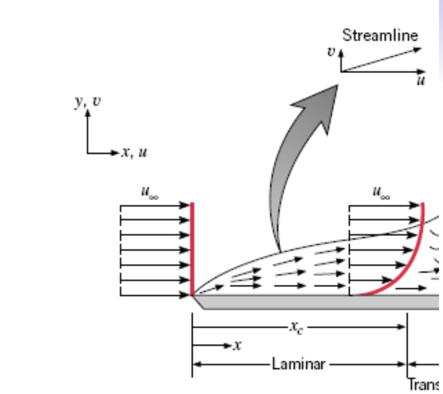
Aproximações da Camada limite:
Equações Gerais



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_x$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g_y$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu\left\{ \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2\left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] \right\} + \dot{q}$$

$$u\frac{\partial C_A}{\partial x} + v\frac{\partial C_A}{\partial y} = D_{AB}\left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} \right) + \dot{N}_A$$

Aproximações da Camada limite

$$X = Y = \dot{q} = \dot{N}_A = 0$$

$$u >> v$$

$$\frac{\partial u}{\partial y} >> \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x}$$

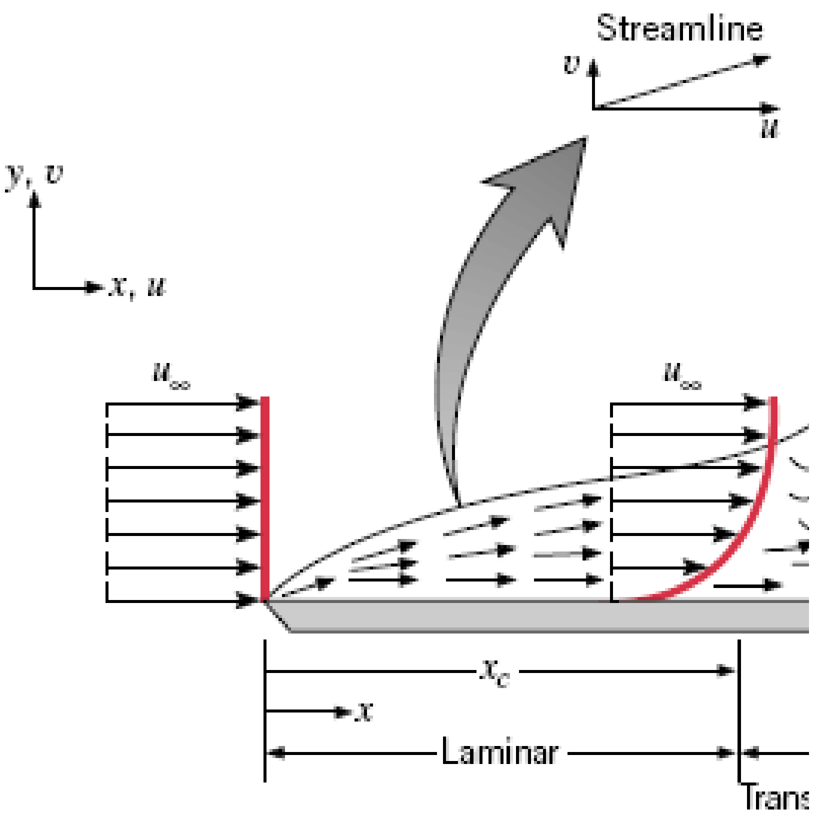
$$\frac{\partial T}{\partial y} >> \frac{\partial T}{\partial x}$$

$$\frac{\partial C_A}{\partial y} >> \frac{\partial C_A}{\partial x}$$

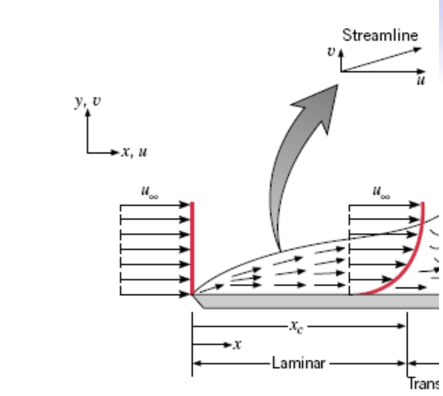
Além disso

$$v = 0 \quad em \ y = 0$$

$$C_A << C_B$$



Resultando



$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial y^2}\right)+\rho g_x$$

$$0=-\frac{\partial p}{\partial y}+\rho g_y$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(+\frac{\partial^2 T}{\partial y^2}\right)+\mu\left\{\left(\frac{\partial u}{\partial y}\right)^2\right\}+\dot{q}$$

$$u\frac{\partial C_A}{\partial x}+v\frac{\partial C_A}{\partial y}=D_{AB}\left(\frac{\partial^2 C_A}{\partial y^2}\right)+\dot{N}_A$$

Camada limite de similaridade: Equações Normalizadas

- Variáveis Dependentes:

$$\tau_s \text{ and } q'' \text{ or } h$$

- Variáveis independentes

Geométrica: Tamanho (L), Posição (x,y)

Hidrodinâmica: Velocidade (V)

Propriedades do Fluido:
$$\begin{cases} MF \rightarrow \rho, \mu \\ TC \rightarrow k, c_p \\ TM \rightarrow D_{AB} \end{cases}$$

$$u = f\left(x, y, V, L, \rho, \mu, \frac{dp}{dx}\right)$$

$$T = f\left(x, y, V, L, \rho, \mu, k, c_p, \frac{dp}{dx}\right)$$

$$C_A = f\left(x, y, V, L, \rho, \mu, D_{AB}, \frac{dp}{dx}\right)$$

Se fixar a geometria não há mais dependência de *dp/dx*

- Forma adimensional

$$x^* \equiv \frac{x}{L} \qquad y^* \equiv \frac{y}{L}$$

$$u^* \equiv \frac{u}{V} \qquad v^* \equiv \frac{v}{V}$$

$$T^* \equiv \frac{T - T_s}{T_\infty - T_s}$$

$$p^* = \frac{p}{\rho V^2}$$

$$C_A^* = \frac{C_A - C_{A,s}}{C_{A,\infty} - C_{A,s}}$$

- Mudando para variáveis adimensionais:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

