# Transferência de calor com Mudança de fase

Ebulição e Condensação

## Considerações Gerais

• Lei de Newton para o resfriamento:

 $q_s'' = h(T_s - T_{sat}) = h \ \varDelta \ T_e$ 

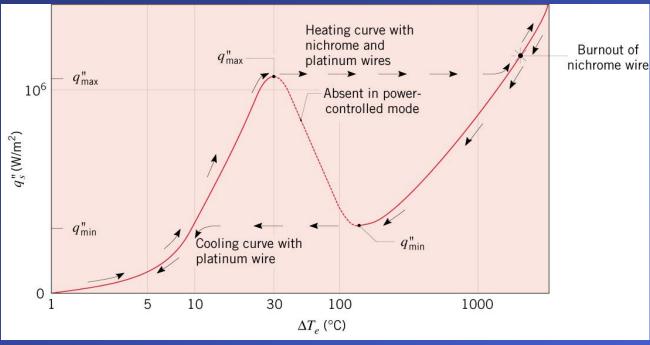
 $\succ$   $T_{sat} \rightarrow$  Temperatura de Saturação

 $\succ \Delta T_e \equiv (T_s - T_{sat}) \rightarrow$  Temperatura de excesso

#### Boiling Curve

## Curvas de ebulição

### Água a pressão atmosférica



Nukiyama, 1934 Drew e Mueller, 1937

Ebulição com convecção livre

$$\left( \varDelta T_e < 5^{\circ} \mathrm{C} \right)$$

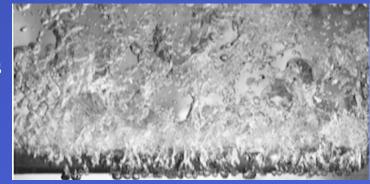
- Pouca formação de vapor.
- Movimentação é devido a convecção livre.
- Início da Ebulição Nucleada

$$ONB\left( \Delta T_e \approx 5^{\circ} \mathrm{C} \right)$$

- Ebulição Nucleada  $(5 < \Delta T_e < 30^{\circ} \text{C})$ > Bolhas individuais  $(5 < \Delta T_e < 10^{\circ} \text{C})$ 
  - Liquid motion is strongly influenced by nucleation of bubbles at the surface.
  - h and  $q_s''$  increase sharply with increasing  $\Delta T_e$ .
  - Heat transfer is principally due to contact of liquid with the surface (single-phase convection) and not to vaporization.

> Jatos e colunas  $(10 < \Delta T_e < 30^\circ \text{C})$ 

- Increasing number of nucleation sites causes bubble interactions and coalescence into jets and slugs.
- Liquid/surface contact is impaired.

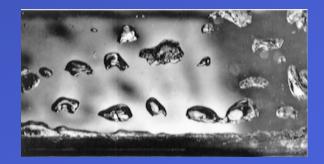


-  $q''_s$  continues to increase with  $\Delta T_e$  while h begins to decrease.

- Fluxo de calor crítico- CHF,  $q''_{\text{max}} \left( \Delta T_e \approx 30^{\circ} \text{C} \right)$ 
  - > Maximum attainable heat flux in nucleate boiling.
  - >  $q''_{\text{max}} \approx 1 \text{ MW/m}^2$  for water at atmospheric pressure.
- Potential Burnout for Power-Controlled Heating
  - An increase in  $q''_s$  beyond  $q''_{max}$  causes the surface to be blanketed by vapor, and the surface temperature can spontaneously achieve a value that potentially exceeds its melting point  $(\Delta T_s > 1000^\circ \text{C})$ .
  - If the surface survives the temperature shock, conditions are characterized by *film boiling*.

#### • Película

- Heat transfer is by conduction and radiation across the vapor blanket.
- A reduction in  $q''_s$  follows the cooling curve continuously to the Leidenfrost point corresponding to the minimum heat flux  $q''_{min}$  for film boiling.



- A reduction in  $q''_s$  below  $q''_{min}$  causes an abrupt reduction in surface temperature to the nucleate boiling regime.
- Transition Boiling for Temperature-Controlled Heating
  - ► Characterized by a continuous decay of  $q''_s$  (from  $q''_{max}$  to  $q''_{min}$ ) with increasing  $\Delta T_e$ .
  - Surface conditions oscillate between nucleate and film boiling, but portion of surface experiencing film boiling increases with  $\Delta T_e$ .
  - > Also termed unstable or partial film boiling.

