Convecção Livre ou Convecção Natural
Placas Verticais ou Horizontais

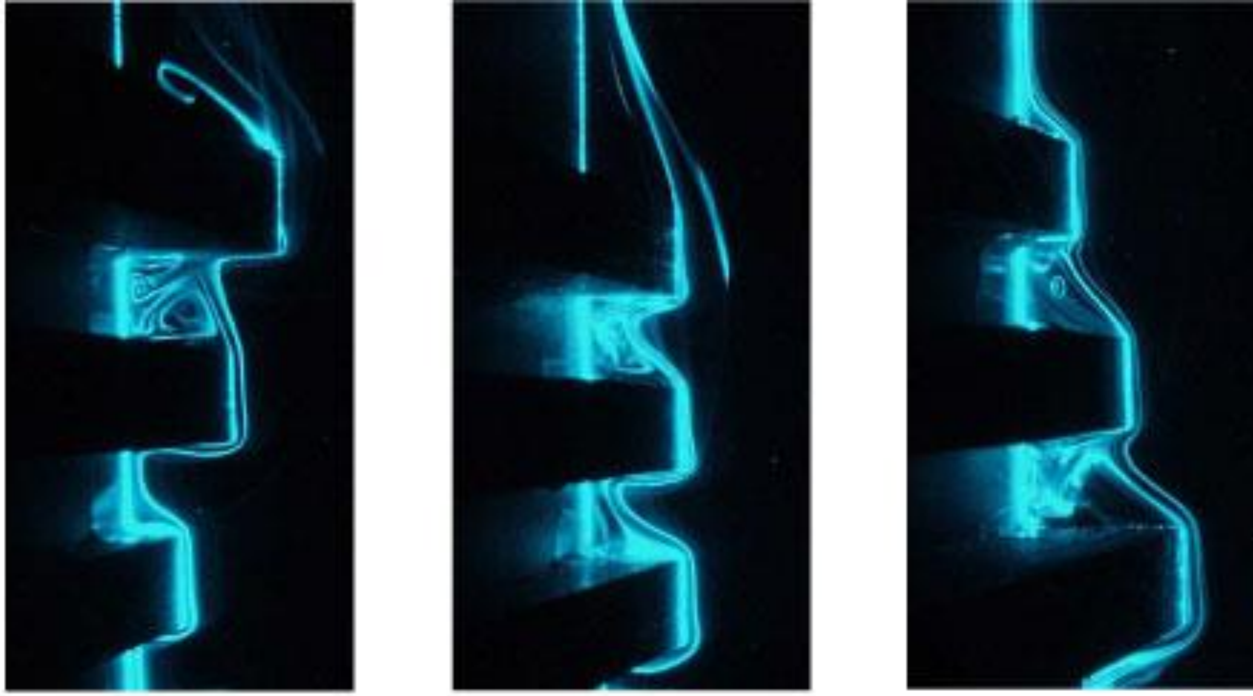

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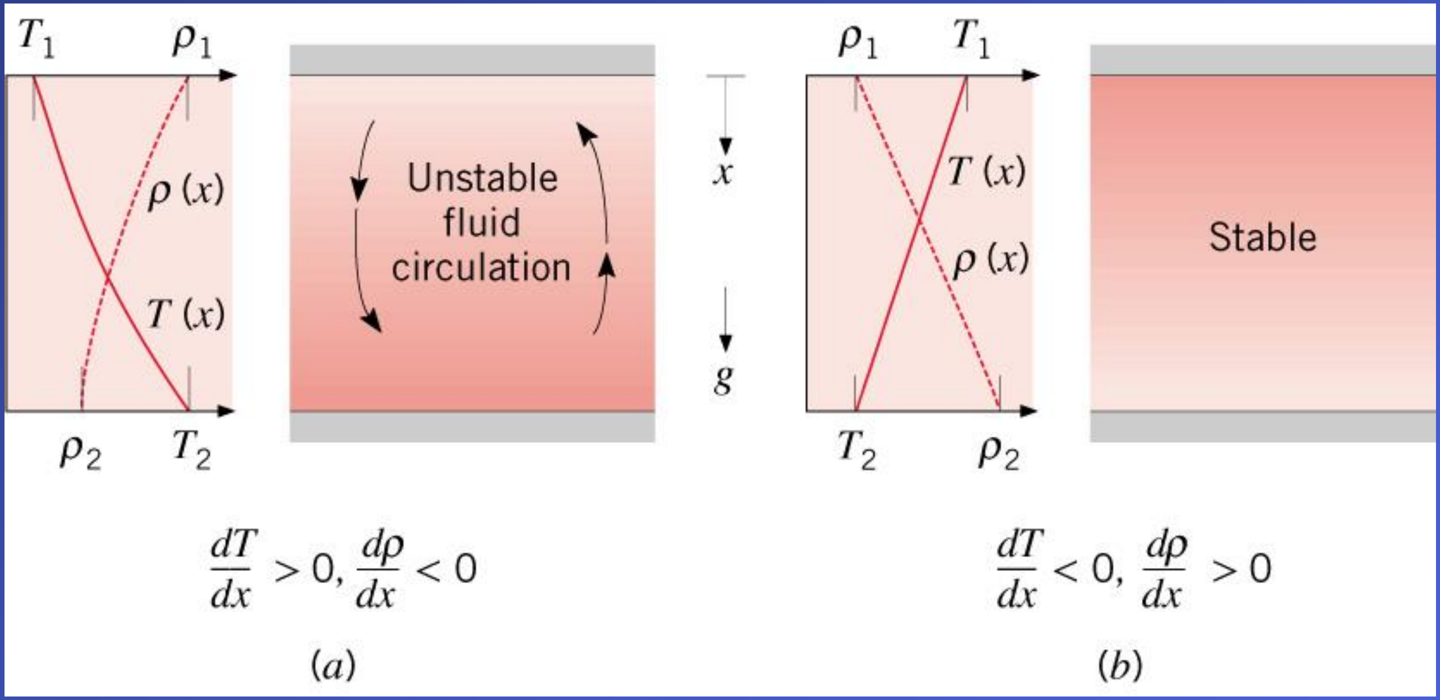
Figure 3.9. Streakline patterns visualised with an electrolytical precipitation method (t = 300s).

