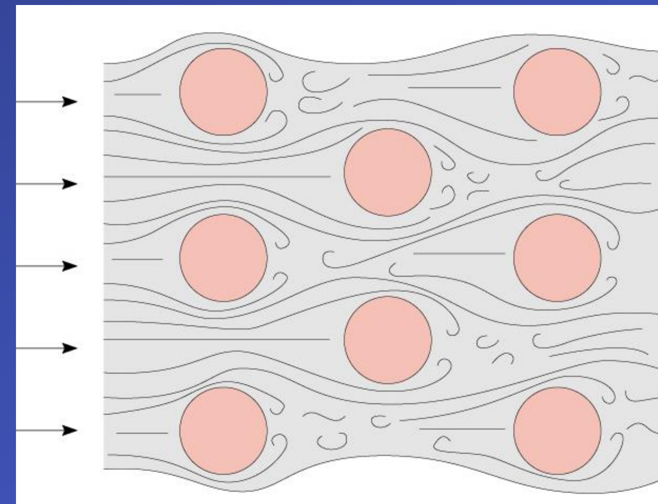
$$V_{max} = \frac{S_T}{S_T - D} V \quad \text{if} \quad 2\left(S_D - D\right) \geq \left(S_T - D\right)$$

or,

$$V_{max} = \frac{S_T}{2\left(S_D - D\right)} V \quad \text{if} \quad 2\left(S_D - D\right) \leq \left(S_T - D\right)$$

- Flow Conditions:





- Average Nusselt Number for an Isothermal Array:

$$\overline{Nu}_D = C_2 \left[C \, \mathrm{Re}_{D,\max}^{m} \, \mathrm{Pr}^{0.36} \left(\mathrm{Pr}/\mathrm{Pr}_s \right)^{1/4} \right]$$

$$C, m \rightarrow \text{Table 7.7}$$

$$C_2 \rightarrow \text{Table 7.8}$$

All properties are evaluated at $\left(T_i + T_o\right)/2$ except for Pr_s.

- Fluid Outlet Temperature (T_o) :

$$\frac{T_s - T_o}{T_s - T_i} = exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right)$$

$$N = N_T \times N_L$$

What may be said about T_o as $N \to \infty$?

- Total Heat Rate:

$$q = \bar{h} A_s \Delta T_{\ell m}$$

$$A_s = N(\pi D L)$$

$$\Delta T_{\ell m} = \frac{(T_s - T_i) - (T_s - T_o)}{\ell n\left(\frac{T_s - T_i}{T_s - T_o}\right)}$$

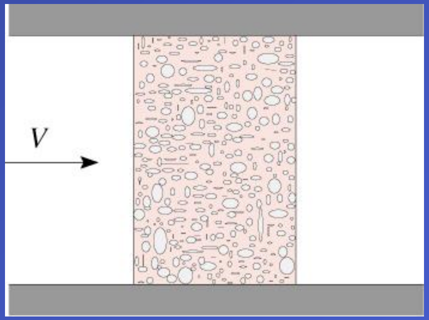
- Pressure Drop:

$$\Delta p = N_L \chi \left(\frac{\rho V_{\max}^2}{2}\right) f$$

$\chi, f \to$ Figures 7.13 and 7.14

The Sphere and Packed Beds

- Flow over a sphere
  - Boundary layer development is similar to that for flow over a cylinder, involving transition and separation.
  - $\overline{Nu}_D = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right)Pr^{0.4}\left(\mu/\mu_s\right)^{1/4}$
  - What are the limiting values of the Nusselt number and the convection coefficient for slow flows over small spheres.
  - $C_D \rightarrow$ Figure 7.8
- Gas Flow through a Packed Bed



  - Flow is characterized by tortuous paths through a bed of fixed particles.
  - Large surface area per unit volume renders configuration desirable for the transfer and storage of thermal energy.

