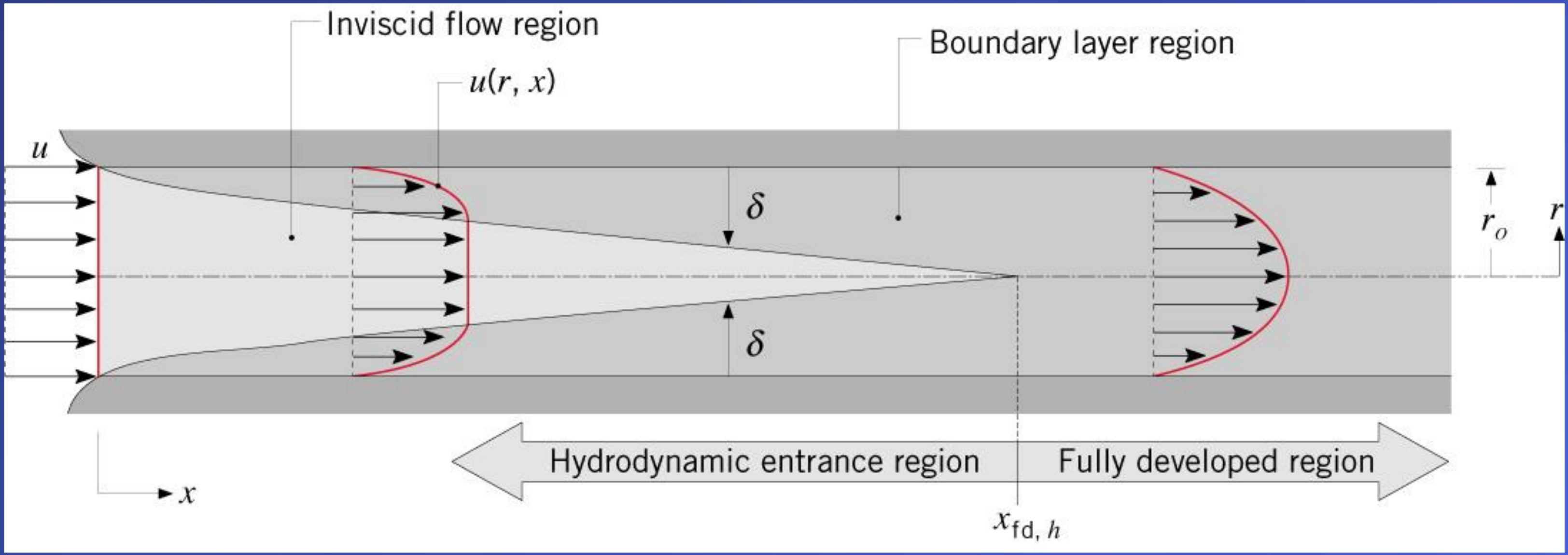
Escoamento em dutos:
Capítulo 8

Região de Entrada e Região com Escoamento Completamente Desenvolvido



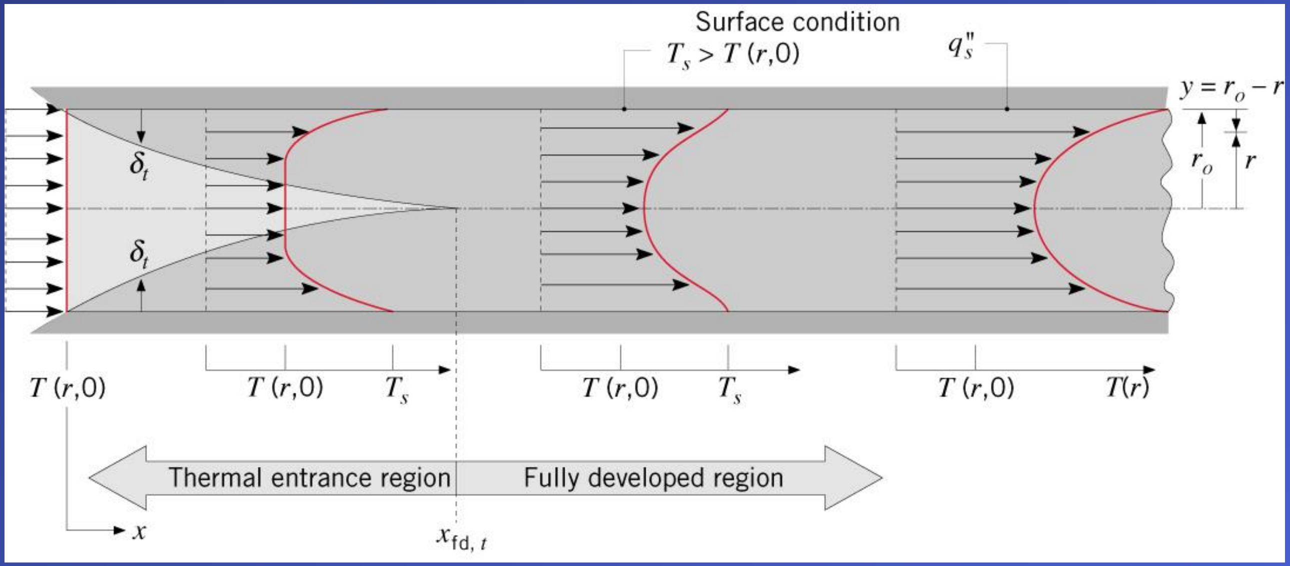
- Velocidade em *x* muda na região de entrada? Por quê?

$$Re = \frac{\rho u_\infty D}{\mu} \quad ; \quad Re_C \approx 2.300$$

Escoamento laminar $\left(\frac{x_{fd,h}}{D}\right) \approx 0{,}05 Re$

Escoamento turbulento $10 \le \left(\frac{x_{fd,h}}{D}\right) \le 60$; *normalmente* $\frac{x_{fd,h}}{D} = 10$

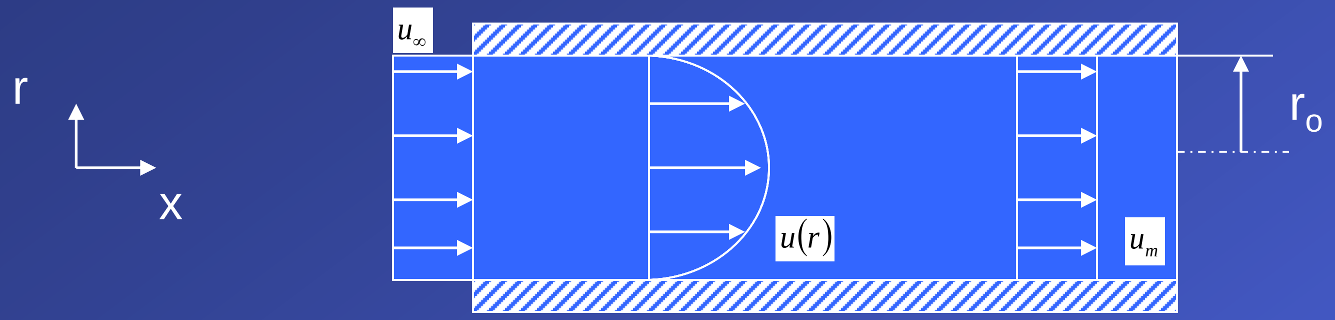
- Problema Térmico: Assumir esc. laminar com temp. uniforme $T(r,0)=T_i$, na
- região de entrada, com temperatura const. na superfície do tubo com $T_s \neq T_i$, ou
- fluxo de calor imposto, q''_s .



– Forma adimensional do perfil de temperatura

Para $\left(T_s \text{ and } q''_s\right)$ torna-se independente de x. => Condições térmicamente desenvolvida

O Conceito de Velocidade Média



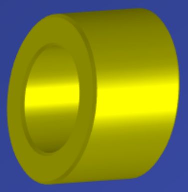
Velocidade média

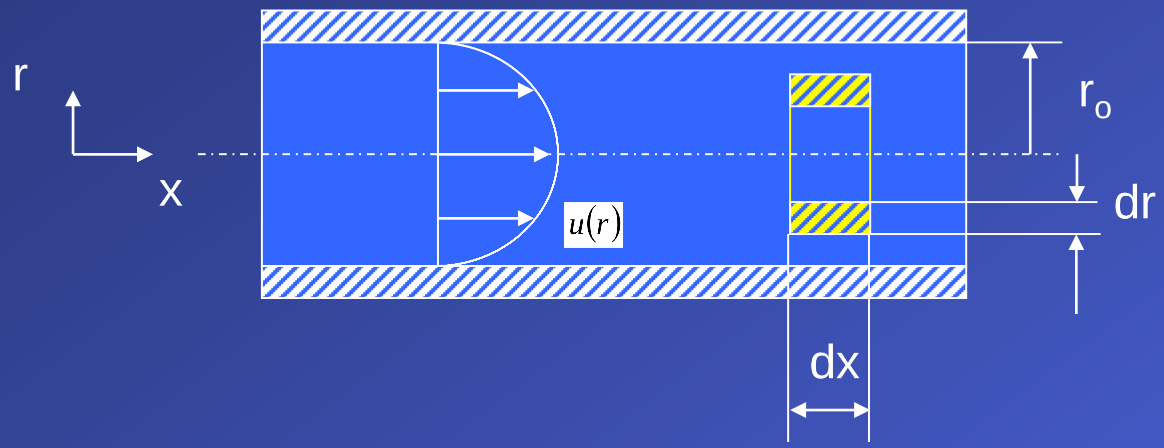
$$\dot{m} = \int_{A_c} \rho u(r,x)\, dA_c = \rho A_C u_m$$

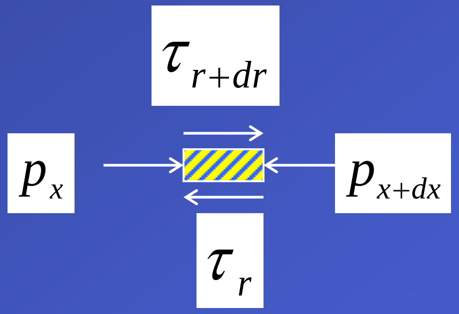
$$u_m = \frac{\int_{A_c} \rho u(r,x)\, dA_c}{\rho A_c}$$

Para escoamento incompressível em um duto circular de raio r_o

$$u_m = \frac{2}{r_o^2} \int_0^{r_o} u(r,x) r\, dr$$







Para escoamento desenvolvido:

$$\begin{cases} v = 0 \\ \dfrac{\partial u}{\partial x} = 0 \end{cases}$$

$$\sum F_{S_x} = 0$$

$$-\tau_r(2\pi r dx) + \left[\tau_r(2\pi r dx) + \frac{d}{dr}(\tau_r(2\pi r dx))dr\right]$$

$$+ p(2\pi r dr) - \left[p(2\pi r dr) + \frac{d}{dx}(p(2\pi r dr))dx\right] = 0$$