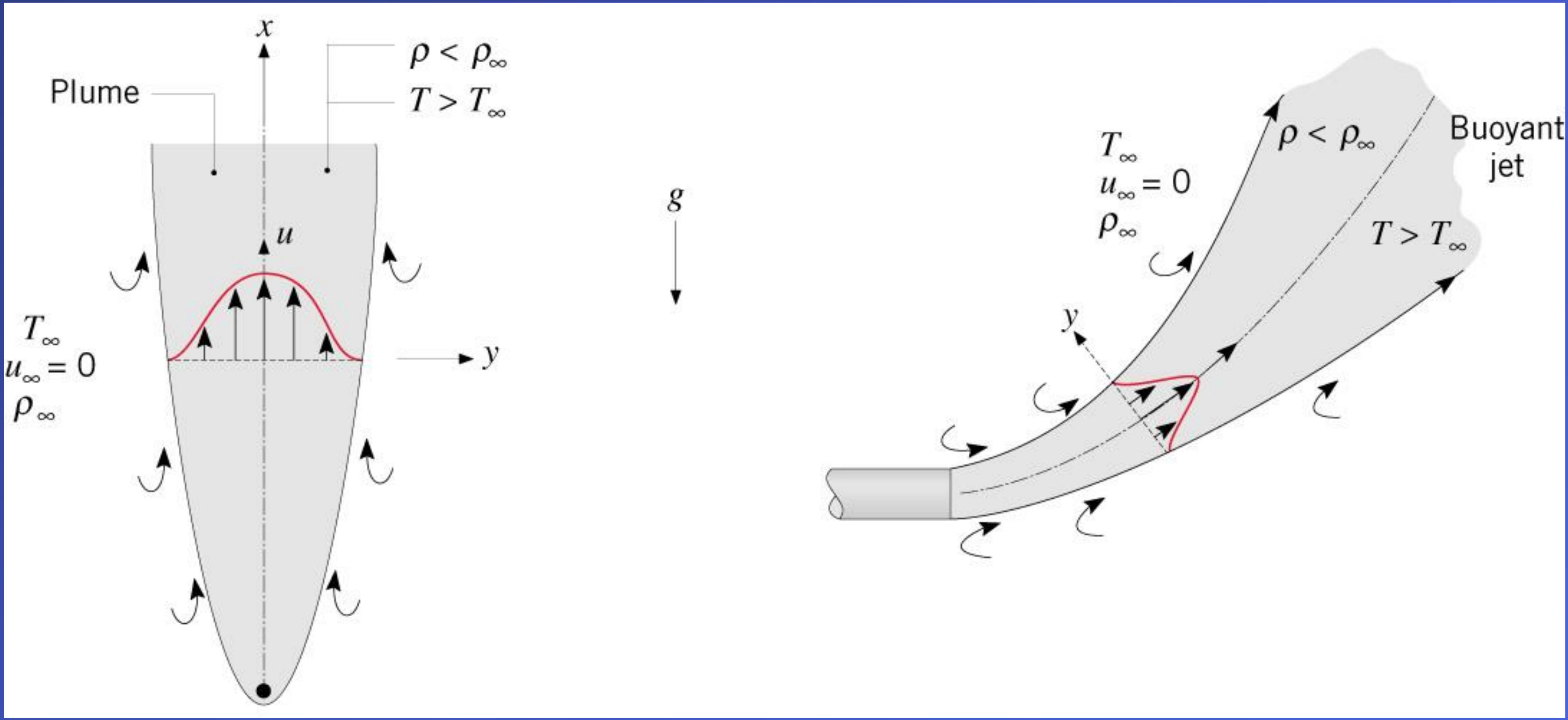
Princípios

- Relacionada com forças de empuxo, densidade e gravidade



- Fluxos de fluido

  - Plumas e jatos de conveção natural:



- Parametros Adimensionais

  - Número de Grashof

$$Gr_L = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu^2} \sim \frac{\text{Buoyancy Force}}{\text{Viscous Force}}$$

$L \rightarrow$ characteristic length of surface

$\beta \rightarrow$ thermal expansion coefficient (a thermodynamic property of the fluid)

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

Liquids: $\beta \rightarrow$ Tables A.5, A.6

Perfect Gas: $\beta = 1/T\,(\mathrm{K})$

  - Número de Rayleigh

$$Ra_L = Gr_L \Pr = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha}$$

- Convecção Mista

➢ Se

$$\left(Gr_L / \mathrm{Re}_L^2\right) \sim o(1)$$

- Free convection $\rightarrow \left(Gr_L / \mathrm{Re}_L^2\right) \gg 1$
- Forced convection $\rightarrow \left(Gr_L / \mathrm{Re}_L^2\right) \ll 1$

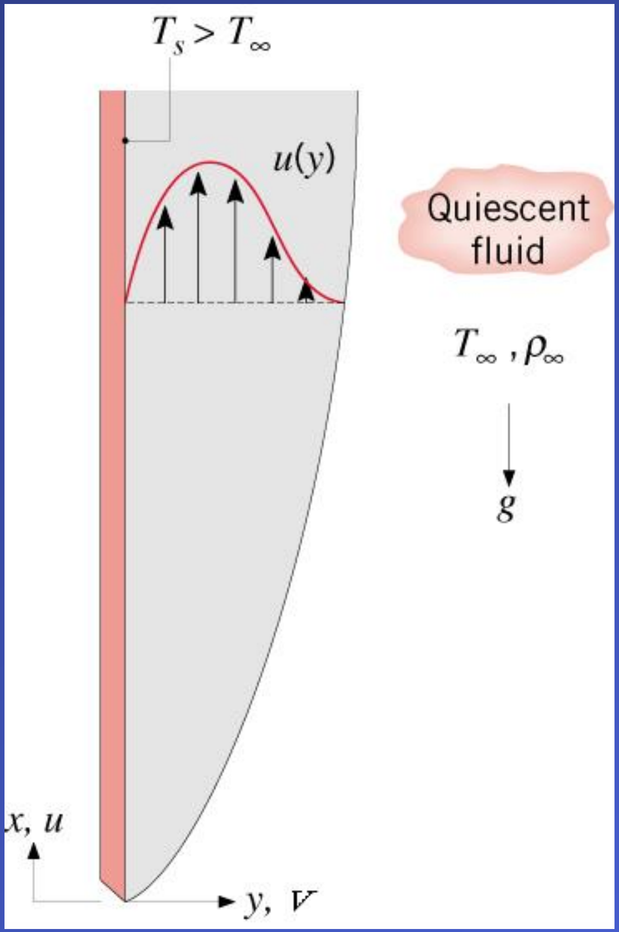
➢ Correlações para convecção Mista

$$Nu^n \approx Nu_{FC}^n \pm Nu_{NC}^n$$

$+ \rightarrow$ assisting and transverse flows
$- \rightarrow$ opposing flows

$$n \approx 3$$

Placas Verticais



- How do conditions differ from those associated with forced convection?
- How do conditions differ for a cooled plate $(T_s < T_\infty)$?

•*x*-Momentum Equação para escoamento Laminar

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\rho g$$

$$\frac{\partial p}{\partial x}=-\rho_\infty g$$

$$g\frac{\rho_\infty}{\rho}-g=g\left(\frac{\rho_\infty-\rho}{\rho}\right)=g\left(\frac{\Delta\rho}{\rho}\right)$$

$$\underbrace{u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}}_{\text{Net Momentum Fluxes (Inertia Forces)}}=\underbrace{g\beta\left(T-T_\infty\right)}_{\text{Buoyancy Force}}+\underbrace{\nu\frac{\partial^2 u}{\partial y^2}}_{\text{Viscous Force}}$$

Aproximação de Boussinesq:

$$\beta=-\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p\approx-\frac{1}{\rho}\frac{\Delta\rho}{\Delta T}=-\frac{1}{\rho}\frac{\rho_\infty-\rho}{T_\infty-T}$$

$$ou\quad\left(\rho_\infty-\rho\right)=-\rho\beta\left(T_\infty-T\right)=\rho\beta\left(T-T_\infty\right)$$

- Solução do campo de temperatura requer o conhecimento de $u(x,y)$ que deve ser
- obtido <u>acoplado</u> com a equação da energia para $T(x,y)$.