## Correlações de Ebulição em vaso

•Ebulição Nucleada

Correlação de Rohsenow

$$\boldsymbol{q_{s}^{\prime\prime}} = \mu_{l}h_{fg}\left[\frac{g\left(\rho_{l}-\rho_{v}\right)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}\boldsymbol{\varDelta}\boldsymbol{T_{e}}}{C_{s,f}h_{fg}\operatorname{Pr}_{l}^{n}}\right)^{3}$$

 $C_{s,f}, n \rightarrow$  Surface/Fluid Combination (Table 10.1)

• Fluxo de calor crítico

$$q_{\max}'' = Ch_{fg}\rho_{v} \left[\frac{\sigma g(\rho_{l} - \rho_{v})}{\rho_{v}^{2}}\right]^{1/4}$$

 $C \rightarrow$  surface geometry dependent

Correlations

#### • Eblulição com formação de película

The cumulative (and coupled effects) of convection and radiation across the vapor layer \_\_\_\_\_

$$\overline{h}^{4/3} \approx \overline{h}_{conv}^{4/3} + \overline{h}_{rad} \overline{h}^{1/3}$$

$$\overline{Nu}_D = \frac{\overline{h}_{conv} D}{k_v} = C \left[ \frac{g(\rho_l - \rho_v) h'_{fg} D^3}{v_v k_v (T_s - T_{sat})} \right]^{1/4}$$

$$\frac{\overline{Geometry}}{\overline{Cylinder(Hor.)}} \qquad \frac{C}{0.62}$$
Sphere 0.67

$$h'_{fg} = h_{fg} + 0.80 c_{p,v} \left(T_s - T_{sat}\right)$$
$$\overline{h}_{rad} = \frac{\varepsilon \sigma \left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}}$$

If  $\overline{h}_{conv} > \overline{h}_{rad}$ ,

$$\overline{h} \approx \overline{h}_{conv} + 0.75 \ \overline{h}_{rad}$$

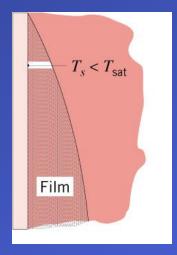


## **General Considerations**

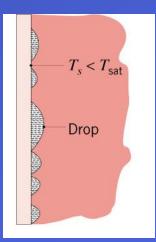
• Heat transfer to a surface occurs by condensation when the surface temperature is less than the saturation temperature of an adjoining vapor.

#### • Film Condensation

Entire surface is covered by the condensate, which flows continuously from the surface and provides a resistance to heat transfer between the vapor and the surface.



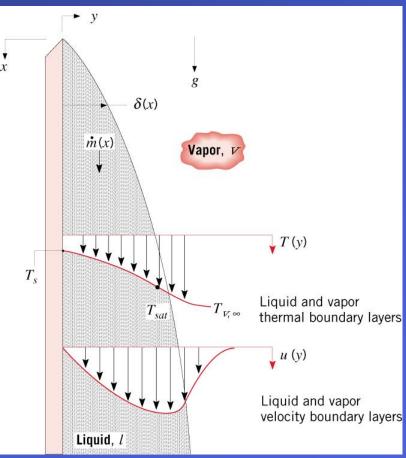
- Thermal resistance is reduced through use of short vertical surfaces and horizontal cylinders.
- > Characteristic of clean, uncontaminated surfaces.
- Dropwise Condensation
  - Surface is covered by drops ranging from a few micrometers to agglomerations visible to the naked eye.