– For a packed bed of spheres:

$$\varepsilon \bar{j}_H = 2.06\ \mathrm{Re}_D^{-0.575}$$

$\varepsilon \rightarrow$ void fraction $(0.3 < \varepsilon < 0.5)$

– $$q = \bar{h} A_{p,t} \Delta T_{\ell m}$$

$A_{p,t} \rightarrow$ total surface area of particles

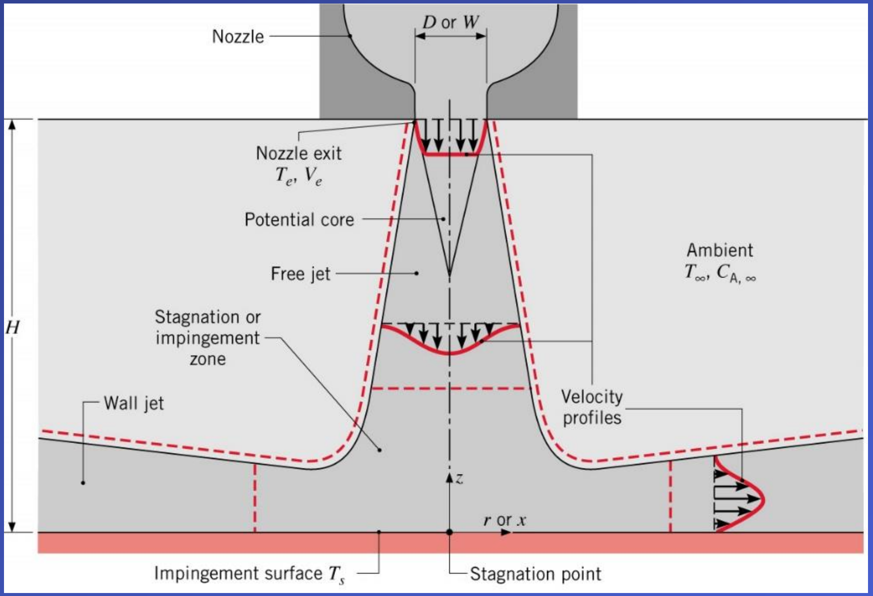
– $$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\bar{h} A_{p,t}}{\rho V A_{c,b} c_p} \right)$$

$A_{c,b} \rightarrow$ cross-sectional area of bed

Gas Jet Impingement

- Characterized by large convection coefficients and used for cooling and heating in numerous manufacturing, electronic and aeronautic applications.

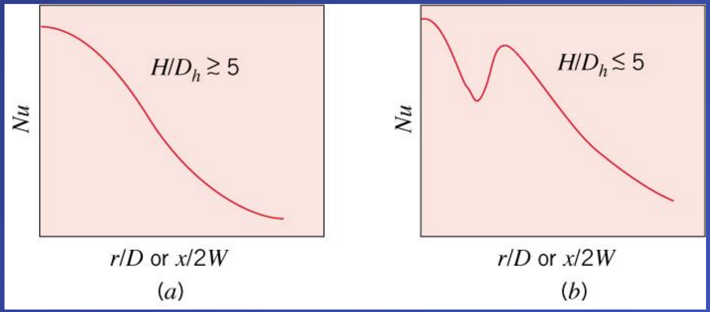
- Flow and Heat Transfer for a Circular or Rectangular Jet:



Significant Features:

- Mixing and velocity profile development in the free jet.
- Stagnation point and zone.
- Velocity profile development in the wall jet.

- Local Nusselt number distribution:



- Average Nusselt number:

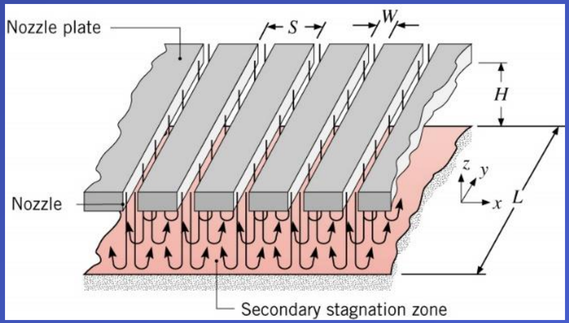
$$\overline{Nu} = \frac{\bar{h} D_h}{k} = f\left(Re, Pr, A_r, H/D_h\right)$$

$$Re = \frac{V_e D_h}{\nu}, A_r \text{ from Fig. 7.17}$$

Correlations $\rightarrow$ Section 7.7.2

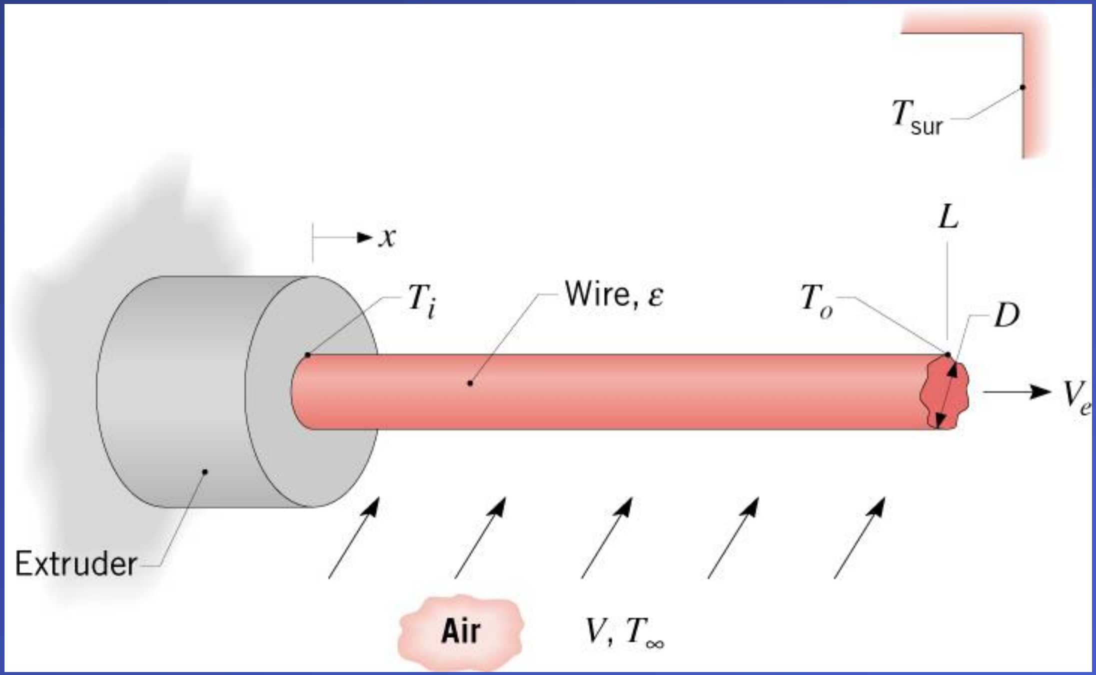
- Jet Arrays

  Slot Jets



- What is the nature and effect of jet interactions and discharge conditions?
- Nusselt number correlations for arrays of circular and slot jets → Section 7.7.2.

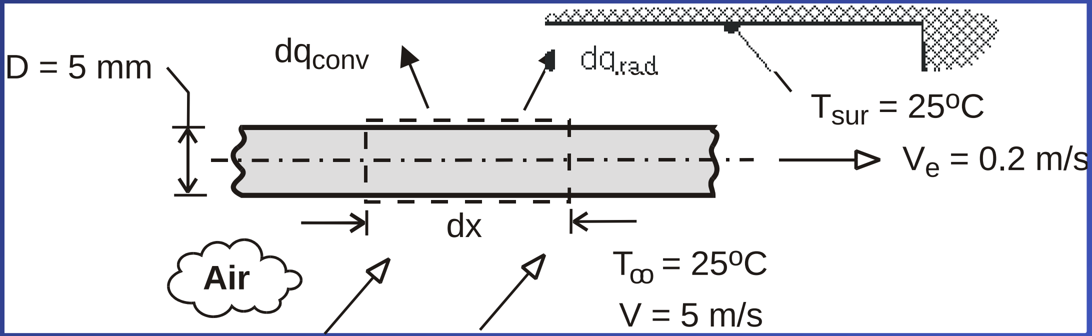
Problem 7.63: Cooling of extruded copper wire by convection and radiation.