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

- Solução por similaridade

$$\eta \equiv \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4}$$

  - Transformando as equações para:

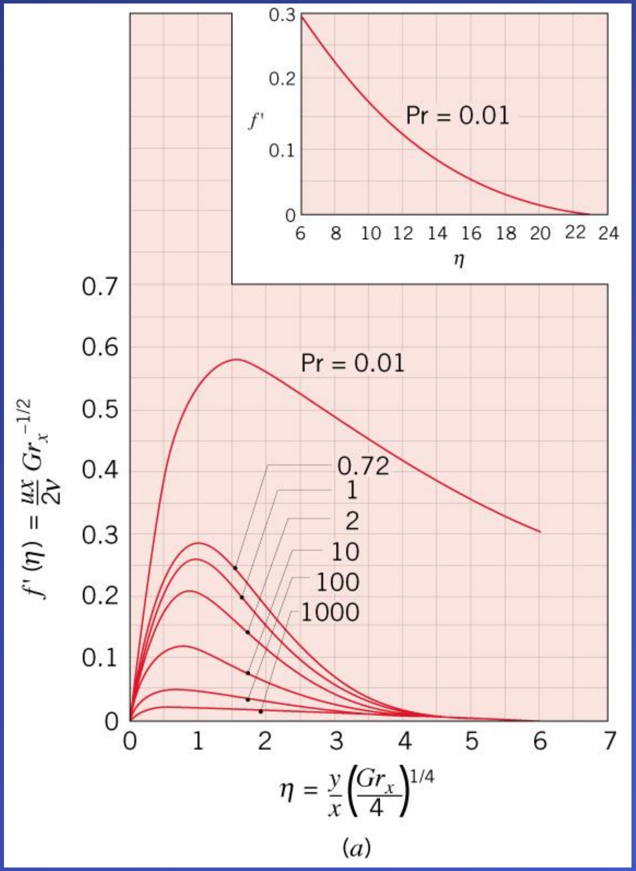
$$f''' + 3ff'' - 2(f')^2 + T^* = 0$$

$$T^{*''} + 3\Pr fT^{*'} = 0$$

$$f'(\eta) \equiv \frac{df}{d\eta} = \frac{x}{2\nu}\left(Gr_x^{-1/2}\right)u \qquad T^* \equiv \frac{T - T_\infty}{T_s - T_\infty}$$

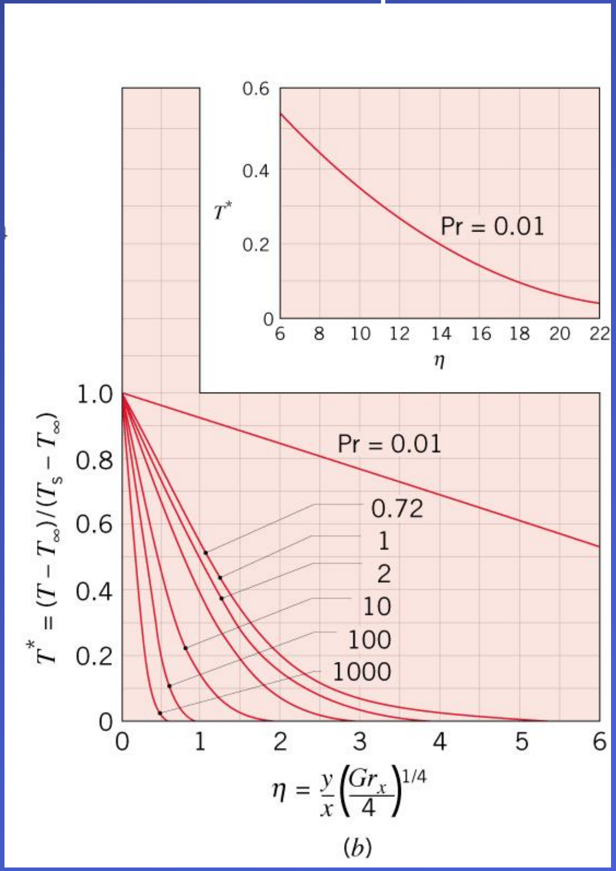
- Soluções numéricas para $f'(\eta)$ and T^* :

Perfil de velocidade



(a)

Perfil de temperatura



(b)

- Camada limite: $(\delta) \rightarrow \eta \approx 5 \text{ for } \Pr > 0.6$

- $\Pr > 0.6: \quad \delta = 5x\left(\frac{Gr_x}{4}\right)^{-1/4} = 7.07\frac{x}{(Gr_x)^{1/4}} \propto x^{1/4}$

- Número de Nusselt $\left(Nu_x \text{ and } \overline{Nu}_L\right)$:

$$Nu_x = \frac{hx}{k} = -\left(\frac{Gr_x}{4}\right)^{1/4} \left.\frac{dT^*}{d\eta}\right|_{\eta=0} = \left(\frac{Gr_x}{4}\right)^{1/4} g\left(\Pr\right)$$

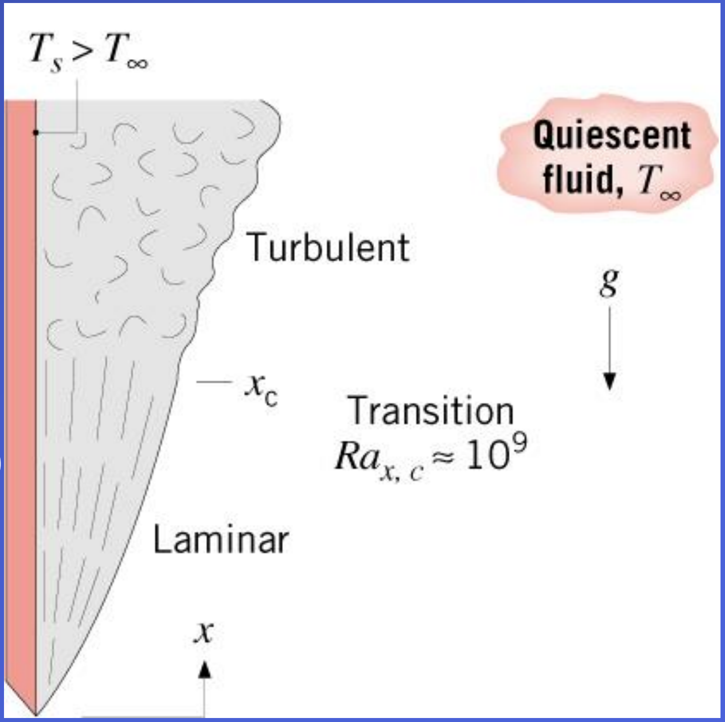
$$g\left(\Pr\right) = \frac{0.75\,\Pr^{1/2}}{\left(0.609 + 1.221\,\Pr^{1/2} + 1.238\,\Pr\right)^{1/4}} \qquad \left(0 < \Pr < \infty\right)$$

$$\bar{h} = \frac{1}{L}\int_o^L h\,dx \rightarrow \overline{Nu}_L = \frac{4}{3} Nu_L$$

- Transição para a Turbulência

  - Transitção ocorre para o Número de Rayleigh crítico.

$$Ra_{x,c} = Gr_{x,c}\Pr = \frac{g\beta\left(T_s - T_\infty\right)x^3}{\nu\alpha} \approx 10^9$$



- Correlações Empíricas
  - Escoamento Laminar $\left(Ra_L < 10^9\right)$:

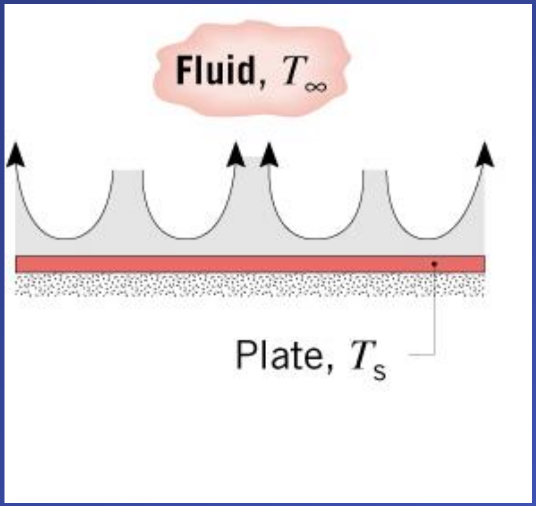
$$\overline{Nu}_L = 0.68 + \frac{0.670\ Ra_L^{1/4}}{\left[1+\left(0.492/\mathrm{Pr}\right)^{9/16}\right]^{4/9}}$$