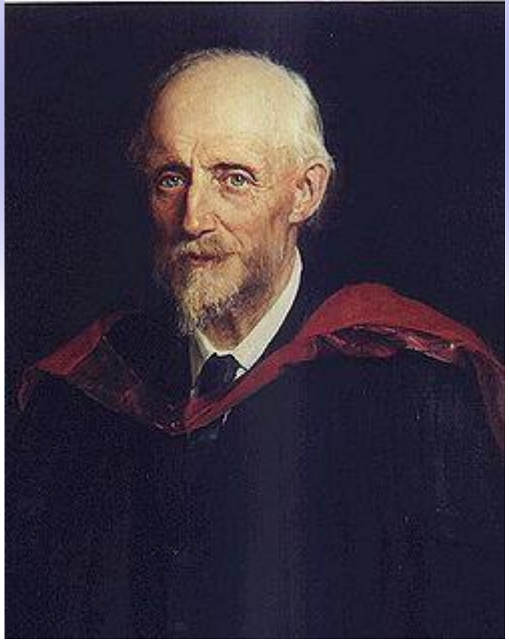
$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L \, Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

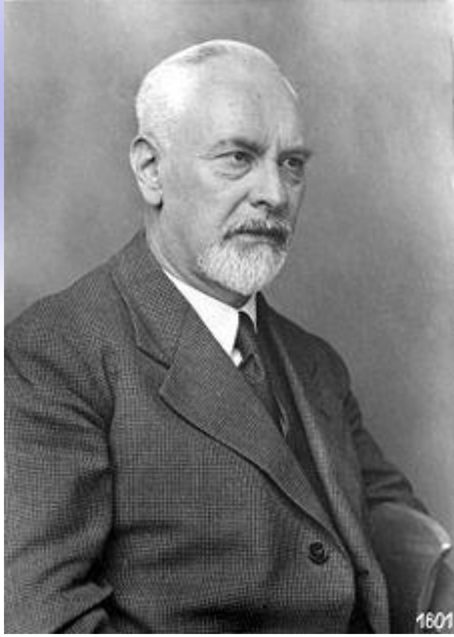
$$u^* \frac{\partial C^*_A}{\partial x^*} + v \frac{\partial C^*_A}{\partial y^*} = \frac{1}{Re \, Sc} \frac{\partial^2 C^*_A}{\partial y^{*2}}$$

$$Re_L \equiv \frac{\rho V L}{\mu} = \frac{VL}{\nu} \rightarrow$$ the Reynolds Number

$$Pr \equiv \frac{c_p \mu}{k} = \frac{\nu}{\alpha} \rightarrow$$ the Prandtl Number

$$Sc = \frac{\nu}{D_{AB}}$$ *Número de Schmidt*





Osborne Reynolds (Belfast, 23 de agosto de 1842 — 21 de fevereiro de 1912) fez matemática em Cambridge completando em 1867. Em 1866, tornou-se o primeiro catedrático em engenharia em Manchester, e o segundo da Inglaterra.
Em 1883, introduziu o mais importante número adimensional da mecânica dos fluidos, hoje conhecido como Número de Reynolds.

Em 1886, formulou a moderna teoria de lubrificação. Três anos depois, formulou, para escoamentos turbulentos, a noção de campos médios e flutuantes, dando origem às equações Reynolds Average Navier-Stokes que hoje sustentam a maior parte dos modelos turbulentos em Fluidodinâmica computacional, CFD.
(fonte:http://pt.wikipedia.org/wiki/Osborne_Reynolds)

Ludwig Prandtl (4 February 1875 – 15 August 1953) was a German scientist. He was a pioneer in the development of rigorous systematic mathematical analyses which he used to underlay the science of aerodynamics, which have come to form the basis of the applied science of aeronautical engineering. Furthermore, he probably built the world's first wind tunnel. In the 1920s he developed the mathematical basis for the fundamental principles of subsonic aerodynamics in particular; and in general up to and including transonic velocities. His studies identified the boundary layer, thin-airfoils, and lifting-line theories.

$$u = f(x, y, L, V, \rho, \mu)$$

$$\tau_s = f(x, L, V, \rho, \mu)$$

Em variáveis adimensionais para uma determinada geometria

$$u^* = f(x^*, y^*, \mathrm{Re}_L)$$

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \left(\frac{\mu V}{L}\right) \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

O coeficiente de atrito local para uma determinada geometria torna-se

$$C_f \equiv \frac{\tau_s}{\rho V^2/2} = \frac{2}{\mathrm{Re}_L} \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = f(x^*, \mathrm{Re}_L)$$

$$C_f = \frac{2}{\mathrm{Re}_L} f(x^*, \mathrm{Re}_L)$$

- Para a transferência de Calor, fixando a geometria,

$$T^* = f\left(x^*, y^*, \mathrm{Re}_L, \mathrm{Pr}\right)$$

$$h = \frac{-k_f \partial T / \partial y\big|_{y=0}}{T_s - T_\infty} = -\frac{k_f}{L}\frac{\left(T_\infty - T_s\right)}{\left(T_s - T_\infty\right)}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = +\frac{k_f}{L}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0}$$