KNOWN: Velocity, diameter, initial temperature and properties of extruded wire. Temperature and velocity of air. Temperature of surroundings.

FIND: (a) Differential equation for temperature distribution T(x) along the wire, (b) Exact solution for negligible radiation and corresponding value of temperature at prescribed length (x = L = 5m) of wire, (c) Effect of radiation on temperature of wire at prescribed length. Effect of wire velocity and emissivity on temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible variation of wire temperature in radial direction, (2) Negligible effect of axial conduction along the wire, (3) Constant properties, (4) Radiation exchange between small surface and large enclosure, (5) Motion of wire has a negligible effect on the convection coefficient (V_e << V).

PROPERTIES: Copper: $\rho = 8900\ \text{kg/m}^3$, $c_p = 400\ \text{J/kg}\cdot\text{K}$, $\varepsilon = 0.55$. Air: $k = 0.037\ \text{W/m}\cdot\text{K}$, $\nu = 3\times10^{-5}\ \text{m}^2/\text{s}$, $\text{Pr} = 0.69$.

ANALYSIS: (a) Applying conservation of energy to a stationary control surface, through which the wire moves, steady-state conditions exist and $\dot{E}_{in} - \dot{E}_{out} = 0$.

Hence, with *inflow* due to *advection* and *outflow* due to *advection*, *convection* and *radiation*,

$$\rho V_e A_c c_p T - \rho V_e A_c c_p (T + dT) - dq_{conv} - dq_{rad} = 0$$

$$-\rho V_e\left(\pi D^2/4\right)c_p dT - \pi D dx\left[\bar{h}\left(T-T_\infty\right)+\varepsilon\sigma\left(T^4-T_{sur}^4\right)\right]=0$$

$$\frac{dT}{dx}=-\frac{4}{\rho V_e D c_p}\left[\bar{h}\left(T-T_\infty\right)+\varepsilon\sigma\left(T^4-T_{sur}^4\right)\right] \qquad (1) \quad <$$

Alternatively, if the control surface is fixed to the wire, conditions are transient and the energy balance is of the form, $-\dot{E}_{out} = \dot{E}_{st}$, or

$$-\pi D\, dx\left[\bar{h}\left(T-T_\infty\right)+\varepsilon\sigma\left(T^4-T_{sur}^4\right)\right]=\rho\left(\frac{\pi D^2}{4}dx\right)c_p\frac{dT}{dt}$$

$$\frac{dT}{dt}=-\frac{4}{\rho D c_p}\left[\bar{h}\left(T-T_\infty\right)+\varepsilon\sigma\left(T^4-T_{sur}^4\right)\right]$$

Dividing the left- and right-hand sides of the equation by dx/dt and $V_e = dx/dt$, respectively, Eq. (1) is obtained.

(b) Neglecting radiation, separating variables and integrating, Eq. (1) becomes

$$\int_{T_i}^{T}\frac{dT}{T-T_\infty}=-\frac{4\bar{h}}{\rho V_e D c_p}\int_0^x dx$$

$$\ln\left(\frac{T-T_\infty}{T_i-T_\infty}\right)=-\frac{4\bar{h}x}{\rho V_e D c_p}$$

$$T=T_\infty+\left(T_i-T_\infty\right)\exp\left(-\frac{4\bar{h}x}{\rho V_e D c_p}\right) \qquad (2) \quad <$$