  - Qualquer condição (laminar e turbulento):

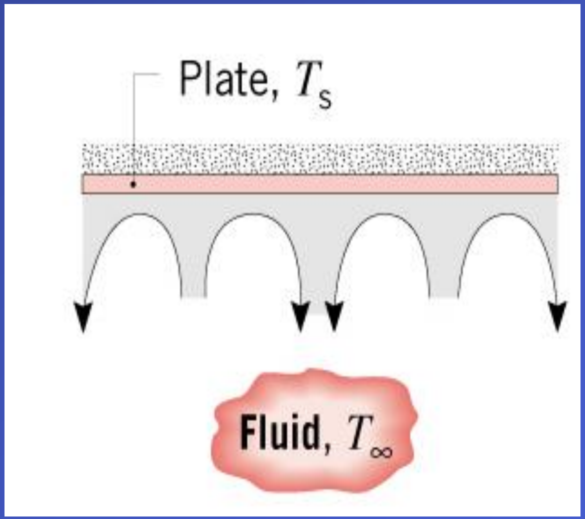
$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\ Ra_L^{1/6}}{\left[1+\left(0.492/\mathrm{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2$$

Placas Horizontais

- Superfície quente na parte inferior ou Superfície fria na parte superior



$T_s > T_\infty$



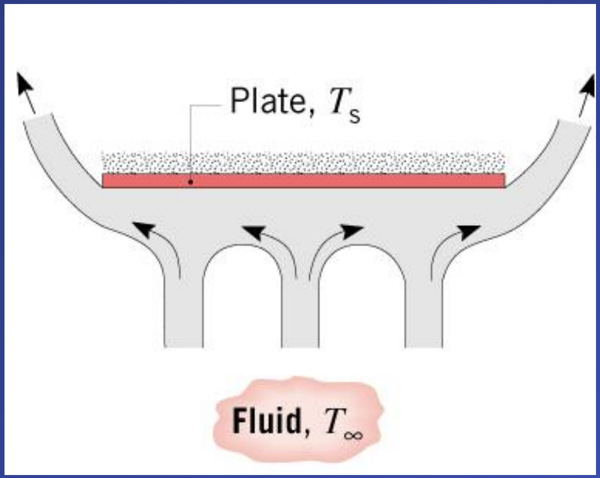
$T_s < T_\infty$

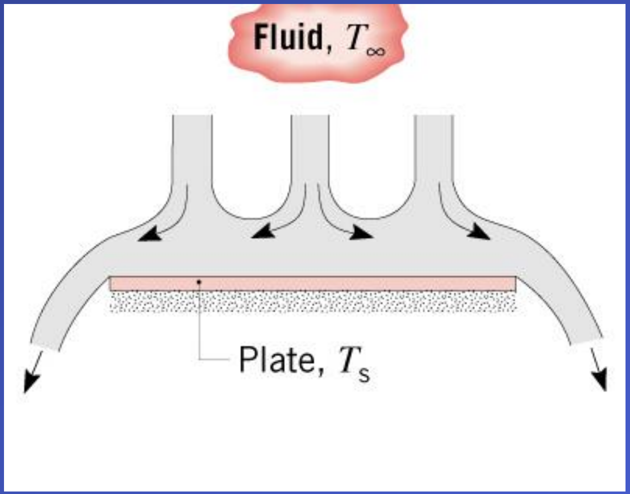
$$\overline{Nu}_L = 0.54\, Ra_L^{1/4} \qquad \left(10^4 < Ra_L < 10^7\right)$$

$$\boxed{L = \frac{A_S}{P}}$$

$$\overline{Nu}_L = 0.15\, Ra_L^{1/3} \qquad \left(10^7 < Ra_L < 10^{11}\right)$$

- invertendo com saídas laterais





$T_s > T_\infty$

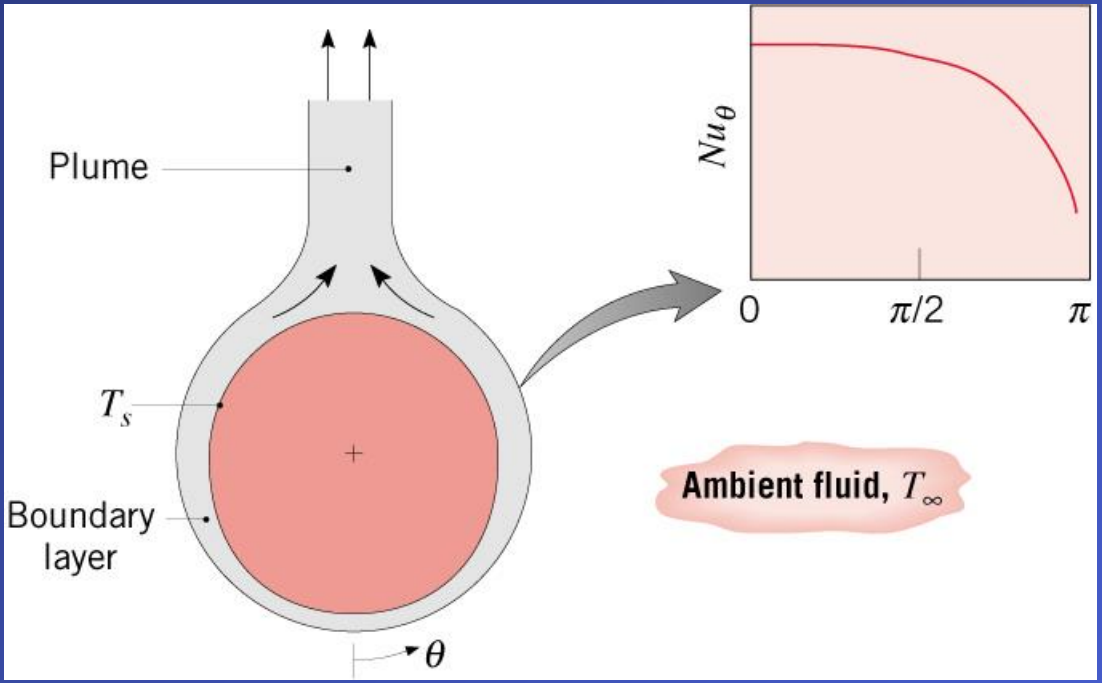
$T_s < T_\infty$

$$\overline{Nu}_L = 0.27 Ra_L^{1/4} \qquad \left(10^5 < Ra_L < 10^{10}\right)$$

$$\boxed{L = \frac{A_s}{P}}$$

Convecção Natural em:
Cilindros, Esferas e Cavidades

Cilindro Horizontal Longo



- Número de Nusselt médio:

$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/\mathrm{Pr})^{9/16}\right]^{8/27}} \right\}^2 \qquad Ra_D < 10^{12}$$

- How do conditions change for a cooled cylinder?