Simplificando os termos

$$\frac{1}{r}\frac{d}{dr}(r\tau_r) = \frac{dp}{dx} = ?$$

$$\frac{1}{r}\frac{d}{dr}\left(r\tau_r\right)=\frac{dp}{dx}=cte$$

sendo $$\tau_r=\mu\frac{du}{dr}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\mu\frac{du}{dr}\right)=\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)=\frac{dp}{dx}=cte$$

integrando

$$\int \frac{d}{dr}\left(r\frac{du}{dr} \right) dr = \int \frac{r}{\mu}\left(\frac{dp}{dx} \right) dr$$

$$r\frac{du}{dr} = \frac{1}{\mu}\left(\frac{dp}{dx} \right)\frac{r^2}{2} + c_1$$

$$\frac{du}{dr} = \frac{1}{\mu}\left(\frac{dp}{dx} \right)\frac{r}{2} + \frac{c_1}{r}$$

$$u(r) = \frac{1}{\mu}\left(\frac{dp}{dx} \right)\frac{r^2}{4} + c_1 \ln r + c_2$$

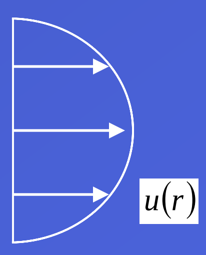
Com condições de contorno

$$\begin{cases} u(r_o) = 0 \rightarrow c_2 = -\dfrac{r_o^{\ 2}}{4\mu}\left(\dfrac{\partial p}{\partial x}\right) \\ \left.\dfrac{\partial u}{\partial r}\right|_{r=0} = 0 \rightarrow c_1 = 0 \end{cases}$$

Desta forma

$$u(r) = -\frac{1}{4\mu}\left(\frac{dp}{dx}\right) r_o^{\ 2}\left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

Esc. laminar

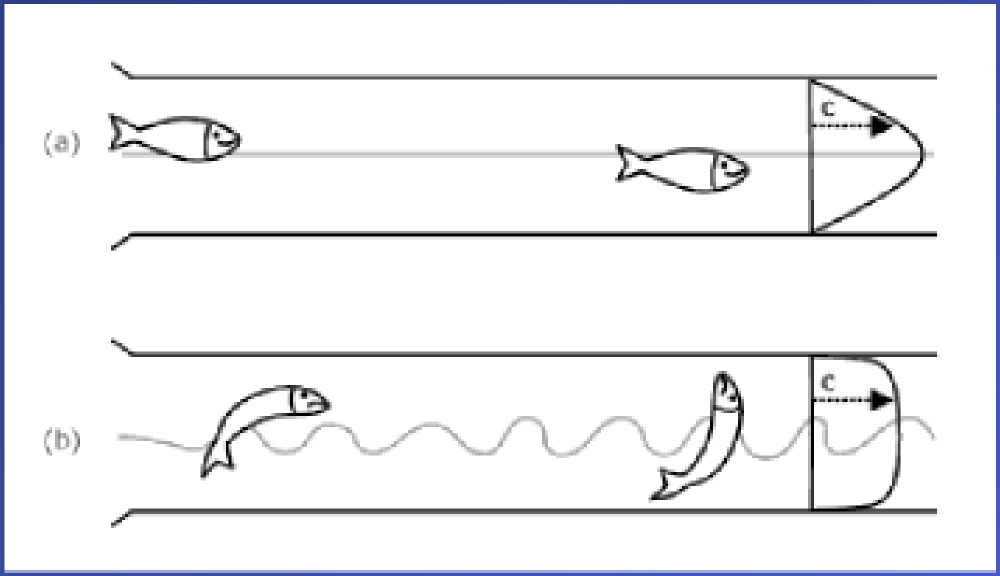


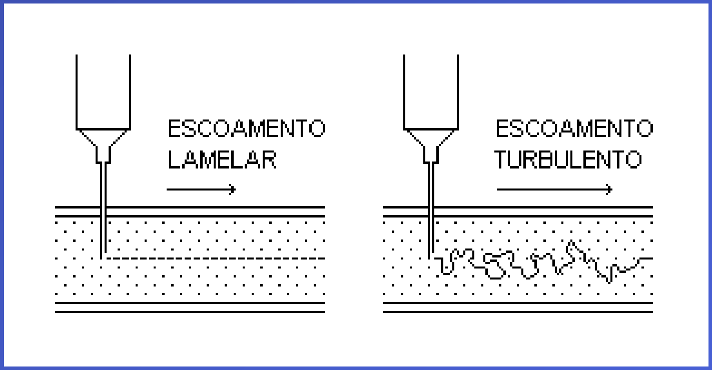
Integrando e calculando a velocidade média

$$u_m = -\frac{r_o^2}{8\mu}\left(\frac{dp}{dx}\right)$$

Esc. laminar

Escoamento turbulento





Condição completamente desenvolvida

- Assumindo propriedades constantes o perfil de velocidades permaneceinalterado na região completamente desenvolvida

- fator de atrito,

$$f \equiv -\frac{(dp/dx)D}{\rho u_m^2/2}$$

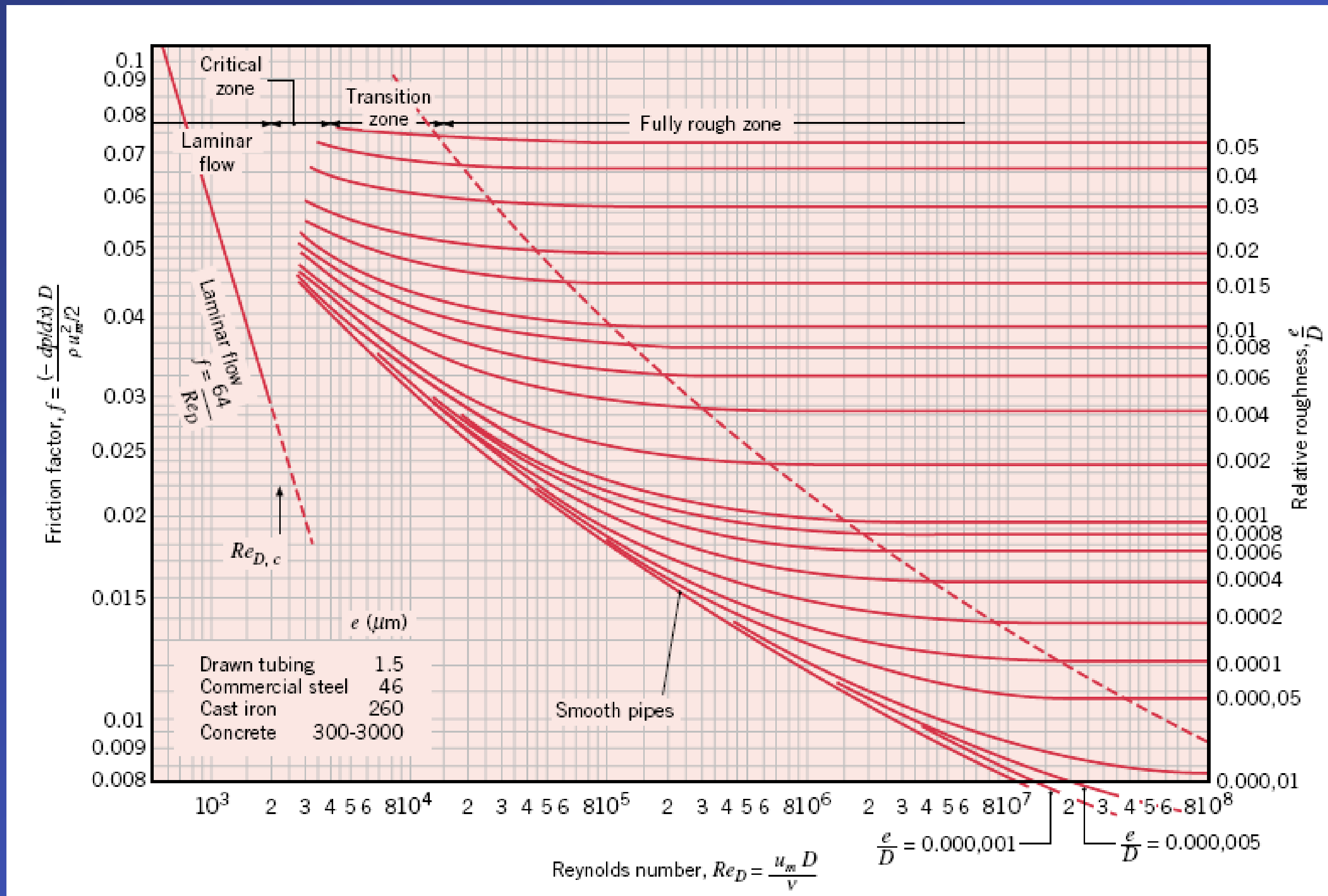
Escoamento laminar em um duto circular:

$$f = \frac{64}{\mathrm{Re}_D}$$

Escoamento turbulento em um tubo circular com paredes lisas:

$$f = (0.790 \ln \mathrm{Re}_D - 1.64)^{-2}$$

Escoamento turbulento com parede rugosa (diagrama de Moody):