O coeficiente local de convecção adimensional é

$$Nu \equiv \frac{hL}{k_f} = \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = f\left(x^*, \mathrm{Re}_L, \mathrm{Pr}\right)$$

Número de Nusselt local

$$\overline{Nu} = \frac{\bar{h}L}{k_f} = f\left(\mathrm{Re}_L, \mathrm{Pr}\right)$$

Número de Nusselt médio

- Para a transferência de Massa, fixando a geometria,

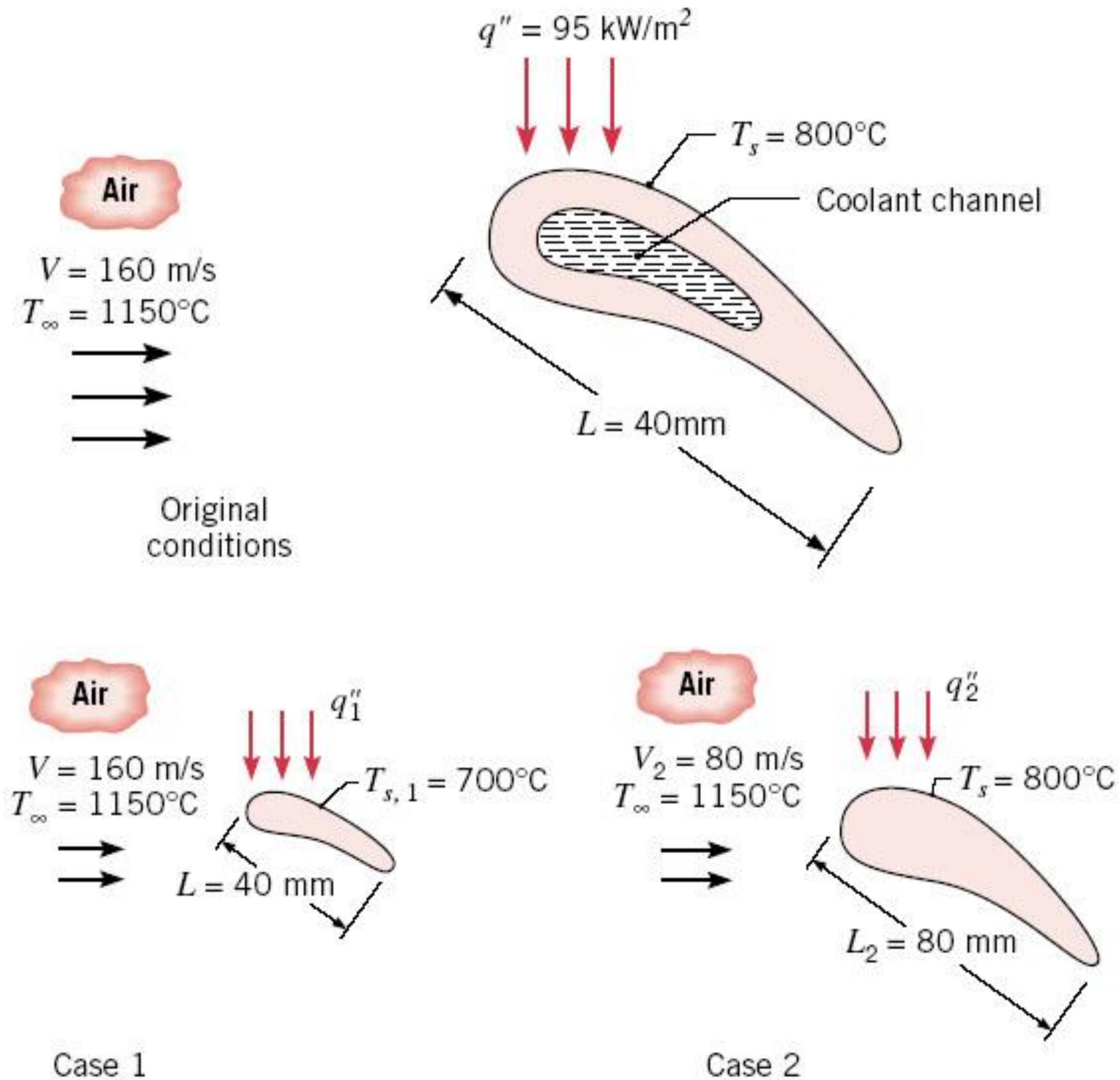
$$C_A = f\left(x, y, V, L, \rho, \mu, D_{AB}\right)$$
$$C^*{}_A = f\left(x^*, y^*, \mathrm{Re}_L, Sc\right)$$

$$h_m = \frac{-D_{AB} \left.\partial C_A \middle/ \partial y\right|_{y=0}}{C_{A,S} - C_{A,\infty}} = -\frac{D_{AB}}{L} \frac{C^*{}_{A,\infty} - C^*{}_{A,S}}{C^*{}_{A,S} - C^*{}_{A,\infty}} \left.\frac{\partial C^*{}_A}{\partial y^*}\right|_{y^*=0} = \frac{D_{AB}}{L} \left.\frac{\partial C^*{}_A}{\partial y^*}\right|_{y^*=0}$$

$$Sh = \frac{h_m L}{D_{AB}} = \left.\frac{\partial C^*{}_A}{\partial y^*}\right|_{y^*=0} = f\left(x^*, \mathrm{Re}_L, Sc\right)$$ Número de Sherwood local

$$\overline{Sh} = \frac{\overline{h_m} L}{D_{AB}} = f\left(\mathrm{Re}_L, Sc\right)$$ Número de Sherwood médio

Ex. 6.5: Lâmina de turbina .