## Film Condensation on a Vertical Plate

- Distinguishing Features
  - > Thickness( $\delta$ ) and flow rate  $\binom{\square}{m}$  of condensate increase with increasing *x*
  - ➢ Generally, the vapor is superheated ( $T_{\nu,\infty} > T_{sat}$ ) and may be part of a mixture that includes noncondensibles.
  - A shear stress at the liquid/vapor interface induces a velocity gradient in the vapor, as well as the liquid.
- Nusselt Analysis for Laminar Flow Assumptions:
  - > A pure vapor at  $T_{sat}$ .
  - Negligible shear stress at liquid/vapor interface.

$$\rightarrow \frac{\partial u}{\partial y}\Big|_{y=\delta} = 0$$



Negligible advection in the film. Hence, the steady-state x-momentum and energy equations for the film are

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu_l} \frac{dp_{\infty}}{dx} - \frac{\rho_l g}{\mu_l}$$
$$\frac{\partial^2 T}{\partial v^2} = 0$$

> The boundary layer approximation,  $\partial p / \partial y = 0$ , may be applied to the film. Hence,

$$\frac{dp_{\infty}}{dx} = \rho_{v}g$$

Solutions to momentum and energy equations —

Film thickness:

$$\delta(x) = \left[\frac{4k_l\mu_l(T_{sat} - T_s)x}{g\rho_l(\rho_l - \rho_v)h_{fg}}\right]^{1/4}$$

Flow rate per unit width:

$$\Gamma \equiv \frac{m}{b} = \frac{g\rho_l(\rho_l - \rho_v)\delta^3}{3\mu_l}$$

Average Nusselt Number:

$$\overline{Nu}_{L} = \frac{\overline{h}_{L}L}{k_{l}} = 0.943 \left[ \frac{\rho_{l}g(\rho_{l} - \rho_{v})h'_{fg}L^{3}}{\mu_{l}k_{l}(T_{sat} - T_{s})} \right]^{1/2}$$
$$h'_{fg} = h_{fg}(1 + 0.68 Ja)$$
$$Ja \equiv \frac{c_{p,l}(T_{sat} - T_{s})}{h_{fg}} \rightarrow \text{Jakob number}$$

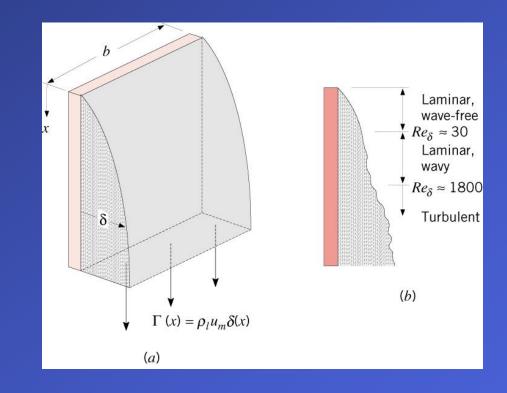
Total heat transfer and condensation rates:

$$q = \overline{h}_L A \big( T_{sat} - T_s \big)$$

$$\stackrel{\square}{m} = \frac{q}{h'_{fg}}$$

- Effects of Turbulence:
  - Transition may occur in the film and three flow regimes may be identified and delineated in terms of a Reynolds number defined as

$$\operatorname{Re}_{\delta} \equiv \frac{4\Gamma}{\mu_{l}} = \frac{4m}{\mu_{l}b} = \frac{4\rho_{l}u_{m}\delta}{\mu_{l}}$$



Vertical Plates (cont)

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 $\text{Re}_{\delta}$  can be determined by calculating the three values below and selecting the one that lies within the range of applicability for that equation.

Wave-free laminar region (Re<sub>$$\delta$$</sub> < 30):  

$$Re_{\delta} = 3.78 \left[ \frac{k_l L (T_{sat} - T_s)}{\mu_l h'_{fg} (v_l^2 / g)^{1/3}} \right]^{3/4}$$
(10.42)

▷ Wavy laminar region  $(30 < \text{Re}_{\delta} < 1800)$ :

$$Re_{\delta} = \left[\frac{3.70k_{l}L(T_{sat} - T_{s})}{\mu_{l}h_{fg}'(v_{l}^{2}/g)^{1/3}} + 4.8\right]^{0.82}$$
(10.43)