Queda de pressão:

$$\Delta p = p_1 - p_2 = f \frac{\rho u_m^2}{2D}(x_2 - x_1)$$

Potência de bombeamento

$$P = \Delta p \dot{\forall} = \frac{\Delta p \dot{m}}{\rho}$$

Região de Entrada Hidrodinâmica e Térmica

•Número de Reynolds :

$$Re_D \equiv \frac{\rho u_m D_h}{\mu}$$

Diametro hidraulico

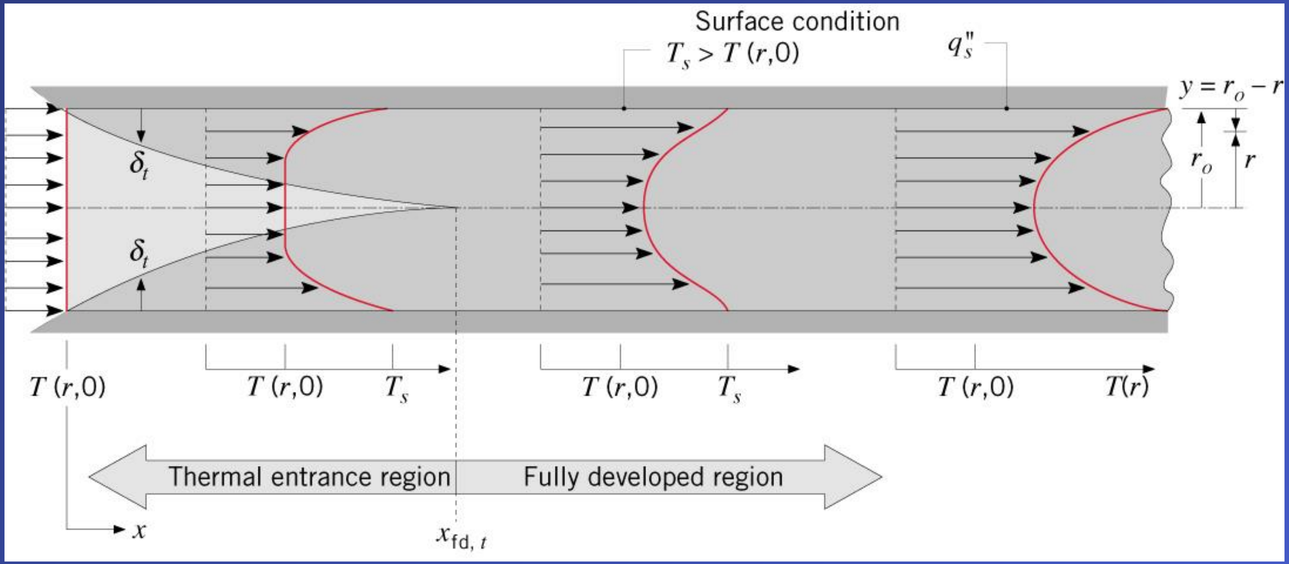
$$D_h \equiv \frac{4A_c}{P}$$

ou

$$Re_D \equiv \frac{\rho u_m D_h}{\mu} = \frac{4\dot{m}}{P\mu}$$

Para um tubo circular,

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{4\dot{m}}{\pi D\mu}$$



$$\mathrm{Re} = \frac{\rho u_\infty D}{\mu} \quad ; \quad \mathrm{Re}_C \approx 2.300$$

Escoamento laminar $\left(\frac{x_{fd,h}}{D}\right) \approx 0{,}05\,\mathrm{Re}$

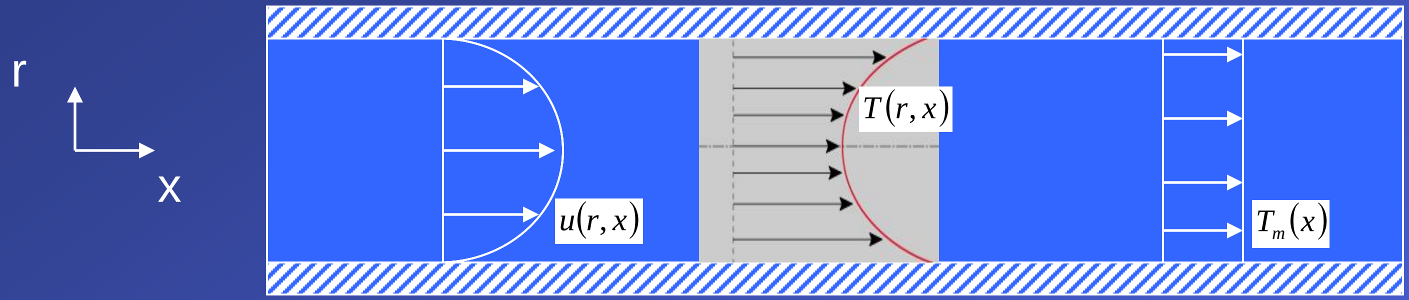
Escoamento turbulento $10 \leq \left(\frac{x_{fd,h}}{D}\right) \leq 60$; *normalmente* $\frac{x_{fd,h}}{D} = 10$

Escoamento laminar $\left(\frac{x_{fd,t}}{D}\right) \approx 0{,}05\,\mathrm{Re}\,\mathrm{Pr}$

$p/\mathrm{Pr}\left(=\frac{\nu}{\alpha}\right) \geq 100 \quad \frac{x_{fd,h}}{D} \approx 0$

Escoamento turbulento $\frac{x_{fd,t}}{D} = 10$

Temperatura média ?



- Energia média = energia da mistura

$$\dot{E}_t = \int_{A_c} \rho u c_p T \, dA_c \equiv \dot{m} c_p T_m$$

Hence,

$$T_m = \frac{\int_{A_c} \rho u c_p T \, dA_c}{\dot{m} c_p}$$

- Para escoamento incompressível, propriedades constante em um duto circular,

$$T_m = \frac{2}{u_m r_o^2} \int_0^{r_o} u(x,r) T(x,r) r \, dr$$

- Lei de Newton para o resfriamento

$$q_s'' = h(T_s - T_m)$$

What is the essential difference between use of T_m for internal flow and T_∞ for external flow?

- CONDIÇÃO PLENAMENTE DESENVOLVIDO (CPD) para temperatura T_S isotérmica:

$$\frac{\partial}{\partial x}\left[\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}\right]_{fd,t} = 0$$

  - considerando que:

$$\frac{T_S - T}{T_S - T_m} \neq f(x)$$

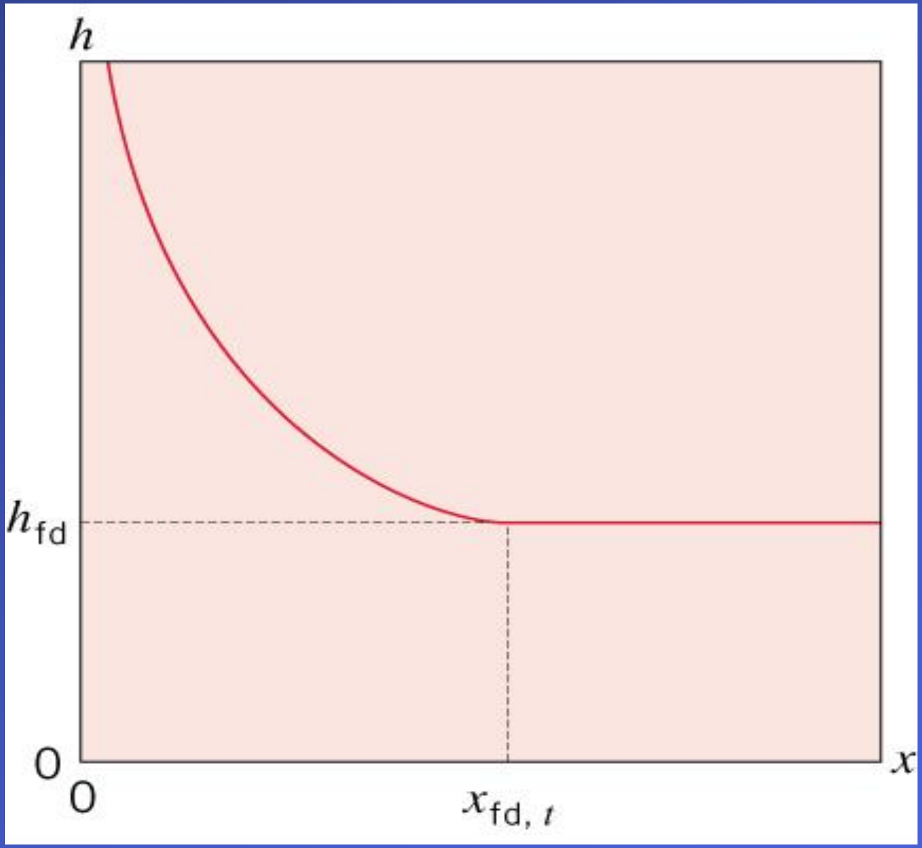
$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)\bigg|_{r=r_o} = \frac{-\partial T/\partial r|_{r=r_o}}{T_s - T_m} \neq f(x)$$

Assumindo propriedades constantes,

$$\frac{q_s''/k}{T_s - T_m} = \frac{h}{k} \neq f(x)$$

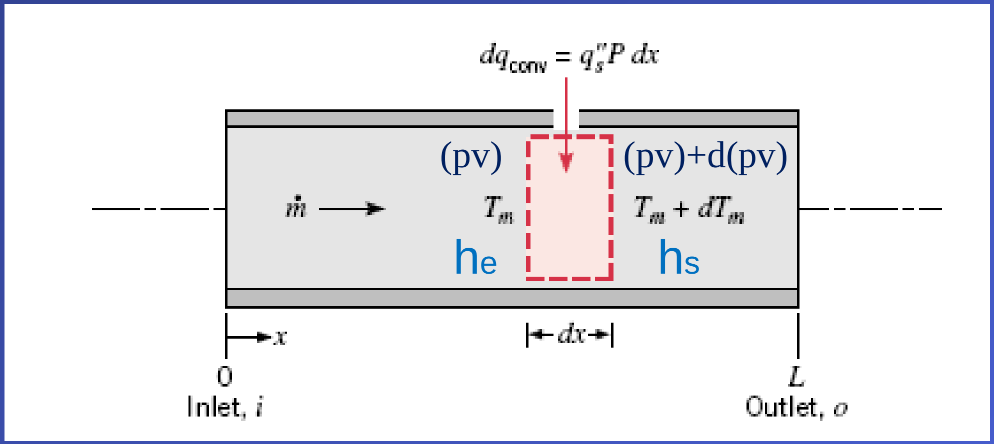
$$h \neq f(x)$$

Variação de *h* na região de entrada e RCP:



Determinação da variação da temperatura de mistura ao longo do tubo

- Determinação de $T_m(x)$



$$\begin{cases} h = u + pv \\ du = c_v dT \\ dh = c_p dT \end{cases}$$

Aplicando conservação de energia:

$$dq_{conv} + \dot{m}(h_e - h_s) = 0$$

$$dq_{conv} + \dot{m}(c_v T_m + pv) - \left[\dot{m}(c_v T_m + pv) + \dot{m}\frac{d(c_v T_m + pv)}{dx} dx \right] = 0$$

desta forma $$dq_{conv} = \dot{m} d(c_v T_m + pv)$$

HIPÓTESES:

1) Gás ideal
$$\begin{cases} pv = RT_m \\ c_p = c_v + R \quad ou \quad c_v = c_p - R \end{cases}$$