With $Re_D = VD/\nu = 5\,m/s \times 0.005\,m / 3\times10^{-5}\,m^2/s = 833$, the Churchill-Bernstein correlation yields

$$\overline{Nu}_D = 0.3 + \frac{0.62(833)^{1/2}(0.69)^{1/3}}{\left[1+(0.4/0.69)^{2/3}\right]^{1/4}}\left[1+\left(\frac{833}{282,000}\right)^{5/8}\right]^{4/5} = 14.4$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.037\,W/m\cdot K}{0.005m}14.4 = 107\,W/m^2\cdot K$$

Hence, applying Eq. (2) at x = L,

$$T_o = 25°C + (575°C)\exp\left(-\frac{4\times107\,W/m^2\cdot K\times5m}{8900\,kg/m^3\times0.2\,m/s\times0.005m\times400\,J/kg\cdot K}\right)$$

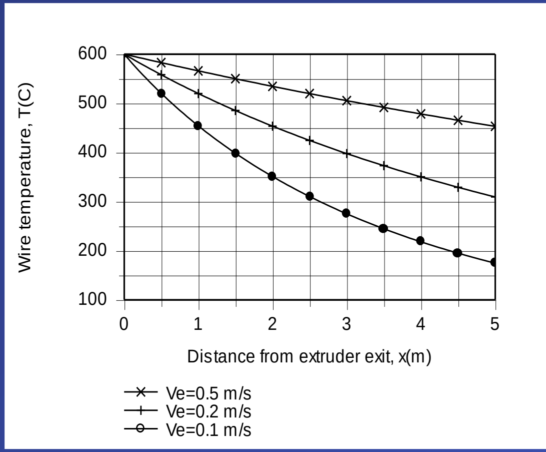
$$T_o = 340°C$$ <

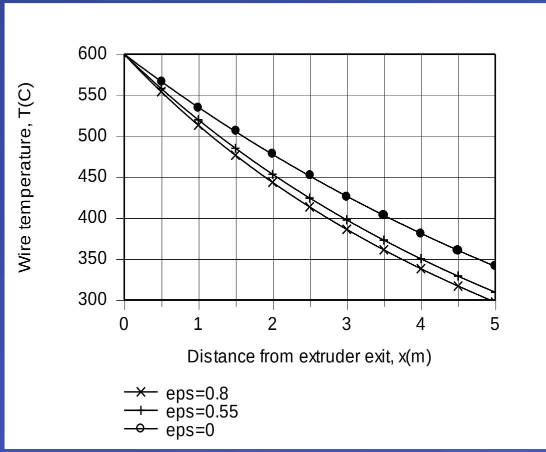
(c) Numerically integrating from x = 0 to x = L = 5.0m, we obtain

$$T_o = 309°C$$ <

Hence, radiation makes a discernable contribution to cooling of the wire.

Parametric conditions reveal the following distributions.





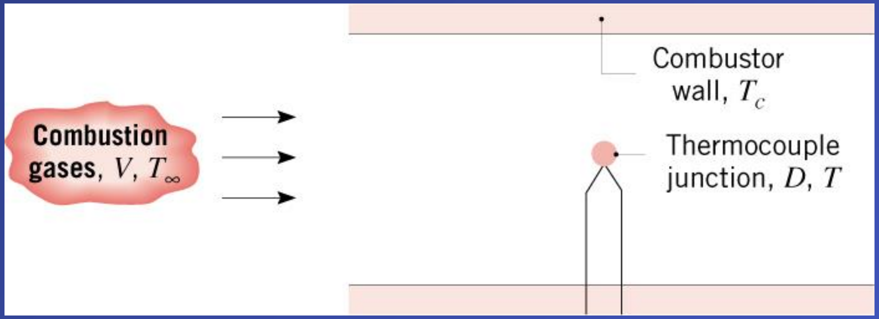
The speed with which the wire is drawn from the extruder has a significant influence on the temperature distribution. The temperature decay decreases with increasing V_e due to the increasing effect of advection on energy transfer in the x direction.

The effect of the surface emissivity is less pronounced, although, as expected, the temperature decay becomes more pronounced with increasing ε.