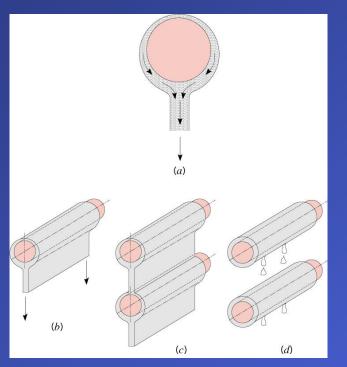
> Turbulent region ( $\text{Re}_{\delta} > 1800$ ):

$$Re_{\delta} = \left[\frac{0.069k_{l}L(T_{sat} - T_{s})}{\mu_{l}h_{fg}'(v_{l}^{2}/g)^{1/3}}Pr_{l}^{0.5} - 151Pr_{l}^{0.5} + 253\right]^{4/3}$$
(10.44)

 $\succ \overline{h}_L$  can then be found from

$$\overline{h}_{L} = \frac{Re_{\delta} \ \mu_{l} h'_{fg}}{4L(T_{sat} - T_{s})} \tag{10.41}$$

## Film Condensation on Radial Systems



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• A single tube or sphere:

$$\overline{h}_{D} = C \left[ \frac{g \rho_{l} (\rho_{l} - \rho_{\nu}) k_{l}^{3} h_{fg}'}{\mu_{l} (T_{sat} - T_{s}) D} \right]^{1}$$

Tube: C = 0.729

Sphere: *C*=0.826

Film Condensation: Radial Systems (cont).

• A vertical tier of *N* tubes:

$$\bar{h}_{D,N} = 0.729 \left[ \frac{g \rho_l (\rho_l - \rho_v) k_l^3 h'_{fg}}{N \mu_l (T_{sat} - T_s) D} \right]^{1/4}$$

> Why does  $\overline{h}_{D,N}$  decrease with increasing N?

- How is heat transfer affected if the continuous sheets (c) breakdown and the condensate *drips* from tube to tube (d)?
- > What other effects influence heat transfer?

## Film Condensation for a Vapor Flow in a Horizontal Tube

• If vapor flow rate is small, condensate flow is circumferential and axial:

$$\operatorname{Re}_{\nu,i} = \left(\frac{\rho_{\nu}u_{m,\nu}D}{\mu_{\nu}}\right)_{i} < 35,000:$$

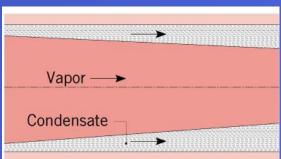
$$\overline{h}_{D} = 0.555 \left[\frac{g\rho_{l}(\rho_{l} - \rho_{\nu})k_{l}^{3}h_{fg}'}{\mu_{l}(T_{sat} - T_{s})D}\right]^{1/4}$$

$$h_{fg}' \equiv h_{fg} + 0.375(T_{sat} - T_{s})$$

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• For larger vapor velocities, flow is principally in the axial direction and characterized by two-phase annular conditions.

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#### Dropwise Condensation

## **Dropwise Condensation**

• Steam condensation on copper surfaces:

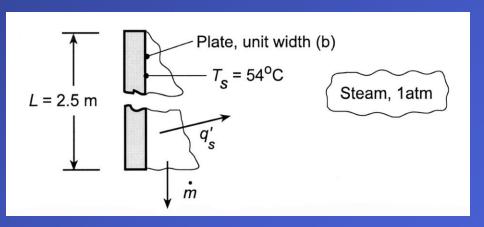
 $q = \bar{h}_{dc} A (T_{sat} - T_s)$  $\bar{h}_{dc} = 51,100 + 2044 T_{sat}$  $22^{\circ} C < T_{sat} < 100^{\circ} C$  $\bar{h}_{dc} = 255,500$  $T_{sat} > 100^{\circ} C$  Problem: Condensation on a Vertical Plate

Problem 10.48 a,b: Condensation and heat rates per unit width for saturated steam at 1 atm on one side of a vertical plate at 54 °C if (a) the plate height is 2.5m and (b) the height is halved.

**KNOWN:** Vertical plate 2.5 m high at a surface temperature  $T_s = 54^{\circ}C$  exposed to steam at atmospheric pressure.

**FIND:** (a) Condensation and heat transfer rates per unit width, (b) Condensation and heat rates if the height were halved.