Significado Físico dos Parâmetros Adimensionais

- Número de Reynolds

$$Re = \frac{F_{inércia}}{F_{viscosa}} \approx \frac{\rho V^2 / L}{\mu V / L^2} = \frac{\rho V L}{\mu}$$

- Número de Prandtl

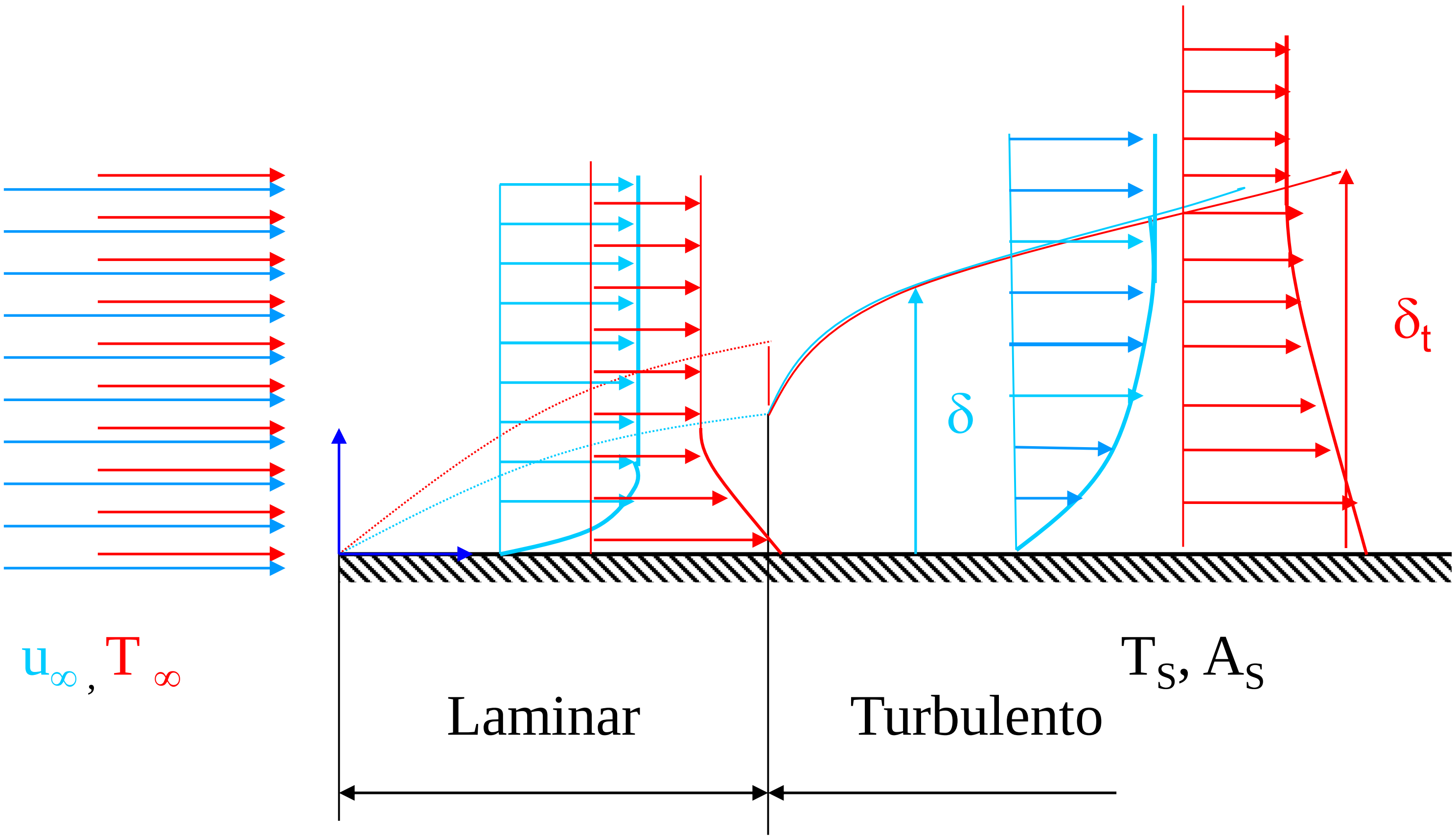
$$Pr = \frac{\nu}{\alpha} = \frac{momento\ de\ difusividade}{difusividade\ térmica}$$

• Gases	Pr≈1
•Metais líquidos	Pr<<1
•Óleos	Pr>>1

- Número de Schmidt

$$Sc = \frac{\nu}{D_{AB}} = \frac{momento\ de\ difusividade}{difusividade\ de\ massa}$$

Espessura da camada limite



Escoamento laminar

- Obtenção experimental

$$\frac{\delta}{\delta_t} \approx \mathrm{Pr}^n$$

Obtido experimentalmente
n≈1/3

$$\frac{\delta}{\delta_C} \approx Sc^n$$

Número de Lewis

$$Le = \frac{Sc}{\mathrm{Pr}} = \frac{\alpha}{D_{AB}}$$

$$\frac{\delta_t}{\delta_C} \approx Le^n$$

Escoamento Turbulento

- Observado experimentalmente

$$\delta \approx \delta_t \approx \delta_c$$

Analogia de camada limite

MF	TC	TM
$u^* = f\left(x^*, y^*, \mathrm{Re}_L, \frac{dp^*}{dx^*}\right)$	$T^* = f\left(x^*, y^*, \mathrm{Re}_L, \mathrm{Pr}, \frac{dp^*}{dx^*}\right)$	$C_A^{\ *} = f\left(x^*, y^*, \mathrm{Re}_L, Sc, \frac{dp^*}{dx^*}\right)$
$C_f = \frac{2}{\mathrm{Re}_L} \frac{\partial u^*}{\partial y^*}\Big\|_{y^*=0}$	$Nu = \frac{hL}{k_f} = \frac{\partial T^*}{\partial y^*}\Big\|_{y^*=0}$	$Sh = D_{AB} = \frac{\partial C_A^{\ *}}{\partial y^*}\Big\|_{y^*=0}$
$C_f = \frac{2}{\mathrm{Re}_L} f\left(x^*, \mathrm{Re}_L\right)$	$Nu = f\left(x^*, \mathrm{Re}_L, \mathrm{Pr}\right)$	$Sh = f\left(x^*, \mathrm{Re}_L, Sc\right)$