2) Líquidos incompressíveis
$$\begin{cases} c_p = c_v \\ d(pv) << d(c_v T_m) \end{cases}$$

logo

$$dq_{conv} = \dot{m} c_p dT_m$$

Integrando da entrada a saída (Análise Integral),

$$q_{conv} = \dot{m} c_p \left(T_{m,o} - T_{m,i} \right) \qquad (1)$$

Análise diferencial para obter $T_m(x)$

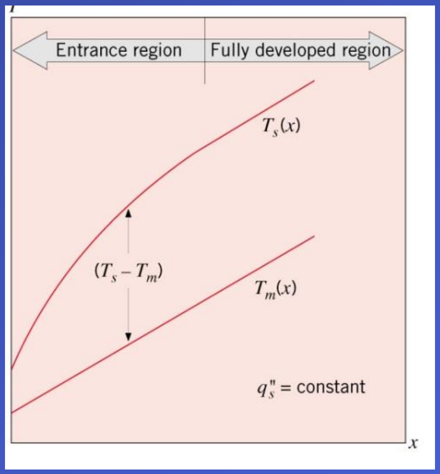
$$dq_{conv} = q_s''(P\,dx) = h(T_s - T_m)P\,dx. \quad = \dot{m}c_p dT_m$$

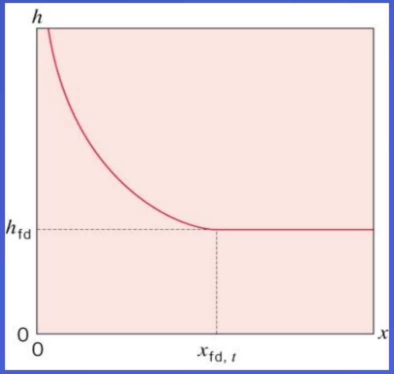
$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p} = \frac{P}{\dot{m}c_p}h(T_s - T_m) \qquad (2)$$

•Fluxo de calor constante

$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p} \neq f(x)$$

$$T_m(x) = T_{m,i} + \frac{q_s''P}{\dot{m}c_p}x$$





Taxa transferida de calor

$$q_{conv} = q_s''\,PL$$

•Temperatura da superfície constante

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} = \frac{P}{\dot{m} c_p} h\left(T_s - T_m\right) \qquad (2)$$

From Eq. (2), with $\Delta T \equiv T_s - T_m$

$$\frac{dT_m}{dx} = -\frac{d\left(\Delta T\right)}{dx} = \frac{P}{\dot{m} c_p} h \Delta T$$

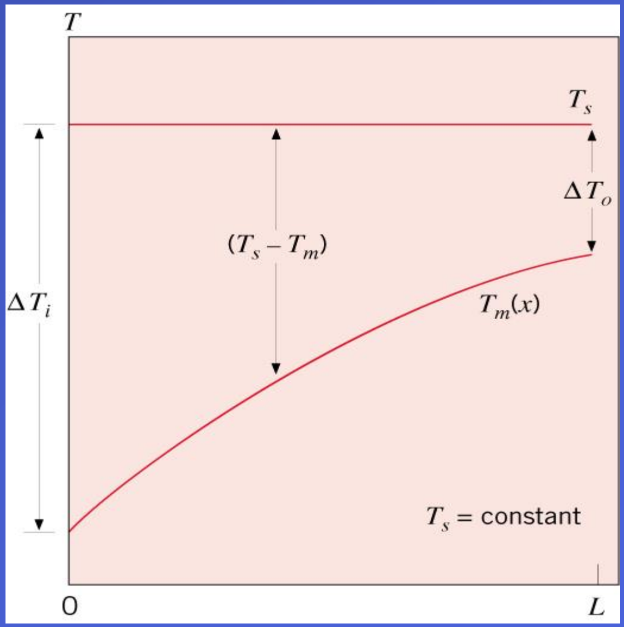
$$\frac{d\left(\Delta T\right)}{\Delta T} = \frac{P h_x}{\dot{m} c_p} dx$$

Integrando de x=0 até uma posição qualquer, x,

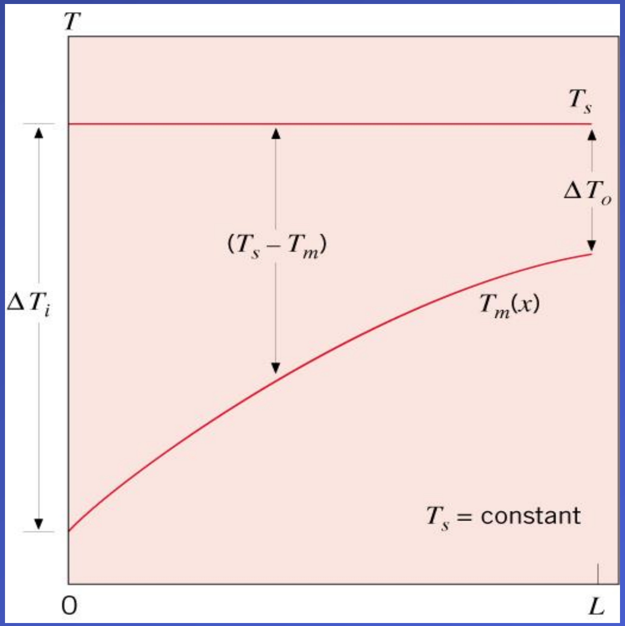
$$\int_{T_S - T_{m,i}}^{T_S - T_m(x)} \frac{d\left(\Delta T\right)}{\Delta T} = \int_0^x \frac{P h_x}{\dot{m} c_p} dx$$

$$\frac{T_s - T_m\left(x\right)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m} c_p} \bar{h}_x\right)$$

com $\qquad \bar{h}_x = \frac{1}{x} \int_0^x h_x \, dx$



$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m} c_p}\bar{h}_x\right)$$



Entre a entrada e a saída:

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m} c_p}\bar{h}\right) = \exp\left(-\frac{\bar{h} A_s}{\dot{m} c_p}\right)$$

cálculo da taxa transferida de calor transferido ao longo do tubo

$$q_{conv} = \int dq_{conv} = \int_0^x \bar{h}(T_S - T_m(x))dA$$

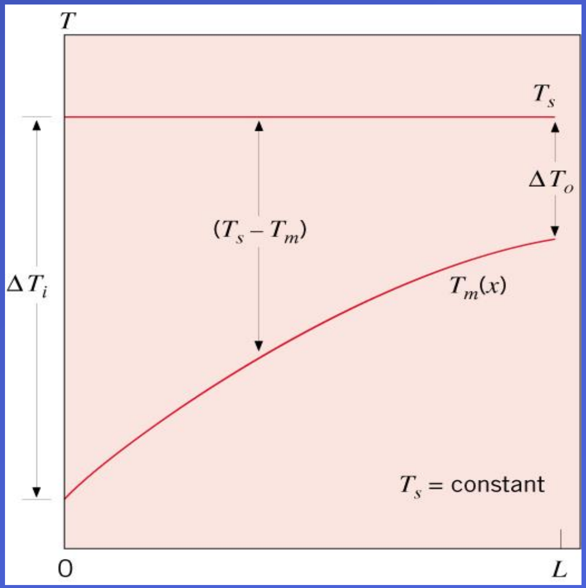
Diferença de temperatura Média logarítmica

$$q_{conv} = \bar{h} A_s \Delta T_{\ell m}$$

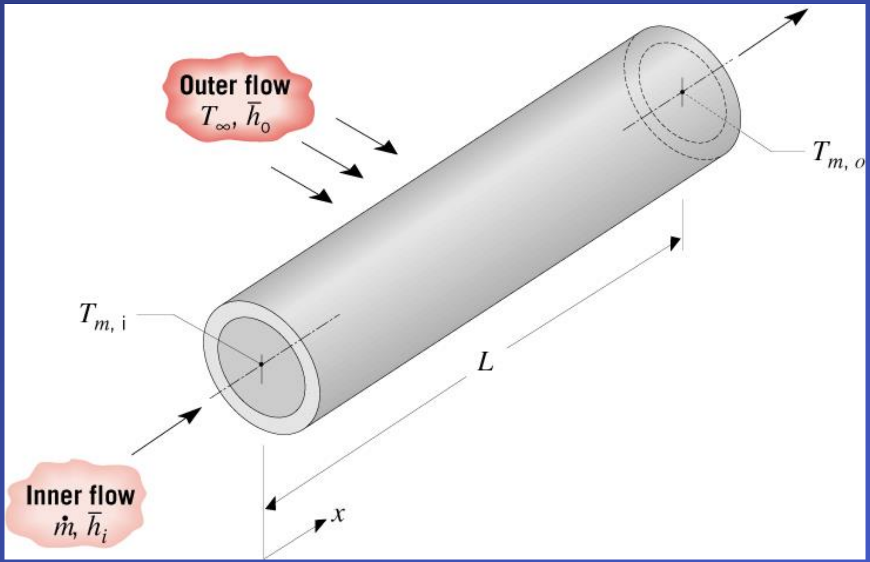
$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m} c_p} \bar{h} \right) = \exp\left(-\frac{\bar{h} A_s}{\dot{m} c_p} \right)$$

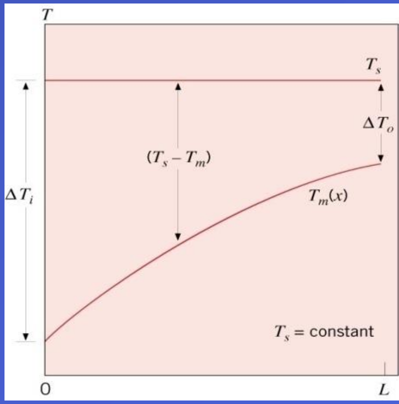
$$\ln \frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m} c_p} \bar{h}_L ; \quad \dot{m} c_p = -\frac{PL}{\ln \dfrac{\Delta T_o}{\Delta T_i}} \bar{h}_L$$

$$\Delta T_{\ell m} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\Delta T_o / \Delta T_i\right)} \qquad (3)$$



•Fluido externo com temperatura uniforme





$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\bar{U} A_s}{\dot{m} c_p}\right) = \exp\left(-\frac{1}{\dot{m} c_p R_{tot}}\right)$$

$$q = \bar{U} A_s \, \Delta T_{\ell m} = \frac{\Delta T_{\ell m}}{R_{tot}}$$

$\Delta T_{\ell m} \rightarrow$ Eq. (3) with T_s replaced by T_∞.

$$\Delta T_{\ell m} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\Delta T_o / \Delta T_i\right)}$$

Note: Replacement of T_∞ by $T_{s,o}$ if outer surface temperature is uniform.

Escoamento completamente desenvolvido

- Escoamento laminar em um Tubo Circular:

O número de Nusselt local é constante na região completamente desenvolvida, mas este valor depende da condição térmica da superfície.