#### **SCHEMATIC:**



**ASSUMPTIONS:** (1) Film condensation, (2) Negligible non-condensables in steam.

**PROPERTIES:** *Table A-6*, Water, vapor (1 atm):  $T_{sat} = 100^{\circ}$ C,  $h_{fg} = 2257$  kJ/kg; *Table A-6*, Water, liquid ( $T_f = (100 + 54)^{\circ}$ C/2 = 350 K):  $\rho_{\ell} = 973.7$  kg/m<sup>3</sup>,  $k_{\ell} = 0.668$  W/m·K,  $\mu_{\ell} = 365 \times 10^{-6}$  N·s/m<sup>2</sup>,  $c_{p,\ell} = 4195$  J/kg·K,  $Pr_{\ell} = 2.29$ ,  $v_{\ell} = \mu_{\ell} / \rho_{\ell} = 3.75 \times 10^{-7}$  m<sup>2</sup>/s.

**ANALYSIS:** (a) For the long plate length, assume turbulent film condensation, Eq. 10.44.

$$Re_{\delta} = \left[\frac{0.069k_{1}L(T_{sat} - T_{s})}{\mu_{1}h'_{fg}(\nu_{1}^{2}/g)^{1/3}}Pr_{1}^{0.5} - 151Pr_{1}^{0.5} + 253\right]^{4/3}$$

$$Re_{\delta} = \left[\frac{0.069 \times 0.668 \text{ W/m} \cdot \text{K} \times 2.5 \text{ m}(100 - 54)\text{K}}{365 \times 10^{-6} \text{ N} \cdot \text{s/m}^{2} \times 2388 \times 10^{3} \text{ J/kg}\left[\left(3.75 \times 10^{-7} \text{ m}^{2}/\text{s}\right)^{2}/9.8 \text{ m/s}^{2}\right]^{1/3}}2.29^{0.5} - 151(2.29)^{0.5} + 288^{-10}\text{ Re}_{\delta} = 2979$$

where  $h'_{fg} = h_{fg} + 0.68c_{p,l} (T_{sat} - T_s) = 2388 \text{ kJ/kg}$ . The turbulent assumption is correct. Then from Eqs. 10.36 and 10.34,

$$\dot{m}' = \frac{\text{Re}_{\delta} \ \mu_{\ell}}{4} = 2979 \times 365 \times 10^{-6} \ \text{N} \cdot \text{s/m}^2/4 = 0.272 \ \text{kg/s} \cdot \text{m}$$
$$q' = \dot{m}' h'_{fg} = 0.272 \ \text{kg/s} \cdot \text{m} \times 2.388 \times 10^6 \ \text{J/kg} = 649 \ \text{kW/m} \qquad <$$

(b) If the length is halved, L = 1.25 m,  $Re_{\delta}$  will decrease and we begin by trying Eq. 10.43,

$$\operatorname{Re}_{\delta} = \left[\frac{3.70k_{\ell}L(T_{\text{sat}} - T_{\text{s}})}{\mu_{\ell}h_{\text{fg}}'(v_{\ell}^{2}/g)^{1/3}} + 4.8\right]^{0.82} = 1375$$

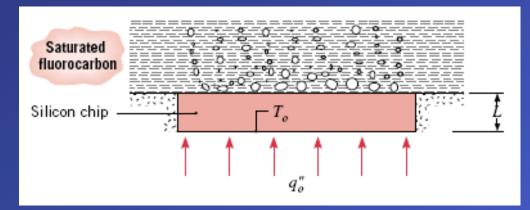
and the assumption of wavy laminar flow was correct. The flow regime changes.

We find 
$$\dot{m}' = \frac{\text{Re}_{\delta} \mu_{\ell}}{4} = 0.125 \text{kg/s} \cdot \text{m}$$
 and  $q' = \dot{m}' h'_{fg} = 300 \text{ kW/m}$ .

#### **COMMENT:**

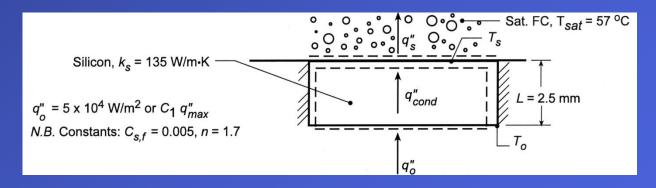
Note that the height was decreased by a factor of 2, while the rates decreased by a factor of 2.2. Would you have expected this result?