Analogia da camada limite

- Experimentalmente

$$Nu = f\left(x^*, \mathrm{Re}_L, \mathrm{Pr}\right) \approx f\left(x^*, \mathrm{Re}_L\right)\mathrm{Pr}^n$$

$$Sh = f\left(x^*, \mathrm{Re}_L, Sc\right) \approx f\left(x^*, \mathrm{Re}_L\right)Sc^n$$

- No Cap. 7 se verá que $f\left(x^*, \mathrm{Re}_L\right)$ são iguais para as duas condições, então

$$\frac{Nu}{\mathrm{Pr}^n} = \frac{Sh}{Sc^n} \qquad \frac{hL/k_f}{\mathrm{Pr}^n} = \frac{h_m L/D_{AB}}{Sc^n} \qquad \frac{h}{h_m} = \frac{k}{D_{AB} Le^n} = \rho c_p Le^{1-n}$$

Analogia de Reynolds

- (dp*/dx*~0) , Pr~1, e Sc~1:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L \Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$u^* \frac{\partial C^*_A}{\partial x^*} + v \frac{\partial C^*_A}{\partial y^*} = \frac{1}{\mathrm{Re}\, Sc} \frac{\partial^2 C^*_A}{\partial y^{*2}}$$

→

$$\underbrace{u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*}}_{\text{Advection terms}} = \underbrace{\frac{1}{\mathrm{Re}} \frac{\partial^2 u^*}{\partial y^{*2}}}_{\text{Diffusion}}$$

$$\overbrace{u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*}}^{\text{Advection terms}} = \overbrace{\frac{1}{\mathrm{Re}} \frac{\partial^2 T^*}{\partial y^{*2}}}^{\text{Diffusion}}$$

- Com condições de contorno equivalente

$$u^* = T^*$$

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

$$C_f \frac{\mathrm{Re}}{2} = Nu$$

Com Pr = Sc=1, a analogia de Reynolds é

$$C_f \frac{\mathrm{Re}_L}{2} = Nu = Sh$$

Definindo

Número de Stanton: $$St = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}} = \frac{h}{\rho V c_p}$$

Número de Stanton de massa: $$St_m = \frac{Sh}{\mathrm{Re}\,Sc} = \frac{h_m}{V}$$

logo $$\frac{C_f}{2} = St = St_m$$

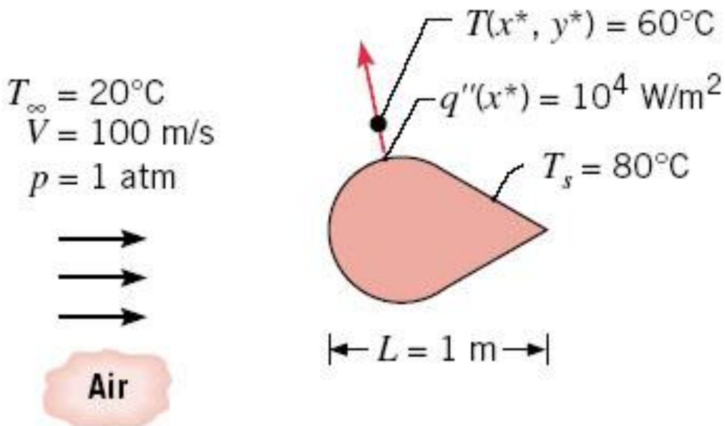
•De modo geral para Pr e Sc ≠1 utiliza-se a analogia modificada de Reynolds (Chilton-Colburn) :

$$\frac{C_f}{2} = St\,\mathrm{Pr}^{2/3} \equiv j_H \quad 0,6 < \mathrm{Pr} < 60$$

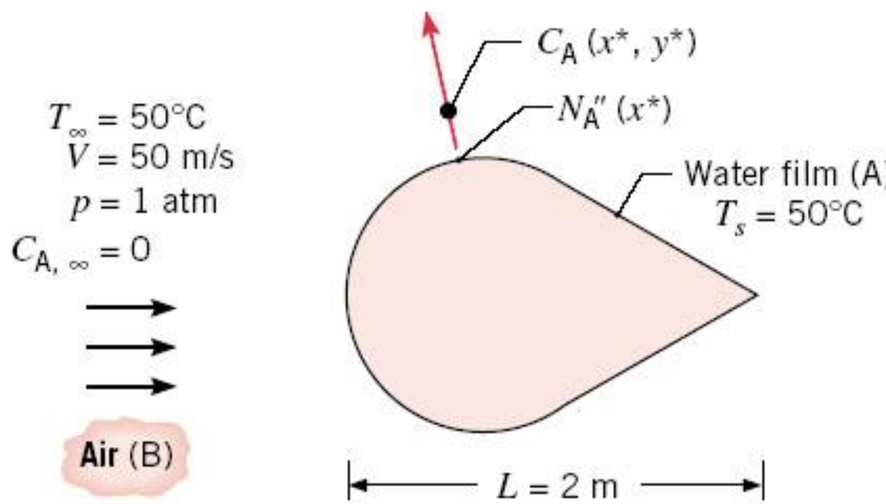
$$\frac{C_f}{2} = St_m Sc^{2/3} \equiv j_m \quad 0,6 < Sc < 3000$$

j=fator de Colburn

– Aplicável para fluido laminar se dp*/dx* ~ 0.
– Geralmente aplicável para escoamento turbulento com restrições para dp*/dx*.