Equação

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$com \quad v=0 \quad e \quad \frac{\partial u}{\partial x}=0$$

$$\frac{u(r)}{u_m}=2\left[1-\left(\frac{r}{R}\right)^2\right]$$

– Fluxo de calor uniforme na superfície (q_s'') :

$$\left.\frac{\partial T}{\partial x}\right|_{fd,t} = \left.\frac{dT_m}{dx}\right|_{fd,t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}2u_m\left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{dT_m}{dx}\right)$$

Sendo que $T_m(x)$ varia linearmente em x e

$$\int \partial\left(r\frac{\partial T}{\partial r}\right) = c\left[r - \frac{r^3}{R^2}\right]dr$$

$$T(r,x) = c\left[\frac{r^2}{4} - \frac{r^4}{16R^2}\right] + c_1 \ln r + c_2$$

$$T(r,x) = T_S(x) - \frac{2u_m R^2}{\alpha}\left(\frac{dT_m}{dx}\right)\left[\frac{3}{16} + \frac{1}{16}\left(\frac{r}{R}\right)^4 - \frac{1}{4}\left(\frac{r}{R}\right)^2\right]$$

sendo

$$T_m = \frac{2}{u_m R^2}\int_0^R uTrdr$$

$$T_m(x) = T_s(x) - \frac{11}{48}\left(\frac{u_m R^2}{\alpha}\right)\left(\frac{dT_m}{dx}\right)$$

Para Fluxo de calor uniforme na superfície

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} \quad , \quad P = \pi D, \;\; \dot{m} = \rho u_m \left(\frac{\pi D^2}{4}\right)$$

$$q_s'' = h\left(T_s - T_m\right)$$

$$h = \frac{48}{11}\left(\frac{k_f}{D}\right)$$

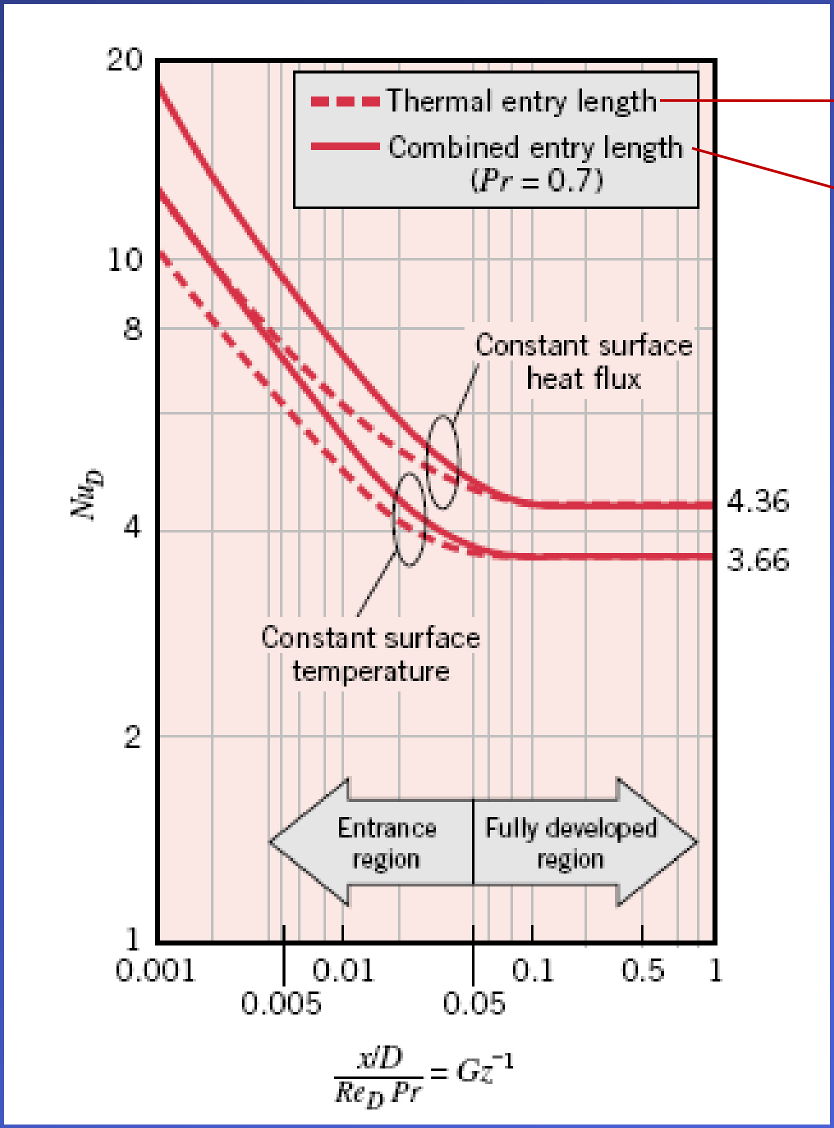
$$ou \quad Nu_D = \frac{hD}{k_f} = 4{,}36$$

–Temperatura Uniforme na Surperfíci e (T_s) , por solução numérica:

$$Nu_D = \frac{hD}{k} = 3.66$$

Região de Entrada

Laminar flow in a circular tube.



Perfil de velocidades estabelecido

Perfil de velocidades desenvolvendo junto com o perfil de temperatura

Gz=Número de Graetz

- Número de Nusselt para escoamento Laminar num duto circular com temperatura uniforme na superfície:

  - Solução combinada:

$$\left[\mathrm{Re}_D \,\mathrm{Pr}/\left(L/D\right)\right]^{1/3}\left(\mu/\mu_s\right)^{0.14} > 2:$$

$$\overline{Nu}_D = 1.86\left(\frac{\mathrm{Re}_D \,\mathrm{Pr}}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$$\left[\mathrm{Re}_D \,\mathrm{Pr}/\left(L/D\right)\right]^{1/3}\left(\mu/\mu_s\right)^{0.14} < 2:$$

  - Comprimento de entrada térmico:

$$\overline{Nu}_D = 3.66 + \frac{0.0668\left(D/L\right)\mathrm{Re}_D \,\mathrm{Pr}}{1 + 0.04\left[\left(D/L\right)\mathrm{Re}_D \,\mathrm{Pr}\right]^{2/3}}$$

- Escoamento Turbulento em um Tubo circular:
  - Para superfície lisa e condições de escoamento turbulento $\left(\mathrm{Re}_D > 10{,}000\right)$, a Equação de Dittus – Boelter pode ser usada:

$$Nu_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^n \qquad \begin{cases} n = 0.3 & \left(T_s < T_m\right) \\ n = 0.4 & \left(T_s > T_m\right) \end{cases}$$

  - Os efeitos da rugosidade da parede e o escoamento de transição $\left(\mathrm{Re}_D > 3000\right)$ pode ser cosiderada utilizando a correlação de Gnielinski :

$$Nu_D = \frac{(f/8)\left(\mathrm{Re}_D - 1000\right)\mathrm{Pr}}{1 + 12.7(f/8)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)}$$

Superfície lisa:

$$f = (0.790 \ln Re_D - 1.64)^{-2}$$

Superfície rugosa $e > 0$

$$f \rightarrow \text{Figure 8.3}$$

- Tubos Não-circulares :

–diametro hidraulico :

$$D_h \equiv \frac{4A_c}{P}$$

– Escoamento Laminar :

O número de Nusselt local é constante (Table 8.1) e depende da Condição de contorno térmica da superfície, $(T_s \text{ or } q_s'')$ e da razão de aspecto do duto.

– Escoamento Turbulento:

Dittus-Boelter ou Gnielinski podem ser usadas com
O diametro hidraulico, independente da condição térmica da superfície

- Average Nusselt Number for Turbulent Flow in a Circular Tube :
  - Effects of entry and surface thermal conditions are less pronounced for turbulent flow and can be neglected.
  - For long tubes $(L/D > 60)$:

$$\overline{Nu}_D \approx Nu_{D,fd}$$

  - For short tubes $(L/D < 60)$:

$$\frac{\overline{Nu}_D}{Nu_{D,fd}} \approx 1 + \frac{C}{(L/D)^m}$$

$$C \approx 1$$
$$m \approx 2/3$$

- Noncircular Tubes:
  - Laminar Flow:

    $\overline{Nu}_{D_h}$ depends strongly on aspect ratio, as well as entry region and surface thermal conditions. See references 11 and 12.

– Turbulent Flow:

As a first approximation, correlations for a circular tube may be used with D replaced by D_h .

- When determining $\overline{Nu}_D$ for any tube geometry or flow condition, all properties are to be evaluated at

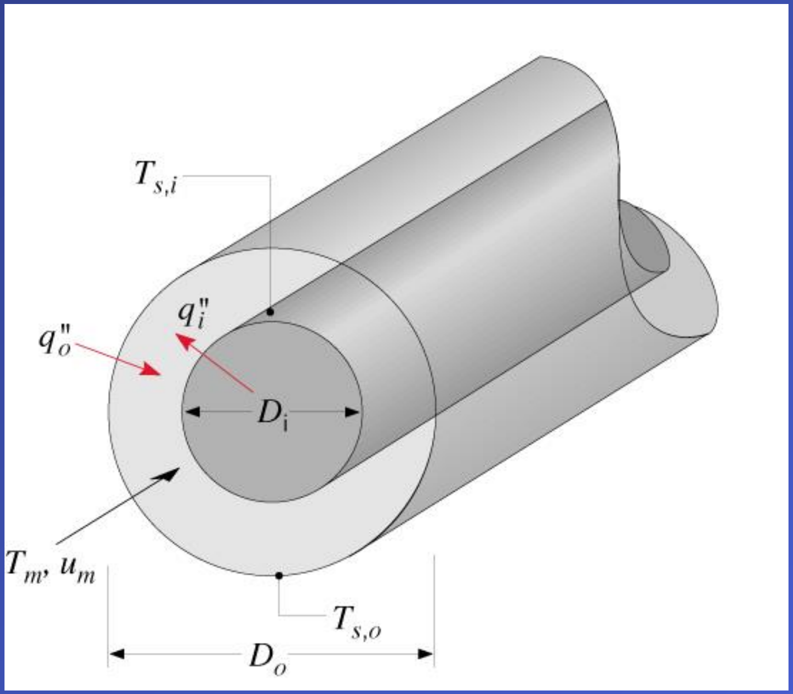
$$\overline{T}_m \equiv \left(T_{m,i} + T_{m,o}\right)/2$$

Why do solutions to internal flow problems often require iteration?

The Concentric Tube Annulus

- Fluid flow through region formed by concentric tubes.

- Convection heat transfer may be from or to inner surface of outer tube and outer surface of inner tube.