## Problem 10.23: Chip thermal conditions associated with cooling by immersion in a fluorocarbon.



**KNOWN:** Thickness and thermal conductivity of a silicon chip. Properties of saturated fluorocarbon liquid.

**FIND:** (a) Temperature at bottom surface of chip for a prescribed heat flux and for a flux that is 90% of CHF, (b) Effect of heat flux on chip surface temperatures; maximum allowable heat flux for a surface temperature of  $80^{\circ}$ C.



**ASSUMPTIONS:** (1) Steady-state conditions, (2) Uniform heat flux and adiabatic sides, hence one-dimensional conduction in chip, (3) Constant properties, (4) Nucleate boiling in liquid.

**PROPERTIES:** Saturated fluorocarbon (given):  $c_{p,\ell} = 1100 \text{ J/kg} \cdot \text{K}$ ,  $h_{fg} = 84,400 \text{ J/kg}$ ,  $\rho_{\ell} = 1619.2 \text{ kg/m}^3$ ,  $\rho_v = 13.4 \text{ kg/m}^3$ ,  $\sigma = 8.1 \times 10^{-3} \text{ kg/s}^2$ ,  $\mu_{\ell} = 440 \times 10^{-6} \text{ kg/m} \cdot \text{s}$ ,  $Pr_{\ell} = 9.01$ .

**ANALYSIS:** (a) Energy balances at the top and bottom surfaces yield  $q''_{o} = q''_{cond} = k_{s} (T_{o} - T_{s})/L = q''_{s}$ ; where  $T_{s}$  and  $q''_{s}$  are related by the Rohsenow correlation,

$$T_{s} - T_{sat} = \frac{C_{s,f} h_{fg} Pr_{\ell}^{n}}{c_{p,\ell}} \left(\frac{q_{s}''}{\mu_{\ell} h_{fg}}\right)^{1/3} \left[\frac{\sigma}{g(\rho_{\ell} - \rho_{v})}\right]^{1/6}$$

Hence, for  $q_8'' = 5 \times 10^4 \text{ W/m}^2$ ,

$$T_{s} - T_{sat} = \frac{0.005(84,400 \,\text{J/kg})9.01^{1.7}}{1100 \,\text{J/kg} \cdot \text{K}} \left(\frac{5 \times 10^{4} \,\text{W/m^{2}}}{440 \times 10^{-6} \,\text{kg/m} \cdot \text{s} \times 84,400 \,\text{J/kg}}\right)^{1/3}$$

$$\times \left[\frac{8.1 \times 10^{-3} \text{ kg/s}^2}{9.807 \text{ m/s}^2 (1619.2 - 13.4) \text{ kg/m}^3}\right]^{1/6} = 15.9^{\circ} \text{ C}$$

 $T_s = (15.9 + 57)^\circ C = 72.9^\circ C$ 

Problem: Electronic Chip Cooling (cont)

From the rate equation,

$$T_{o} = T_{s} + \frac{q_{o}''L}{k_{s}} = 72.9^{\circ}C + \frac{5 \times 10^{4} \text{ W/m}^{2} \times 0.0025 \text{ m}}{135 \text{ W/m} \text{ K}} = 73.8^{\circ}C$$

For a heat flux which is 90% of the critical heat flux ( $C_1 = 0.9$ ),

$$q_{0}'' = 0.9q_{max}'' = 0.9 \times 0.149 h_{fg} \rho_{v} \left[ \frac{\sigma g(\rho_{\ell} - \rho_{v})}{\rho_{v}^{2}} \right]^{1/4} = 0.9 \times 0.149 \times 84,400 \text{ J/kg} \times 13.4 \text{ kg/m}^{3}$$
$$\times \left[ \frac{8.1 \times 10^{-3} \text{ kg/s}^{2} \times 9.807 \text{ m/s}^{2} (1619.2 - 13.4) \text{ kg/m}^{3}}{(13.4 \text{ kg/m}^{3})^{2}} \right]^{1/4}$$

$$q_0'' = 0.9 \times 15.5 \times 10^4 \text{ W/m}^2 = 13.9 \times 10^4 \text{ W/m}^2$$