COMMENTS: (1) A critical parameter in wire extrusion processes is the *coiling temperature*, that is, the temperature at which the wire may be safely coiled for subsequent storage or shipment. The larger the production rate (V_e), the longer the cooling distance needed to achieve a desired coiling temperature.

(2) Cooling may be enhanced by increasing the cross-flow velocity, and the specific effect of V may also be explored.

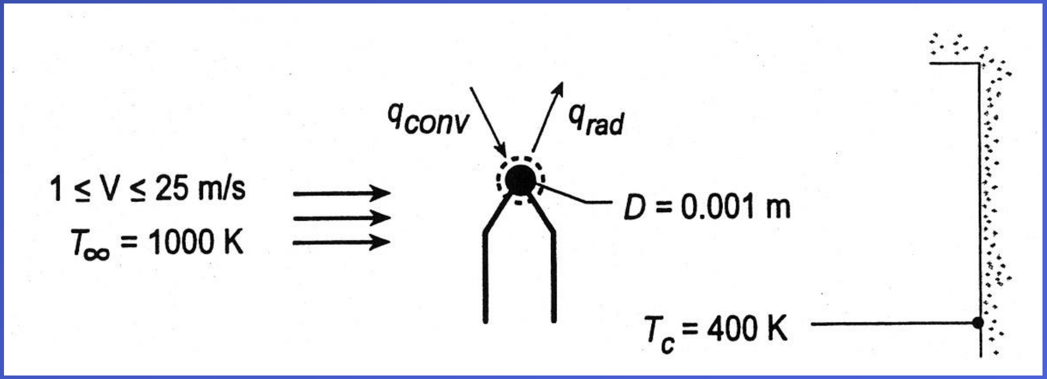
Problem: 7.78 Measurement of combustion gas temperature with a spherical thermocouple junction.



KNOWN: Velocity and temperature of combustion gases. Diameter and emissivity of thermocouple junction. Combustor temperature.

FIND: (a) Time to achieve 98% of maximum thermocouple temperature rise for negligible radiation, (b) Steady-state thermocouple temperature, (c) Effect of gas velocity and thermocouple emissivity on measurement error.

SCHEMATIC:



ASSUMPTIONS: (1) Validity of lumped capacitance analysis, (2) Constant properties, (3) Negligible conduction through lead wires, (4) Radiation exchange between small surface and a large enclosure (parts b and c).

PROPERTIES: Thermocouple: $0.1 \le \varepsilon \le 1.0$, k = 100 W/m·K, c = 385 J/kg·K, ρ = 8920 kg/m^3; Gases: k = 0.05 W/m·K, $\nu = 50 \times 10^{-6}$ m^2/s, Pr = 0.69.

ANALYSIS: (a) If the lumped capacitance analysis may be used, it follows from Equation 5.5 that

$$t = \frac{\rho V c}{\bar{h} A_s} \ln \frac{T_i - T_\infty}{T - T_\infty} = \frac{D \rho c}{6 \bar{h}} \ln(50).$$

Neglecting the viscosity ratio correlation for variable property effects, use of V = 5 m/s with the Whitaker correlation yields

$$\overline{Nu}_D = \left(\bar{h} D / k\right) = 2 + \left(0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\right) Pr^{0.4} \qquad Re_D = \frac{VD}{\nu} = \frac{5\,\text{m/s}\,(0.001\,\text{m})}{50 \times 10^{-6}\,\text{m}^2/\text{s}} = 100$$

$$\bar{h} = \frac{0.05\,\text{W/m}\cdot\text{K}}{0.001\,\text{m}} \left[2 + \left(0.4(100)^{1/2} + 0.06(100)^{2/3}\right)(0.69)^{0.4}\right] = 328\,\text{W/m}^2\cdot\text{K}$$

Since $Bi = \bar{h}(r_o/3)/k = 5.5 \times 10^{-4}$, the lumped capacitance method may be used.

$$t = \frac{0.001\,\text{m}\left(8920\,\text{kg/m}^3\right) 385\,\text{J/kg}\cdot\text{K}}{6 \times 328\,\text{W/m}^2\cdot\text{K}} \ln(50) = 6.83\text{s}$$

(b) Performing an energy balance on the junction, $q_{conv} = q_{rad}$.