- Surface thermal conditions may be characterized by uniform temperature $(T_{s,i}, T_{s,o})$ or uniform heat flux (q''_i, q''_o).

- Convection coefficients are associated with each surface, where

$$q''_i = h_i\left(T_{s,i} - T_m\right)$$

$$q''_o = h_o\left(T_{s,o} - T_m\right)$$

$$Nu_i \equiv \frac{h_i D_h}{k} \qquad Nu_o \equiv \frac{h_o D_h}{k}$$

$$D_h = D_o - D_i$$

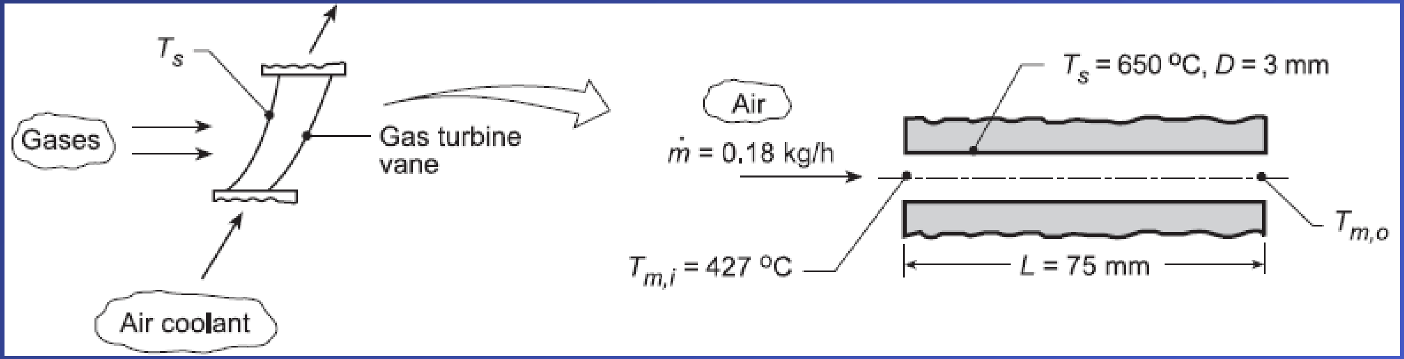
- Fully Developed Laminar Flow

Nusselt numbers depend on D_i / D_o and surface thermal conditions (Tables 8.2, 8.3)

- Fully Developed Turbulent Flow

Correlations for a circular tube may be used with D replaced by D_h .

Problem 8.43: For an air passage used to cool a gas turbine vane, calculate the air outlet temperature and heat removed from the vane.



KNOWN: Diameter and length of copper tubing. Temperature of collector plate to which tubing is soldered. Water inlet temperature and flow rate.

FIND: (a) Outlet temperature of the air coolant for the prescribed conditions and (b) Compute and plot the air outlet temperature $T_{m,o}$ as a function of flow rate, $0.1 \le \dot{m} \le 0.6$ kg/h. Compare this result with those for vanes having passage diameters of 2 and 4 mm.

ASSUMPTIONS: (1) Steady-state conditions, (2) Ideal gas with negligible viscous dissipation and pressure variation.

PROPERTIES: *Table A.4*, Air (assume $\bar{T}_m$ = 780 K, 1 atm): c_p = 1094 J/kg·K, k = 0.0563 W/m·K, $\mu = 363.7 \times 10^{-7}$ N·s/m^2, Pr = 0.706; (T_s = 650°C = 923 K, 1 atm): $\mu = 404.2 \times 10^{-7}$ N·s/m^2.

ANALYSIS: (a) For constant wall temperature heating, from Eq. 8.41b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\bar{h}}{\dot{m}c_p}\right) \qquad (1)$$

where P = πD. For flow in a circular passage,

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.18\,kg/h\,(1/3600\,s/h)}{\pi(0.003\,m)363.7\times 10^{-7}\,N\cdot s/m^2} = 584\,. \qquad (2)$$

The flow is laminar, and since L/D = 75 mm/3 mm = 25, the Sieder-Tate correlation including combined entry length yields

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 1.86\left(\frac{Re_D\,Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \qquad (3)$$

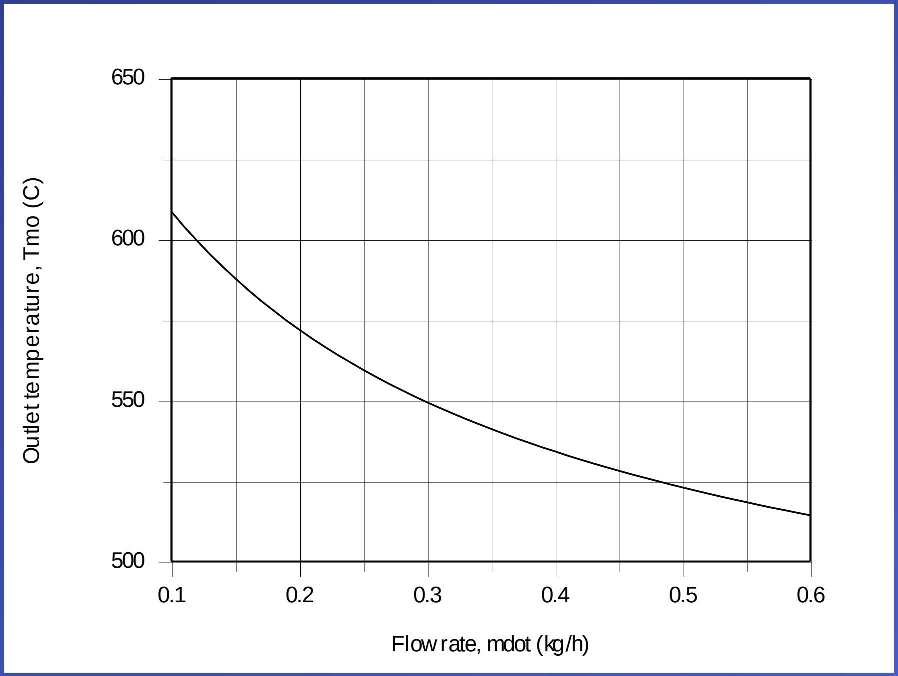
$$\bar{h} = \frac{0.0563\,W/m\cdot K}{0.003\,m}1.86\left(\frac{584\times 0.706}{25}\right)^{1/3}\left(\frac{363.7\times 10^{-7}}{404.2\times 10^{-7}}\right)^{0.14} = 87.5\,W/m^2\cdot K\,.$$

Hence, the air outlet temperature is

$$\frac{650 - T_{m,o}}{(650-427)^\circ C} = \exp\left(-\frac{\pi(0.003\,m)\times 0.075\,m\times 87.5\,W/m^2\cdot K}{(0.18/3600)\,kg/s\times 1094\,J/kg\cdot K}\right)$$

$$T_{m,o} = 578^\circ C$$

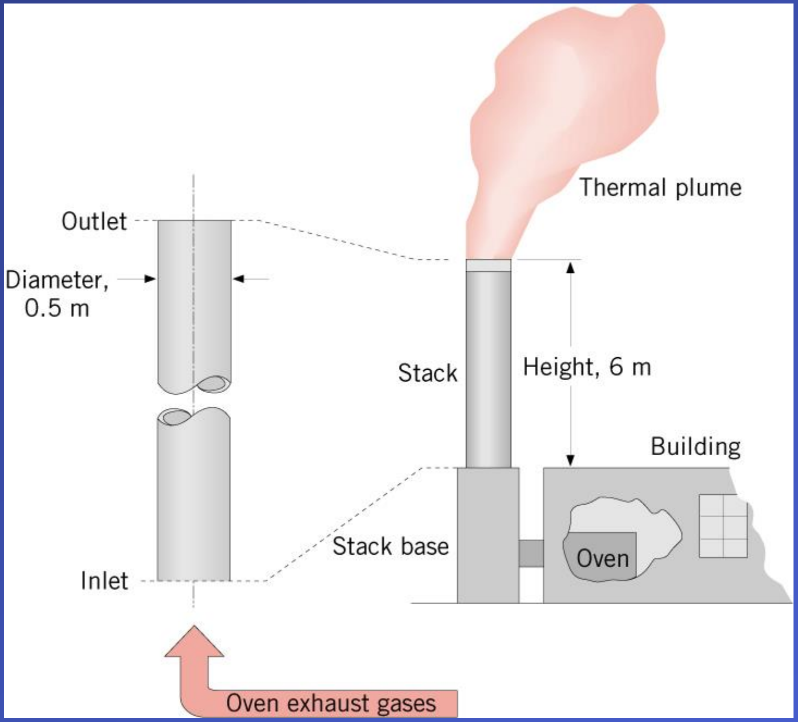
(b) Using the *IHT Correlations Tool, Internal Flow*, for *Laminar Flow* with *combined entry length*, along with the energy balance and rate equations above, the outlet temperature $T_{m,o}$ was calculated as a function of flow rate for diameters of D = 2, 3 and 4 mm. The plot below shows that $T_{m,o}$ decreases nearly linearly with increasing flow rate, but is independent of passage diameter.



Based upon the calculation for $T_{m,o}$ = 578°C, $\bar{T}_m$ = 775 K which is in good agreement with our assumption to evaluate the thermophysical properties.

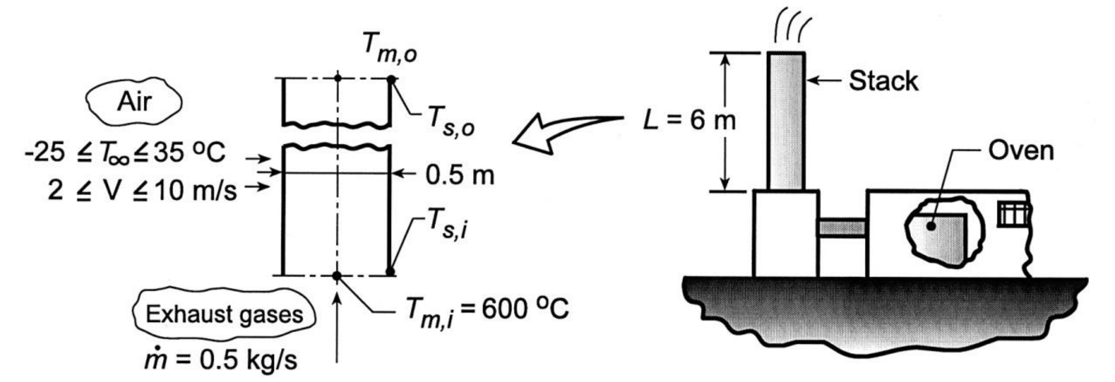
Why is $T_{m,o}$ independent of D? From Eq. (3), note that $\bar{h}$ is inversely proportional to D, $\bar{h} \sim D^{-1}$. From Eq. (1), note that on the right-hand side the product $P \cdot \bar{h}$ will be independent of D. Hence, $T_{m,o}$ will depend only on $\dot{m}$. This is, of course, a consequence of the laminar flow condition and will not be the same for turbulent flow.