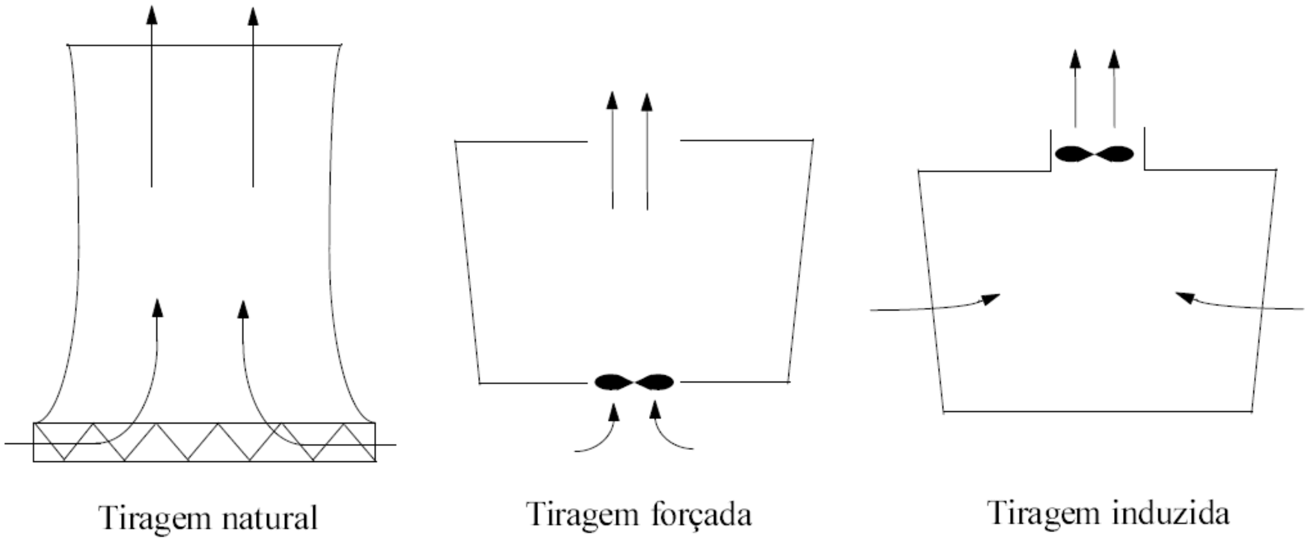


Case 1: heat transfer



Case 2: mass transfer

Resfriamento Evaporativo



Resfriamento Evaporativo

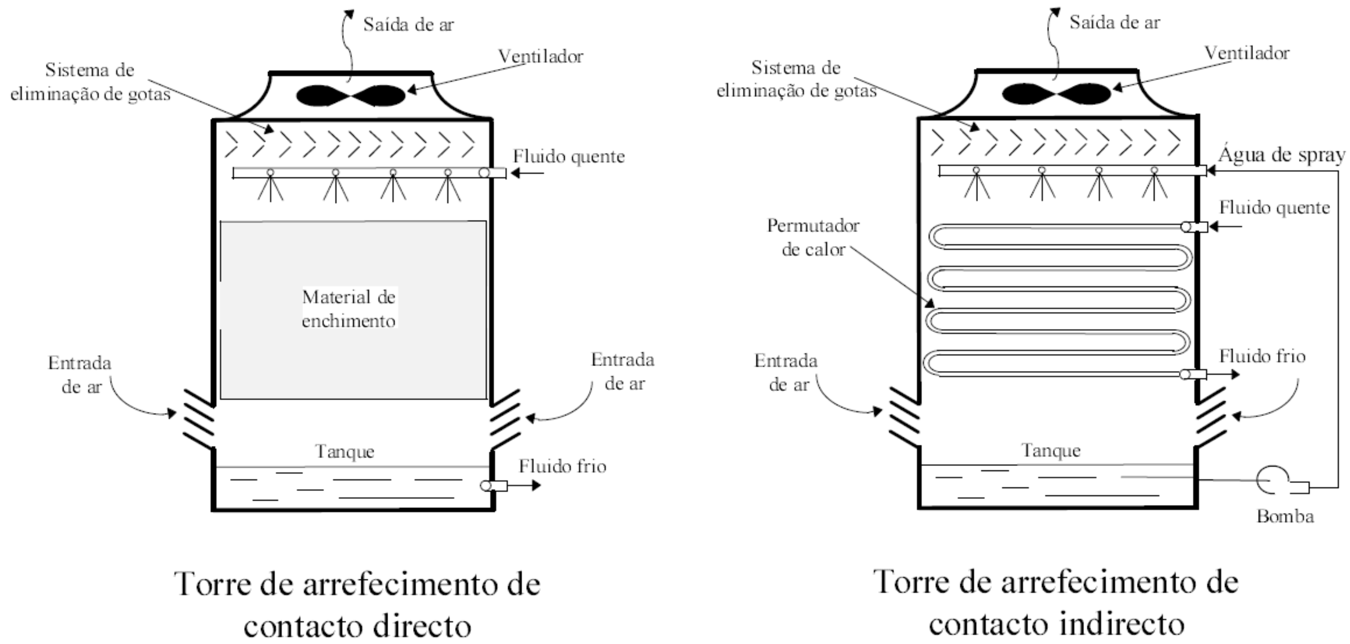


Figura 1.3: Classificação das torres de arrefecimento.