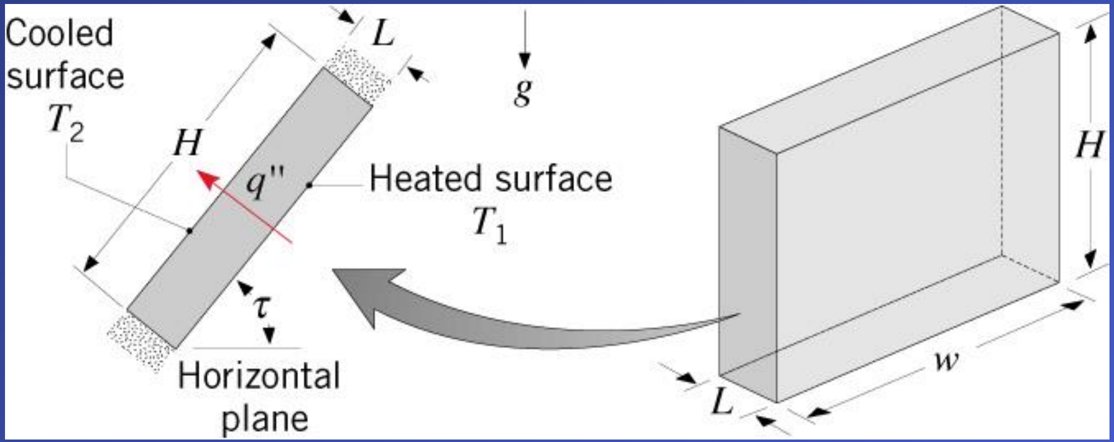
Esferas

- Número de Nusselt médio:

$$\overline{Nu}_D = 2 + \frac{0.589\ Ra_D^{1/4}}{\left[1 + \left(0.469/\Pr\right)^{9/16}\right]^{4/9}}$$

Confinamentos

- Cavidades retangulares



$$Ra_L \equiv \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu}$$

➢ $q'' = h(T_1 - T_2)$

➢ Cavidade Horizontal $\rightarrow \tau = 0, 180\,\text{deg}$

➢ Cavidade Vertical $\rightarrow \tau = 90\ \text{deg}$

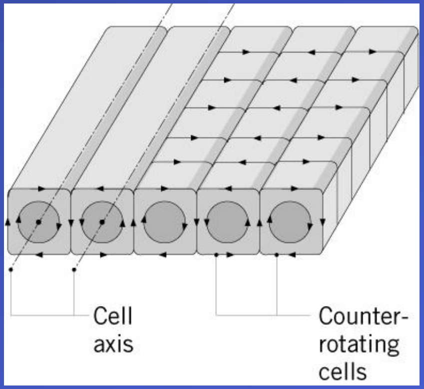
- Cavidade Horizontal
  - Aquecimento inferior $(\tau = 0)$
    - $Ra_L < Ra_{L,c} = 1708:$

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = 1$$

Fluid layer is thermally stable.

$-1708 < \mathrm{Ra}_L < 5 \times 10^4 :$

Thermal instability yields a regular convection pattern in the form of roll cells.



– $3 \times 10^5 < Ra_L < 7 \times 10^9 :$

Buoyancy driven flow is turbulent

$$\overline{Nu}_L = 0.069\ Ra_L^{1/3}\,\mathrm{Pr}^{0.074}$$

- Aquecimento superior $(\tau = 180 \text{ deg})$
  - Sem movimento.

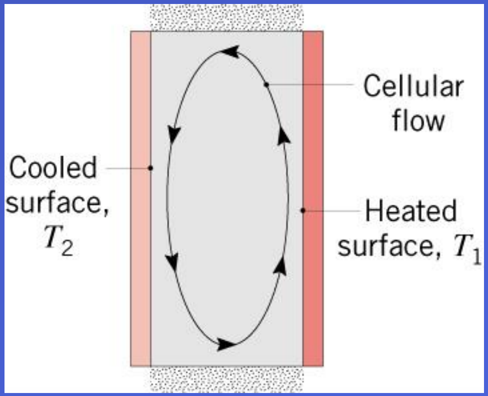
$$\overline{Nu}_L = 1$$

- Cavidade Vertical

- $Ra_L < 10^3$:

$$\overline{Nu}_L = 1$$

- $Ra_L > 10^3$:
  - A primary cellular flow is established, as the core becomes progressively more quiescent, and secondary (corner) cells develop with increasing Ra_L.

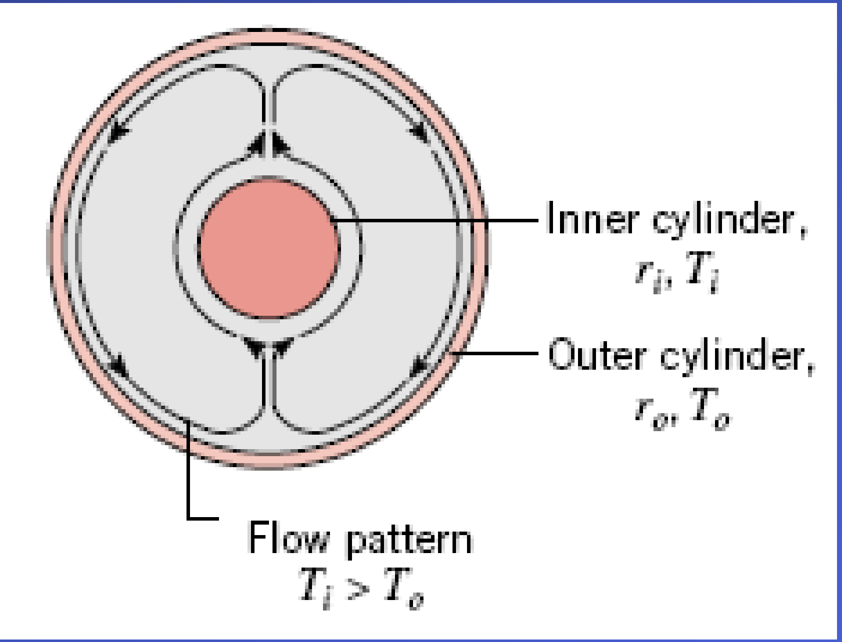


- Correlations for $\overline{Nu}_L \rightarrow$ Eqs. (9.50) - (9.53).

- Cavidade inclinada
  - Relevant to flat plate solar collectors.
  - Heat transfer depends on the magnitude of τ relative to a critical angle τ^*, whose value depends on *H/L* (Table 9.4).
  - Heat transfer also depends on the magnitude of Ra_L relative to a critical Rayleigh number of $Ra_{L,c} = 1708/\cos\tau$.
  - Heat transfer correlations $\longrightarrow$ Eqs. (9.54) – (9.57).

Cavidades Anulares

- Cilindros Concentricos



- $q' = \dfrac{2\pi k_{eff}}{\ln(r_o/r_i)}(T_i - T_o)$
- $\dfrac{k_{eff}}{k} = 0.386\left(\dfrac{Pr}{0.861+Pr}\right)^{1/4} Ra_c^{1/4}$

or $k_{eff}/k = 1$ if the value calculated above is less than unity.

- The length scale in Ra_c is given by

$$L_c = \frac{2\left[ln\left(r_o/r_i\right)\right]^{4/3}}{\left(r_i^{-3/5}+r_o^{-3/5}\right)^{5/3}}$$

- Concentric Spheres
  - $$q = \frac{4\pi k_{eff}\left(T_i - T_o\right)}{\left(1/r_i\right)-\left(1/r_o\right)}$$
  - $$\frac{k_{eff}}{k} = 0.74\left(\frac{Pr}{0.861+Pr}\right)^{1/4} Ra_s^{1/4}$$

  or $k_{eff}/k = 1$ if the value calculated above is less than unity.