Problem 8.52: Determine effect of ambient air temperature and wind velocity on temperature at which oven gases are discharged from a stack.



KNOWN: Thin-walled, tall stack discharging exhaust gases from an oven into the environment.

FIND: (a) Outlet gas and stack surface temperatures, $T_{m,o}$ and $T_{s,o}$, for V=5 m/s and $T_\infty = 4^\circ$C; (b) Effect of wind temperature and velocity on $T_{m,o}$.



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible wall thermal resistance, (3) Exhaust gas properties approximately those of atmospheric air, (4) Negligible radiative exchange with surroundings, (5) Ideal gas with negligible viscous dissipation and pressure variation, (6) Fully developed flow, (7) Constant properties.

PROPERTIES: *Table A.4*, air (assume $T_{m,o}$ = 773 K, $\bar{T}_m$ = 823 K, 1 atm): c_p = 1104 J/kg·K, $\mu = 376.4 \times 10^{-7}$ N·s/m^2, k = 0.0584 W/m·K, Pr = 0.712; *Table A.4*, air (assume T_s = 523 K, T_∞ = 4°C = 277 K, T_f = 400 K, 1 atm): $\nu = 26.41 \times 10^{-6}$ m^2/s, k = 0.0338 W/m·K, Pr = 0.690.

ANALYSIS: (a) From Eq. 8.45a,

$$T_{m,o} = T_\infty - \left(T_\infty - T_{m,i}\right)\exp\left[-\frac{PL}{\dot{m}c_p}\bar{U}\right] \qquad \bar{U} = 1\Big/\left(\frac{1}{\bar{h}_i} + \frac{1}{\bar{h}_o}\right)$$

where $\bar{h}_i$ and $\bar{h}_o$ are average coefficients for internal and external flow, respectively.

Internal flow: With a Reynolds number of

$$Re_{D_i} = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.5\,\text{kg/s}}{\pi \times 0.5\,\text{m} \times 376.4 \times 10^{-7}\,\text{N} \cdot \text{s/m}^2} = 33{,}827$$

The flow is turbulent, and assuming fully developed conditions throughout the stack, the Dittus-Boelter correlation may be used to determine $\bar{h}_i$.

$$\overline{Nu}_D = \frac{\bar{h}_i D}{k} = 0.023\,Re_{D_i}^{4/5}\,Pr^{0.3}$$

$$\bar{h}_i = \frac{58.4 \times 10^{-3}\,\text{W/m} \cdot \text{K}}{0.5\,\text{m}} \times 0.023(33{,}827)^{4/5}(0.712)^{0.3} = 10.2\,\text{W/m}^2 \cdot \text{K}$$

External flow: Working with the Churchill/Bernstein correlation and

$$Re_{D_o} = \frac{VD}{\nu} = \frac{5\,\text{m/s} \times 0.5\,\text{m}}{26.41 \times 10^{-6}\,\text{m}^2/\text{s}} = 94{,}660$$

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} = 205$$

Hence,

$$\bar{h}_o = \left(0.0338\,\text{W/m}\cdot\text{K}/0.5\,\text{m}\right)\times 205 = 13.9\,\text{W}/\text{m}^2\cdot\text{K}$$

The outlet gas temperature is then

$$T_{m,o} = 4^\circ\text{C} - \left(4 - 600\right)^\circ\text{C}\exp\left[-\frac{\pi\times 0.5\,\text{m}\times 6\,\text{m}}{0.5\,\text{kg/s}\times 1104\,\text{J/kg}\cdot\text{K}}\left(\frac{1}{1/10.2 + 1/13.9}\,\text{W}/\text{m}^2\cdot\text{K}\right)\right] = 543^\circ\text{C}$$

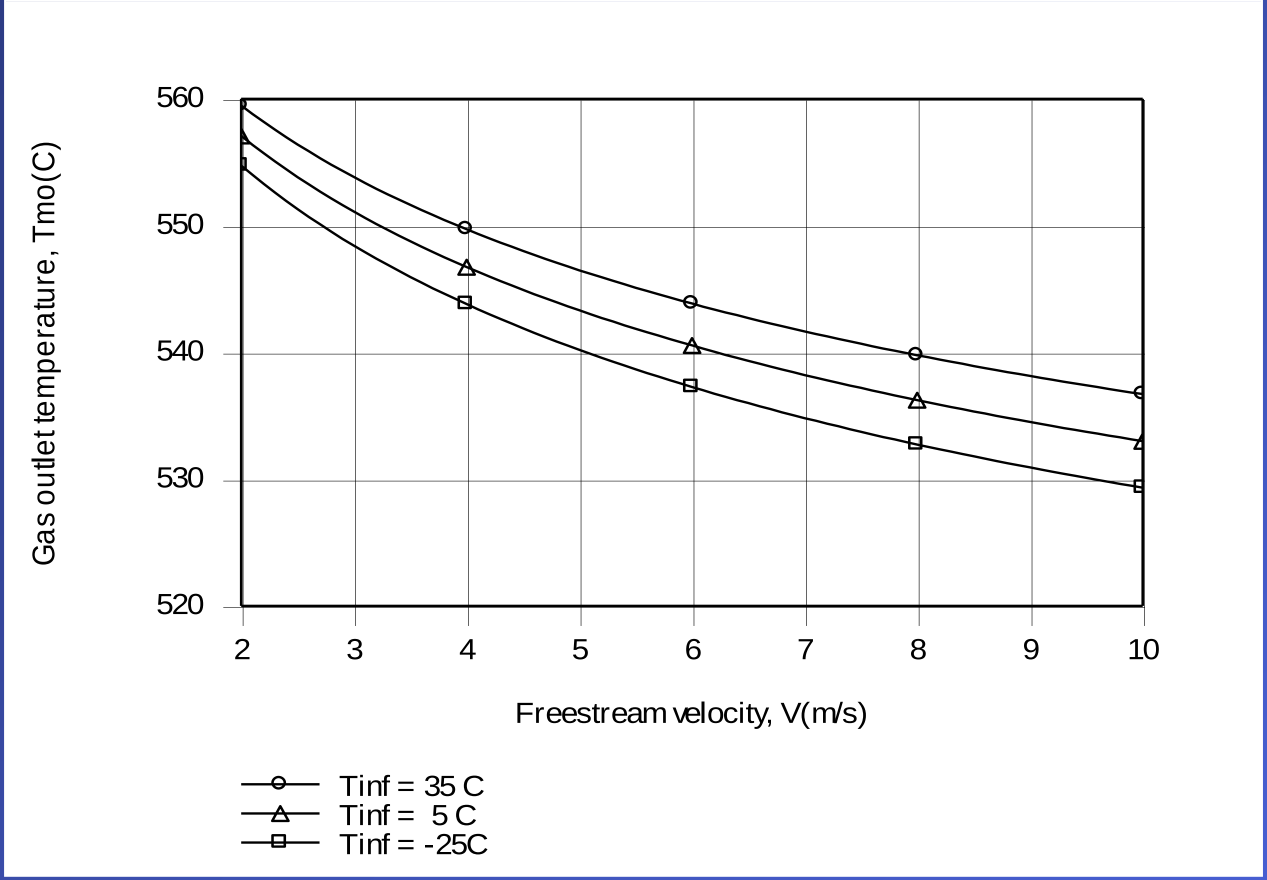
The outlet stack surface temperature can be determined from a local surface energy balance of the form,

$$h_i(T_{m,o} - T_{s,o}) = h_o(T_{s,o} - T_\infty),$$

which yields

$$T_{s,o} = \frac{h_i T_{m,o} + h_o T_\infty}{h_i + h_o} = \frac{\left(10.2\times 543 + 13.9\times 4\right)\text{W}/\text{m}^2}{\left(10.2 + 13.9\right)\text{W}/\text{m}^2\cdot\text{K}} = 232^\circ\text{C}$$

b) The effects of the air temperature and velocity are as follows



Due to the elevated temperatures of the gas, the variation in ambient temperature has only a small effect on the gas exit temperature. However, the effect of the freestream velocity is more pronounced. Discharge temperatures of approximately 530 and 560°C would be representative of cold/windy and warm/still atmospheric conditions, respectively.

COMMENTS: If there are constituents in the gas discharge that condense or precipitate out at temperatures below $T_{s,o}$, related operating conditions should be avoided.

Internal Flow: Mass Transfer

Chapter 8

Section 8.9

General Considerations

- Convection mass transfer occurs for gas flow through a tube whose inner surface is coated with a volatile liquid (evaporation) or solid (sublimation) and characterized by a uniform concentration, $\rho_{A,s}$.

- A species concentration boundary layer develops in an entry region and a fully developed condition is eventually reached.

$$\left.\frac{x_{fd,c}}{D}\right)_{1am} \approx 0.05\,\mathrm{Re}_D\, Sc$$

$$10 < \left(\frac{x_{fd,c}}{D}\right)_{turb} < 60$$

- The *mean species concentration* is

$$\rho_{A,m} = \frac{\int_{A_c} (\rho_A u) dA_c}{u_m A_c}$$

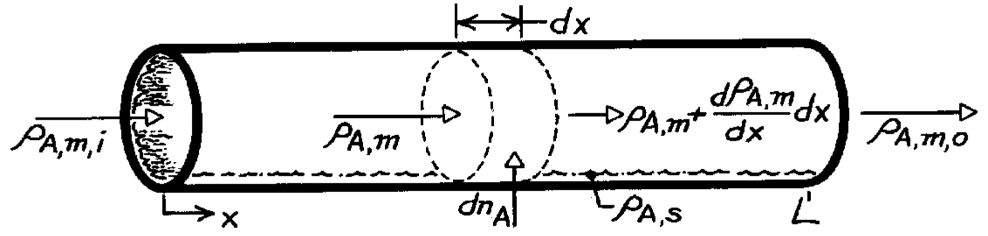
Species Transfer Rates

- Local Species Mass Flux:

$$n''_A = h_m\left(\rho_{A,s} - \rho_{A,m}\right)$$

- Longitudinal Distribution of Mean Species Concentration:

Applying conservation of species to a differential control volume in the tube,



the following expression may be obtained (Problem 8.112).

$$\frac{\rho_{A,s} - \rho_{A,m}(x)}{\rho_{A,s} - \rho_{A,m,i}} = \exp\left(-\frac{\bar{h}_m P}{u_m A_c}x\right)$$

where

$$u_m A_c = \frac{\dot{m}}{\rho}$$

$\dot{m}$ — Total mass flow rate.