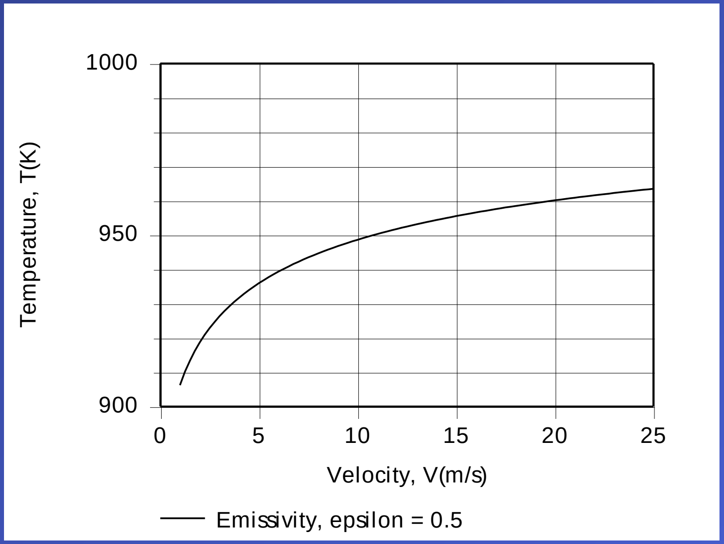
Hence, evaluating radiation exchange from Equation 1.7 and with ε = 0.5,

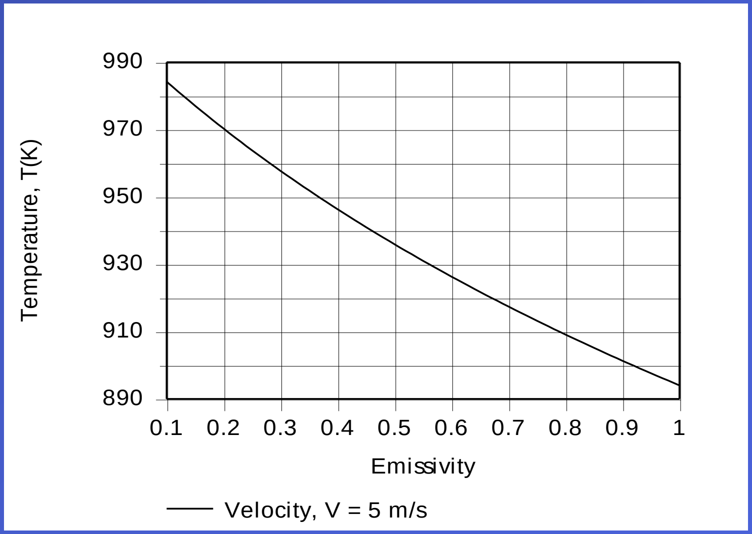
$$\bar{h}A_s\left(T_\infty - T\right) = \varepsilon A_s \sigma\left(T^4 - T_c^4\right)$$

$$\left(1000 - T\right)K = \frac{0.5 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4}{328\,W/m^2 \cdot K}\left[T^4 - \left(400\right)^4\right]K^4$$

T = 936 K

Parametric calculations to determine the effects of V and ε yield the following results:





Since the temperature recorded by the thermocouple junction increases with increasing V and decreasing ε, the measurement error, T_∞ - T, decreases with increasing V and decreasing ε. The error is due to net radiative transfer from the junction (which depresses T) and hence should decrease with decreasing ε.

For a prescribed heat loss, the temperature difference (T_∞ - T) decreases with decreasing convection resistance, and hence with increasing h(V).

COMMENTS: To infer the actual gas temperature (1000 K) from the measured result (936 K), correction would have to be made for radiation exchange with the cold surroundings.

What measures may be taken to reduce the error associated with radiation effects?