Resfriamento evaporativo

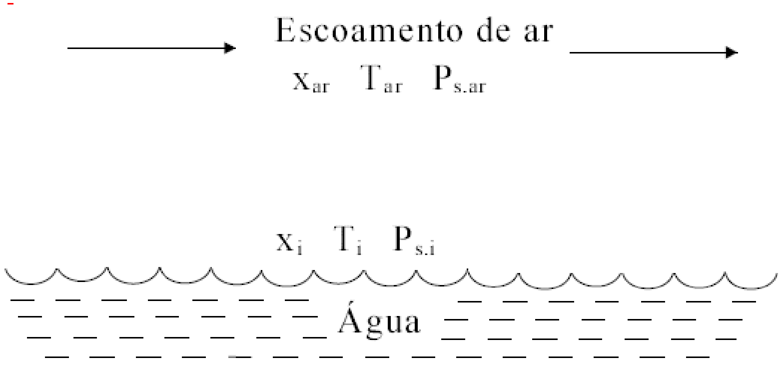
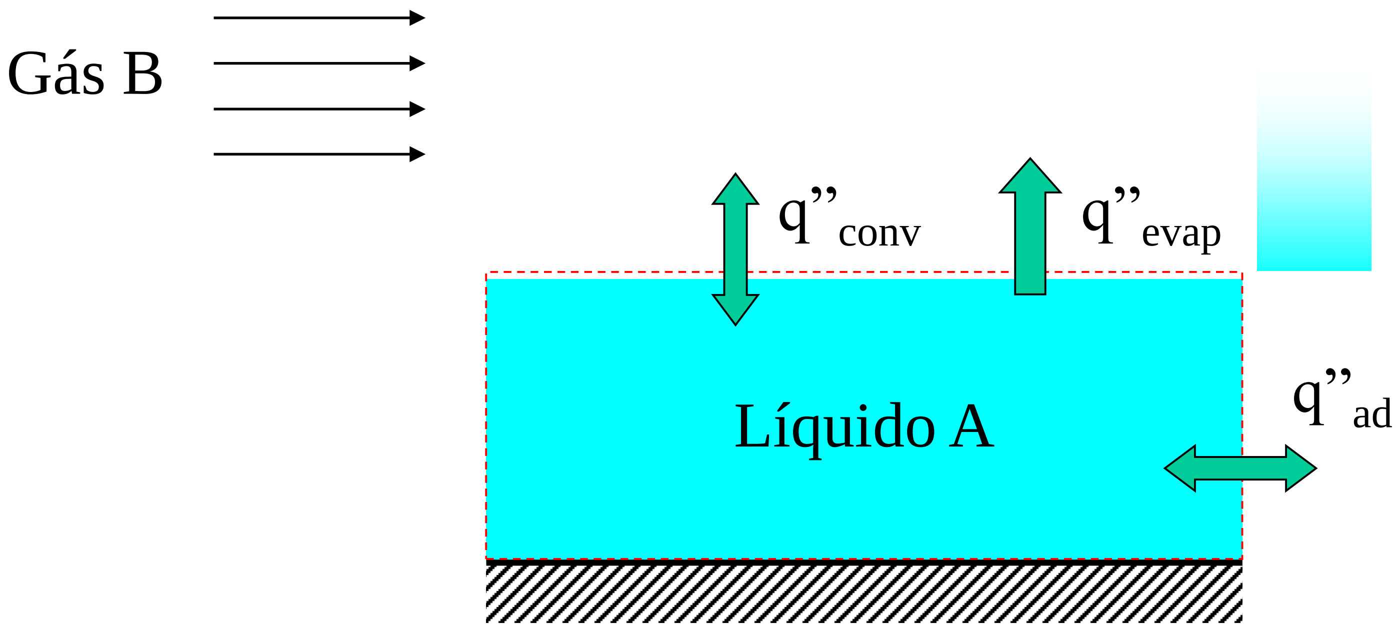
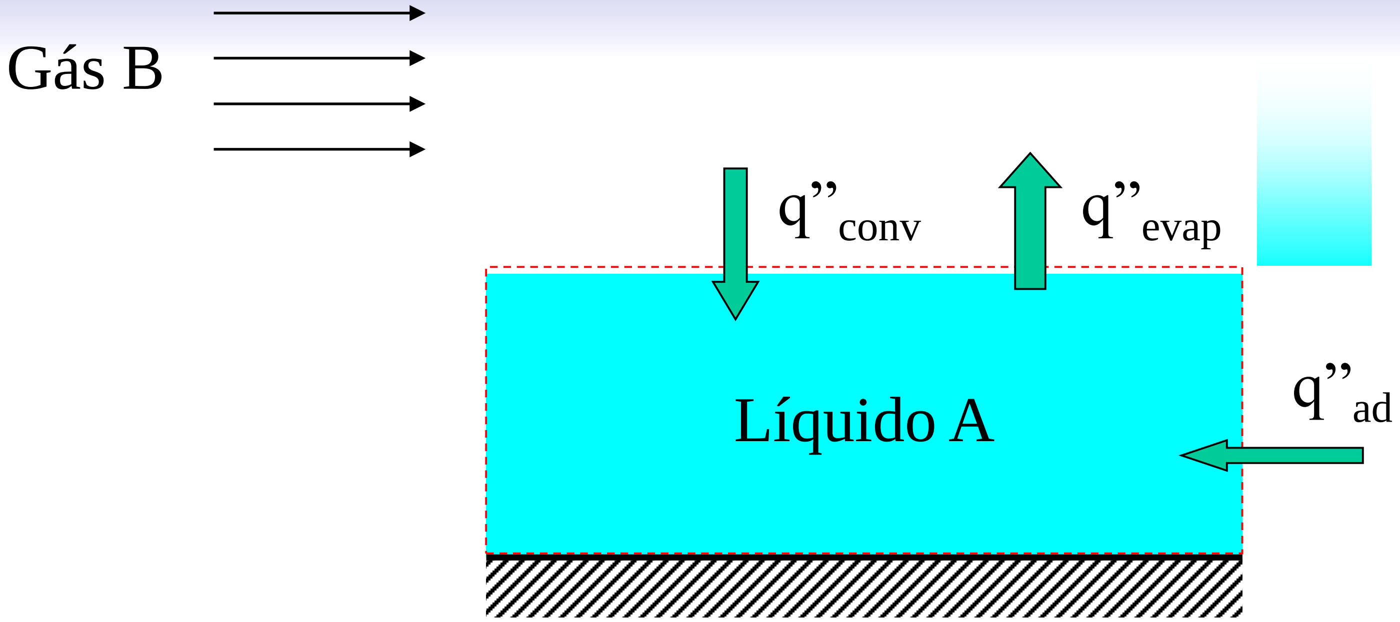