ρ — Total mass density.

- Species Transfer Rate:

$$n_A = \frac{\dot{m}}{\rho}\left(\rho_{A,m,o} - \rho_{A,m,i}\right)$$

or

$$n_A = \overline{h}_m A_s \,\Delta\, \rho_{A,1m}$$

where

$$\Delta\, \rho_{A,1m} = \frac{\Delta\, \rho_{A,o} - \Delta\, \rho_{A,i}}{1\text{n}\left(\Delta\, \rho_{A,o} / \Delta\, \rho_{A,i}\right)}$$

and

$$\Delta\rho_A \equiv \rho_{A,s} - \rho_{A,m}$$

Convection Mass Transfer Coefficients

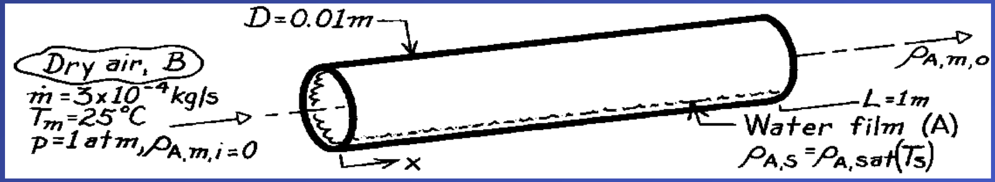
Obtained by analogy to heat transfer correlations for internal flow:

$$Sh_D \equiv \frac{h_m D}{D_{AB}} \leftrightarrow Nu_D$$

$$Sc \equiv \frac{\nu}{D_{AB}} \leftrightarrow \Pr$$

Problem 8.113: Evaporation rate and outlet vapor density for airflow through wetted circular tube.

KNOWN: Flow rate and temperature of air. Tube diameter and length. Presence of water film on tube inner surface.



FIND: (a) Vapor density at tube outlet, (b) Evaporation rate.

ASSUMPTIONS: (1) Steady, incompressible flow, (2) Constant flow rate, (3) Isothermal system (water film maintained at 25°C), (4) Fully developed flow.

PROPERTIES: *Table A-4*, Air (1 atm, 298K): $\rho = 1.1707$ kg/m^3, $\mu = 183.6 \times 10^{-7}$ N·s/m^2, $\nu = 15.71 \times 10^{-6}$ m^2/s; *Table A-6*, Water vapor (298K): $\rho_{A,sat} = 1/v_g = (1/44.25$ m^3/kg$) = 0.0226$ kg/m^3; *Table A-8*, Air-vapor (298K): $D_{AB} = 26 \times 10^{-6}$ m^2/s; $Sc = \nu/D_{AB} = 0.60$.

ANALYSIS: (a) From Equation 8.84,

$$\rho_{A,m,o} = \rho_{A,s} - \left(\rho_{A,s} - \rho_{A,m,i}\right)\exp\left(-\frac{\pi DL}{\dot{m}}\overline{h}_m\right)$$

$$Re_D = \frac{4\,\dot{m}}{\pi\, D\mu} = \frac{4\times 3\times 10^{-4}\,\text{kg/s}}{\pi\left(0.01\text{ m}\right)183.6\times 10^{-7}\,\text{N}\cdot\text{s/m}^2} = 2080.$$

Flow is laminar and from the mass transfer analogy to Eq. 8.57,

$$\overline{Sh}_D = 1.86\left(\frac{Re_D Sc}{L/D}\right)^{1/3} = 1.86\left(\frac{2080\times 0.60}{100}\right)^{1/3} = 4.31$$

$$\bar{h}_m = \frac{\overline{Sh}_D\; D_{AB}}{D} = \frac{4.31\times 26\times 10^{-6}\,\text{m}^2/\text{s}}{0.01\text{ m}} = 0.0112\text{ m/s}$$

$$\rho_{A,m,o} = 0.0226\text{ kg/m}^3 -$$

$$0.0226\text{ kg/m}^3\exp\left(-\frac{\pi\times 0.01\text{ m}\times 1\text{ m}\times 1.17\text{ kg/m}^3\times 0.0112\text{ m/s}}{3\times 10^{-4}\text{ kg/s}}\right) = 0.0169\text{ kg/m}^3$$

(b) The evaporation rate is

$$n_A = u_m A_c\left(\rho_{A,m,o} - \rho_{A,m,i}\right) = \frac{\dot{m}}{\rho}\left(\rho_{A,m,o}\right) = \frac{3\times 10^{-4}\text{ kg/s}}{1.1707\text{ kg/m}^3}\,0.0169\,\frac{\text{kg}}{\text{m}^3} = 4.33\times 10^{-6}\text{ kg/s}.$$

COMMENTS: With

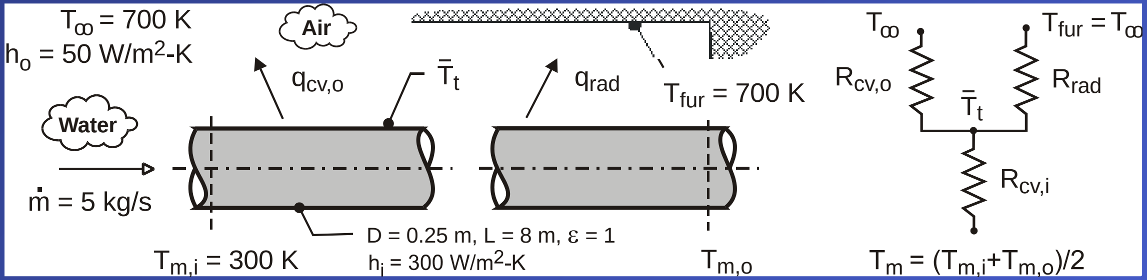
$$\Delta\rho_{A,o} = \Delta\rho_{A,i}\exp\left(-\frac{\pi\, DL\rho}{\dot{m}}\bar{h}_m\right) = 0.0226\ \exp\left(-\frac{\pi\times 0.01\times 1\times 1.17}{3\times 10^{-4}\ \text{kg/s}}0.0112\frac{\text{m}}{\text{s}}\right) = 5.73\times 10^{-3}\ \text{kg/m}^3$$

the evaporation rate is

$$n_A = \bar{h}_m \pi\, DL\frac{\Delta\rho_{A,o} - \Delta\rho_{A,i}}{\ell n\left(\Delta\rho_{A,o}/\Delta\rho_{A,i}\right)} = 0.0112\frac{\text{m}}{\text{s}}\pi\left(0.01\ \text{m}\right)1\text{m}\frac{\left(0.00573 - 0.0226\right)\text{kg/m}^3}{\ell n\left(0.00573/0.0226\right)} = 4.33\times 10^{-6}\,\text{kg/s}$$

which agrees with the result of part (b).

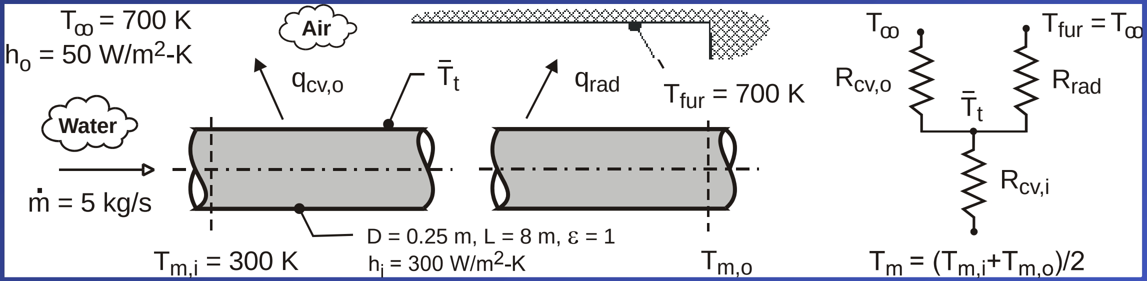
Problem 8.17: Estimate temperature of water emerging from a thin-walled tube heated by walls and air of a furnace. Inner and outer convection coefficients are known.



KNOWN: Water at prescribed temperature and flow rate enters a 0.25 m diameter, black thin-walled tube of 8-m length, which passes through a large furnace whose walls and air are at a temperature of $T_{fur} = T_\infty = 700$ K. The convection coefficients for the internal water flow and external furnace air are 300 $W/m^2 \cdot K$ and 50 $W/m^2 \cdot K$, respectively.

FIND: The outlet temperature of the water, $T_{m,o}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions; (2) Tube is small object with large, isothermal surroundings; (3) Furnace air and walls are at the same temperature; (4) Tube is thin-walled with black surface, and (5) Incompressible liquid with negligible viscous dissipation.

PROPERTIES: *Table A-6*, Water: $c_p \approx 4180$ J/kg·K.

ANALYSIS: The linearized radiation coefficient may be estimated from Eq. 1.9 with ε = 1,

$$\bar{h}_{rad} \approx \sigma\left(\bar{T}_t + \bar{T}_{fur}\right)\left(\bar{T}_t^2 + T_{fur}^2\right)$$

where $\bar{T}_t$ represents the average tube wall surface temperature, which can be estimated from an energy balance on the tube.

As represented by the thermal circuit, the energy balance may be expressed as

$$\frac{T_m - \bar{T}_t}{R_{cv,i}} = \frac{\bar{T}_t - T_{fur}}{1/R_{cv,o} + 1/R_{rad}}$$

The thermal resistances, with A_s = PL = πDL, are

$$R_{cv,i} = 1/h_i A_s \qquad R_{cv,o} = 1/h_o A_s \qquad R_{rad} = 1/\bar{h}_{rad}$$

and the mean temperature of the water is approximated as

$$T_m = \left(T_{m,i} + T_{m,o}\right)/2$$

The outlet temperature can be calculated from Eq. 8.45b, with $T_{fur} = T_\infty$,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{1}{\dot{m}\, c_p\, R_{tot}}\right)$$

where

$$R_{tot} = R_{cv,i} + \frac{1}{1/R_{cv,o} + 1/R_{rad}}$$

with

$$R_{cv,i} = 6.631 \times 10^{-5}\ \text{K/W} \qquad R_{cv,o} = 3.978 \times 10^{-4}\ \text{K/W} \qquad R_{rad} = 4.724 \times 10^{-4}\ \text{K/W}$$

it follows that

$$T_m = 331\ \text{K} \qquad \bar{T}_t = 418\ \text{K} \qquad T_{m,o} = 362\ \text{K}$$