  - The length scale in Ra_s is given by

$$L_s = \frac{\left(1/r_i - 1/r_o\right)^{4/3}}{2^{1/3}\left(r_i^{-7/5}+r_o^{-7/5}\right)^{5/3}}$$

Convecção natural:
Transferência de Massa

Transferência de Massa

- A analogia entre transferência de calor e massa pode ser utilizada se as variações da densidade com a temperatura são muito inferiores as variações oriundas dos gradientes de concentração. Nestes casos:

$$\overline{Sh}_L \equiv \frac{\overline{h}_m L}{D_{AB}} = f\left(Gr_L, Sc\right)$$

$$Gr_L = \frac{g\left(\rho_s - \rho_\infty\right)L^3}{\rho \nu^2}$$

$$\rho_s = \rho_{s,A} + \rho_{s,B}$$

$$\rho_\infty = \rho_{\infty,A} + \rho_{\infty,B}$$

$$\rho = \left(\rho_s + \rho_\infty\right)/2$$

Placas Verticais (analogous to Eq. 9.24):

$$\overline{Sh}_L = C\left(Gr_L Sc\right)^n$$

laminar - $C = 0.59, n = 1/4 \quad \left(10^4 < Ra_L < 10^9\right)$

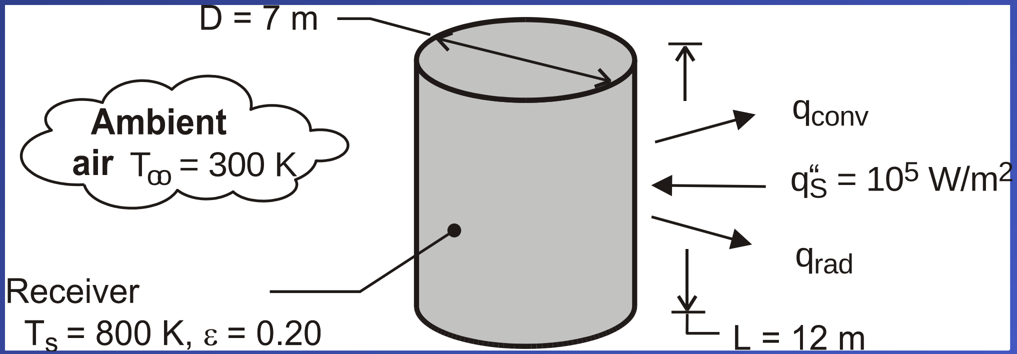
turbulent - $C = 0.10, n = 1/3 \quad \left(10^9 < Ra_L < 10^{13}\right)$

Placas Horizontais: $\rho_s < \rho_\infty$ at lower surface or $\rho_s > \rho_\infty$ at upper surface.

$$\overline{Sh}_L = 0.27\left(GR_L Sc\right)^{1/4} \quad \left(10^5 < Ra_L < 10^{10}\right)$$

Analoga a Eq. 9.32.

Problem 9.31: Convection and radiation losses from the surface of a central solar receiver.



KNOWN: Dimensions and emissivity of cylindrical solar receiver. Incident solar flux. Temperature of ambient air.

FIND: (a) Heat loss and collection efficiency for a prescribed receiver temperature, (b) Effect of receiver temperature on heat losses and collector efficiency.

ASSUMPTIONS: (1) Steady-state, (2) Ambient air is quiescent, (3) Incident solar flux is uniformly distributed over receiver surface, (4) All of the incident solar flux is absorbed by the receiver, (5) Negligible irradiation from the surroundings, (6) Uniform receiver surface temperature, (7) Curvature of cylinder has a negligible effect on boundary layer development, (8) Constant properties

PROPERTIES: *Table A-4*, air (T_f = 550 K): k = 0.0439 W/m·K, $\nu = 45.6 \times 10^{-6}$ m^2/s, α = 66.7×10^{-6} m^2/s, Pr = 0.683, $\beta = 1.82 \times 10^{-3}$ K^{-1}.

ANALYSIS: (a) The total heat loss is

$$q = q_{rad} + q_{conv} = A_s \varepsilon\sigma T_s^4 + \bar{h} A_s \left(T_s - T_\infty\right)$$

With $Ra_L = g\beta(T_s - T_\infty)L^3/\nu\alpha = 9.8\ \text{m/s}^2\ (1.82 \times 10^{-3}\ \text{K}^{-1})\ 500\text{K}\ (12\text{m})^3/(45.6 \times 66.7 \times 10^{-12}\ \text{m}^4/\text{s}^2) = 5.07 \times 10^{12}$, the Churchill and Chu correlation yields

$$\bar{h} = \frac{k}{L}\left\{0.825 + \frac{0.387\,Ra_L^{1/6}}{\left[1 + \left(0.492/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \frac{0.0439\ \text{W/m}\cdot\text{K}}{12\text{m}}\left\{0.825 + 42.4\right\}^2 = 6.83\ \text{W/m}^2\cdot\text{K}$$

Hence, with $A_s = \pi DL = 264\ \text{m}^2$

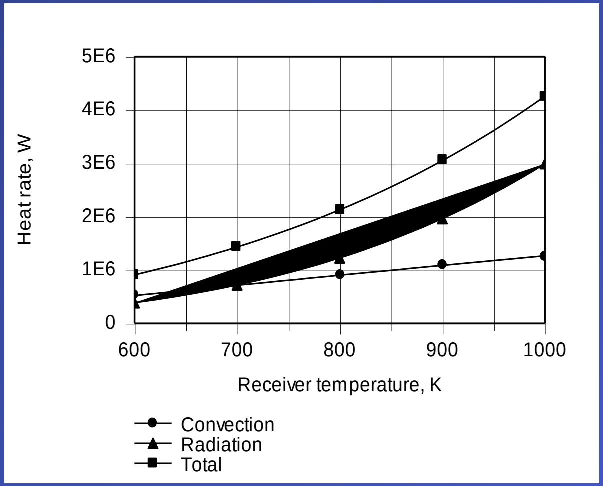
$$q = 264\,\text{m}^2 \times 0.2 \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(800\,\text{K}\right)^4 + 264\,\text{m}^2 \times 6.83\,\text{W/m}^2\cdot\text{K}\left(500\,\text{K}\right)$$

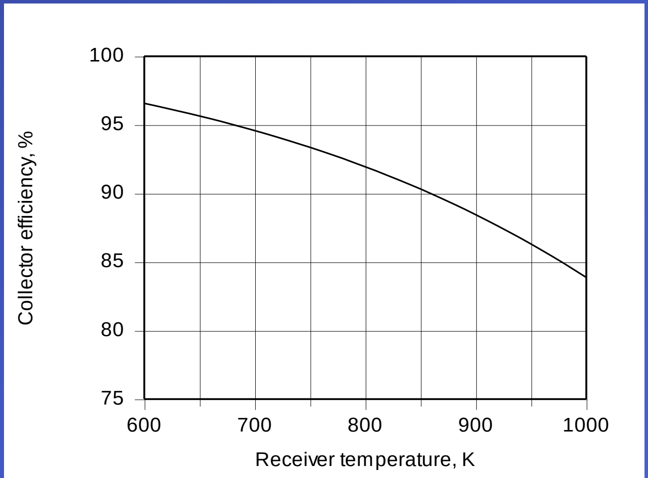
$$q = q_{rad} + q_{conv} = 1.23 \times 10^6\,\text{W} + 9.01 \times 10^5\,\text{W} = 2.13 \times 10^6\,\text{W}$$

With $A_s\, q_s'' = 2.64\times10^7$ W, the collector efficiency is

$$\eta = \left(\frac{A_s\, q_s'' - q}{A_s\, q_s''}\right)100 = \frac{\left(2.64\times10^7 - 2.13\times10^6\right)\mathrm{W}}{2.64\times10^7\ \mathrm{W}}(100) = 91.9\%$$