Figura 2.1: Tranferência de calor entre uma superfície molhada e o escoamento de ar

MODELO



Balanço de energia para $T_S<T_\infty$



Balanço de energia para $T_S<T_\infty$

$$q''_{conv} + q''_{ad} = q''_{evap}$$

$$q''_{conv} + q''_{ad} = q''_{evap}$$

$$\begin{cases} q_{evap} = n''_A h_{lv} = n''_A h_{fg} \\ q_{conv} = h\left(T_\infty - T_S\right) \end{cases}$$

substituindo

$$h\left(T_\infty - T_S\right) + q''_{ad} = n''_A h_{lv} = h_{lv} h_m \left(\rho_{A,sat(T)} - \rho_{A,\infty}\right)$$

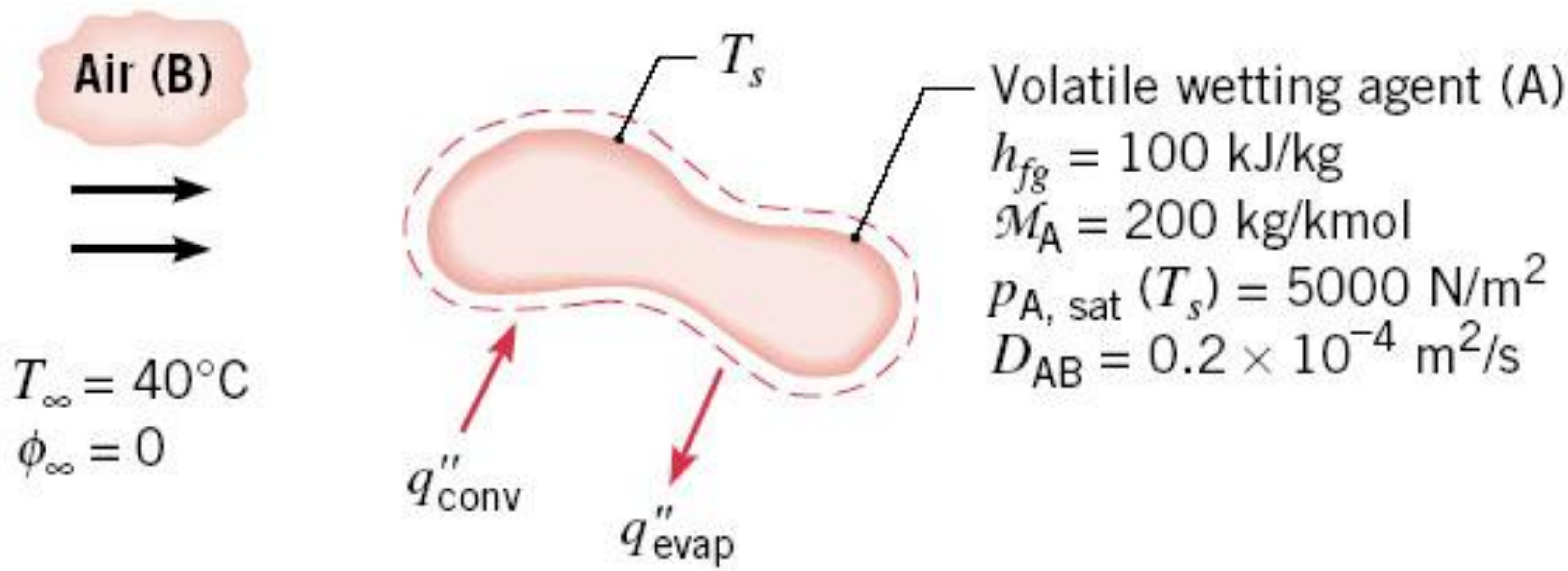
- Para um problema com $q''_{ad} = 0$

$$h(T_\infty - T_S) = h_{lv} h_m \left(\rho_{A,sat(T)} - \rho_{A,\infty}\right)$$

$$T_\infty - T_S = h_{lv} \frac{h_m}{h} \left(\rho_{A,sat(T)} - \rho_{A,\infty}\right)$$

ou para GASES IDEAIS

$$T_\infty - T_S = \frac{M_{mol_A} h_{lv}}{R \rho c_p Le^{2/3}} \left(\frac{p_{A,sat(T)}}{T_s} - \frac{p_{A,\infty}}{T_\infty} \right);$$


Air (B)
T_s
Volatile wetting agent (A)
h_{fg} = 100 kJ/kg
$\mathcal{M}_A$ = 200 kg/kmol
$p_{A,\,sat}\,(T_s)$ = 5000 N/m^2
$D_{AB} = 0.2 \times 10^{-4}$ m^2/s
$T_\infty = 40°C$
$\phi_\infty = 0$
q''_{conv}
q''_{evap}