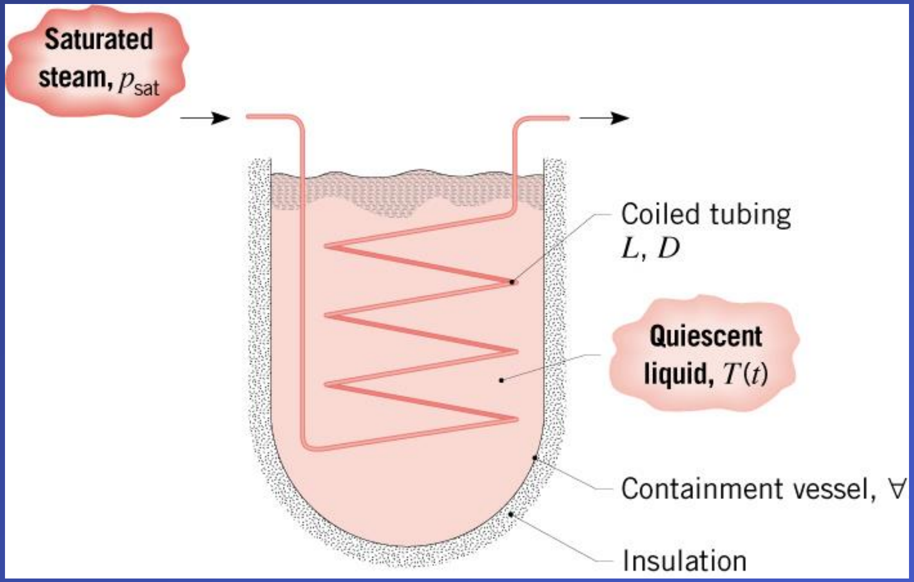
(b) As shown below, because of its dependence on temperature to the fourth power, q_{rad} increases more significantly with increasing T_s than does q_{conv}, and the effect on the efficiency is pronounced





COMMENTS: The collector efficiency is also reduced by the inability to have a perfectly absorbing receiver. Partial reflection of the incident solar flux will reduce the efficiency by at least several percent.

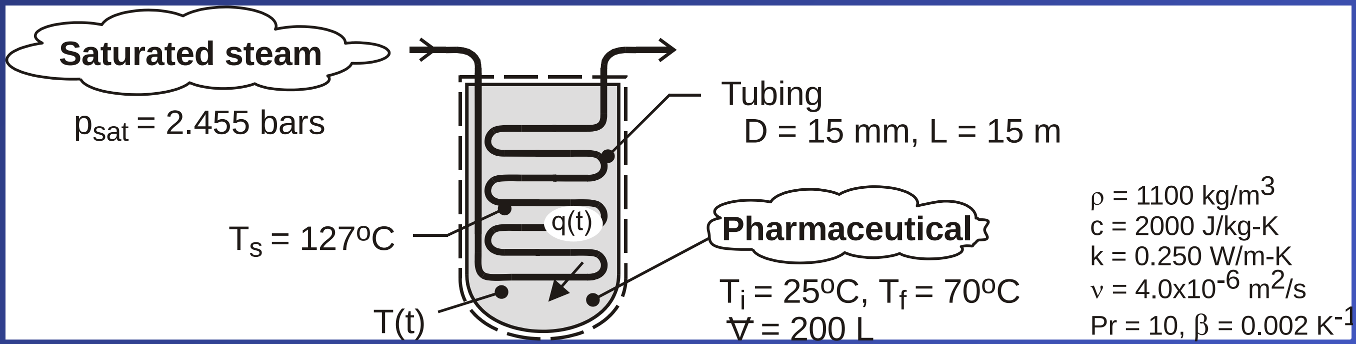
Problem 9.74: Use of saturated steam to heat a pharmaceutical in a batch reactor.



KNOWN: Volume, thermophysical properties, and initial and final temperatures of a pharmaceutical. Diameter and length of submerged tubing. Pressure of saturated steam flowing through the tubing.

FIND: (a) Initial rate of heat transfer to the pharmaceutical, (b) Time required to heat the pharmaceutical to 70°C and the amount of steam condensed during the process.

SCHEMATIC:



ASSUMPTIONS: (1) Pharmaceutical may be approximated as an infinite, quiescent fluid of uniform, but time-varying temperature, (2) Free convection heat transfer from the coil may be approximated as that from a heated, horizontal cylinder, (3) Negligible thermal resistance of condensing steam and tube wall, (4) Negligible heat transfer from tank to surroundings, (5) Constant properties.

PROPERTIES: *Table A-4*, Saturated water (2.455 bars): T_{sat} = 400K = 127°C, $h_{fg} = 2.183 \times 10^6$ J/kg. Pharmaceutical: See schematic.

ANALYSIS: (a) The initial rate of heat transfer is $q = \bar{h} A_s \left(T_s - T_i\right)$, where $A_s = \pi DL = 0.707$ m^2 and $\bar{h}$ is obtained from Eq. 9.34.

With $\alpha = \nu/Pr = 4.0 \times 10^{-7}\ m^2/s$ and $Ra_D = g\beta(T_s - T_i)\,D^3/\alpha\nu = 9.8\ m/s^2\ (0.002\ K^{-1})\ (102K)\ (0.015m)^3/16 \times 10^{-13}\ m^4/s^2 = 4.22 \times 10^6$,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\,Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + \frac{0.387\left(4.22\times10^6\right)^{1/6}}{\left[1+(0.559/10)^{9/16}\right]^{8/27}}\right\}^2 = 27.7$$

Hence, $\bar{h} = Nu_D\, k/D = 27.7 \times 0.250\,W/m\cdot K/0.015m = 462\,W/m^2\cdot K$

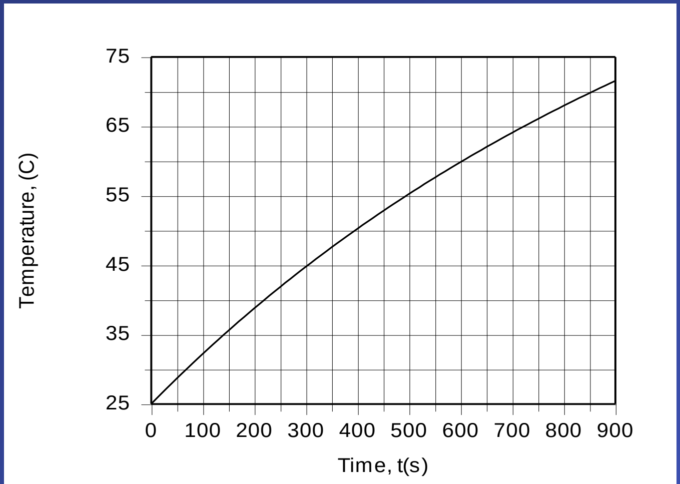
and $q = \bar{h}A_s\left(T_s - T_i\right) = 462\,W/m^2\cdot K \times 0.707\,m^2\,(102°C) = 33{,}300\,W$

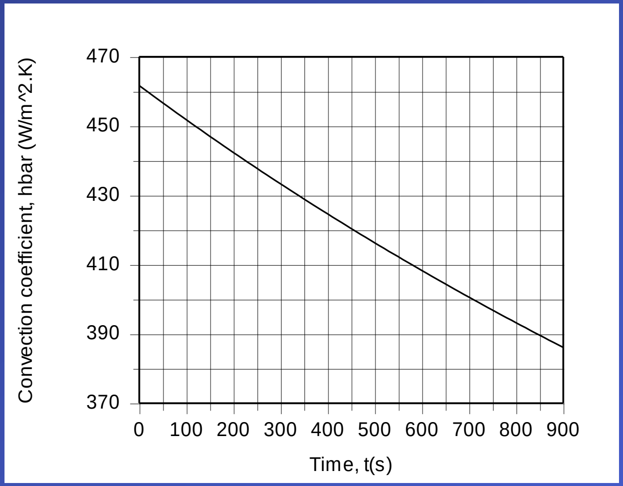
(b) Performing an energy balance at an instant of time for a control surface about the liquid,

$$\frac{d\left(\rho\forall cT\right)}{dt} = q(t) = \bar{h}(t)A_s\left(T_s - T(t)\right)$$

where the Rayleigh number, and hence $\bar{h}$, changes with time due to the change in the temperature of the liquid.

Integrating the foregoing equation numerically, the following results are obtained for the variation of T and $\bar{h}$ with t.





The time at which the liquid reaches 70°C is

$$t_f \approx 855\,\mathrm{s}$$ <

The rate at which T increases decreases with increasing time due to the corresponding reduction in $(T_s - T)$, and hence reductions in Ra_D, $\bar{h}$ and q.

The Rayleigh number decreases from 4.22×10^6 to 2.16×10^6, while the heat rate decreases from 33,300 to 14,000 W.

The convection coefficient decreases approximately as $(T_s - T)^{1/3}$, while $q \sim (T_s - T)^{4/3}$.

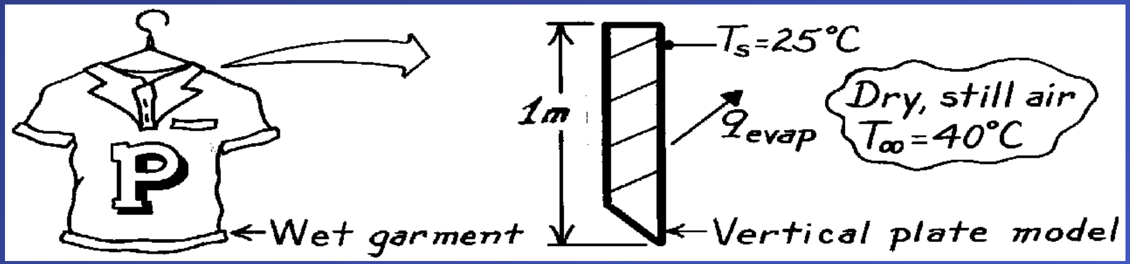
The latent energy released by the condensed steam corresponds to the increase in thermal energy of the pharmaceutical. Hence, $m_c h_{fg} = \rho \forall c \left(T_f - T_i \right)$,

and

$$m_c = \frac{\rho \forall c \left(T_f - T_i \right)}{h_{fg}} = \frac{1100\,\text{kg/m}^3 \times 0.2\,\text{m}^3 \times 2000\,\text{J/kg} \cdot \text{K} \times 45°\text{C}}{2.183 \times 10^6\,\text{J/kg}} = 9.07\,\text{kg} \qquad <$$

COMMENTS: (1) Over such a large temperature range, the fluid properties are likely to vary significantly, particularly ν and Pr. A more accurate solution could therefore be performed if the temperature dependence of the properties were known. (2) Condensation of the steam is a significant process expense, which is linked to the equipment (capital) and energy (operating) costs associated with steam production.

Problem 9.113: Determination of drying rate per unit width of a garment hanging in dry air.



KNOWN: Wet garment at 25°C hanging in a room with still, dry air at 40°C.

FIND: Drying rate per unit width of garment.

ASSUMPTIONS: (1) Analogy between heat and mass transfer applies, (2) Water vapor at garment surface is saturated at T_s, (3) Perfect gas behavior of vapor and air.

PROPERTIES: *Table A-4*, Air ($T_f \approx (T_s + T_\infty)/2 = 305K$, 1 atm): $\nu = 16.39 \times 10^{-6}\ m^2/s$; *Table A-6*, Water vapor ($T_s = 298K$, 1 atm): $p_{A,s} = 0.0317$ bar, $\rho_{A,s} = 1/v_f = 0.02660\ kg/m^3$; *Table A-8*, Air-water vapor (305 K): $D_{AB} = 0.27 \times 10^{-4}\ m^2/s$, $Sc = \nu/DAB = 0.607$.

ANALYSIS: The drying rate per unit width of the garment is

$$\dot{m}'_A = \bar{h}_m \cdot L\left(\rho_{A,s} - \rho_{A,\infty}\right)$$

where $\bar{h}_m$ is the mass transfer coefficient associated with a vertical surface that models the garment. From the heat and mass transfer analogy, Eq. 9.24 yields

$$\overline{Sh}_L = C\left(Gr_L\, Sc\right)^n$$

where $Gr_L = g\Delta\rho L^3/\rho\nu^2$ and $\Delta\rho = \rho_s - \rho_\infty$. Since the still air is dry, $\rho_\infty = \rho_{B,\infty} = p_{B,\infty}/R_B T_\infty$, where $R_B = \Re / M_B = 8.314 \times 10^{-2}$ m^3·bar/kmol·K/29 kg/kmol = 0.00287 m^3·bar/kg·K. With $p_{B,\infty}$ = 1 atm = 1.0133 bar,

$$\rho_\infty = \frac{1.0133 \text{ bar}}{0.00287 \text{ m}^3 \cdot \text{bar/kg} \cdot \text{K} \times 313 \text{ K}} = 1.1280 \text{ kg/m}^3$$

The density of the air/vapor mixture at the surface is $\rho_s = \rho_{A,s} + \rho_{B,s}$. With $p_{B,s}$ = 1 atm – $p_{A,s}$ = 1.0133 bar – 0.0317 bar = 0.9816 bar,

$$\rho_{B,s} = \frac{p_{B,s}}{R_B T_s} = \frac{0.9816 \text{ bar}}{0.00287 \text{m}^3 \cdot \text{bar/kg} \cdot \text{K} \times 298 \text{ K}} = 1.1477 \text{ kg/m}^3$$

Hence, $\rho_s = (0.0266 + 1.1477)\ kg/m^3 = 1.1743\ kg/m^3$ and $\rho = (\rho_s + \rho_\infty)/2 = 1.512\ kg/m^3$. The Grashof number is then

$$Gr_L = \frac{9.8\ m/s^2 \times (1.1743 - 1.1280)\,kg/m^3\,(1\ m)^3}{1.1512\ kg/m^3 \times \left(16.39 \times 10^{-6}\,m^2/s\right)^2} = 1.467 \times 10^9$$

and $(Gr_L\ Sc) = 8.905 \times 10^8$. Hence, from Section 9.6.1, C and n are 0.59 and 1 / 4 and the convection coefficient is then

$$\bar{h}_m = \frac{D_{AB}}{L}\overline{Sh}_L = \frac{0.27 \times 10^{-4}\,m^2/s}{1\ m} \times 0.59\left(8.905 \times 10^8\right)^{1/4} = 0.00275\ m/s$$

The drying rate is then

$$\dot{m}'_A = 2.750 \times 10^{-3}\,m/s \times 1.0m\,(0.0226 - 0)\,kg/m^3 = 6.21 \times 10^{-5}\,kg/s \cdot m.$$

COMMENTS: Since $\rho_s > \rho_\infty$, the buoyancy driven flow *descends* along the garment.