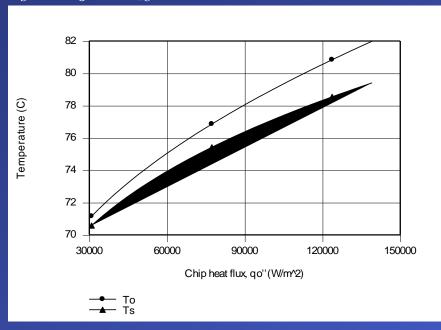
From the results of the previous calculation and the Rohsenow correlation, it follows that

$$\Delta T_{e} = 15.9^{\circ} C \left( q_{0}'' / 5 \times 10^{4} W / m^{2} \right)^{1/3} = 15.9^{\circ} C \left( 13.9 / 5 \right)^{1/3} = 22.4^{\circ} C$$

Hence,  $T_s = 79.4$ °C and

$$T_{o} = 79.4^{\circ}C + \frac{13.9 \times 10^{4} \text{ W/m}^{2} \times 0.0025 \text{ m}}{135 \text{ W/m} \text{ K}} = 82^{\circ}C$$

(b) Parametric calculations for  $0.2 \le C_1 \le 0.9$ , yield the following variations of  $T_8$  and  $T_0$  with  $q''_0$ .



The chip surface temperatures, as well as the difference between temperatures, increase with increasing heat flux. The maximum chip temperature is associated with the bottom surface, and  $T_0 = 80^{\circ}$ C corresponds to

$$I_{0,\max}'' = 11.3 \times 10^4 \text{ W/m}^2$$

which is 73% of CHF ( $q''_{max} = 15.5 \times 10^4 \text{ W/m}^2$ ).

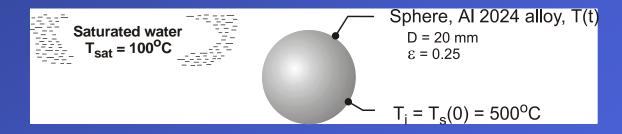
**COMMENTS:** Many of today's VLSI chip designs involve heat fluxes well in excess of 15  $W/cm^2$ , in which case pool boiling in a fluorocarbon would not be an appropriate means of heat dissipation.

# Problem 10.26: Initial heat transfer coefficient for immersion of an aluminum sphere in a saturated water bath at atmospheric pressure and its temperature after immersion for 30 seconds.

**KNOWN:** A sphere (aluminum alloy 2024) with a uniform temperature of 500°C and emissivity of 0.25 is suddenly immersed in a saturated water bath maintained at atmospheric pressure.

**FIND:** (a) The total heat transfer coefficient for the initial condition; fraction of the total coefficient contributed by radiation; and (b) Estimate the temperature of the sphere 30 s after it has been immersed in the bath.

#### **SCHEMATIC**



Problem: Quenching of Aluminum Sphere (cont.)

**ASSUMPTIONS:** (1) Water is at atmospheric pressure and uniform temperature,  $T_{sat}$ , and (2) Lumped capacitance method is valid.

#### **PROPERTIES:**

Table A-4, Water vapor  $(T_{f,i} = 573 \text{K})$ :  $k_v = 0.0399 \text{ W/m} \cdot \text{K}$ ,  $c_{p,v} = 2010 \text{ J/kg} \cdot \text{K}$ ,  $\rho_v = 0.3843 \text{ kg/m}^3$ ,  $v_v = 51.44 \times 10^{-6} \text{ m}^2$  / s, Table A-6, Water  $(T_{\text{sat}} = 373 \text{K})$ :  $\rho_l = 958 \text{ kg/m}^3$ ,  $h_{fg} = 2.257 \times 10^6 \text{ J/kg}$ .

Aluminum Alloy:  $\rho_s = 2700 \text{ kg/m}^3$ ,  $c_{p,s} = 875 \text{ J/kg} \cdot \text{K}$ ,  $k_s = 186 \text{ W/m} \cdot \text{K}$ .

**ANALYSIS:** (a) For the initial condition with  $T_s = 500^{\circ}$ C, *film boiling* will occur and the coefficients due to convection and radiation are estimated using Eqs. 10.8 and 10.11, respectively,

$$\overline{\mathrm{Nu}}_{\mathrm{D}} = \frac{\overline{\mathrm{h}}_{\mathrm{conv}}\mathrm{D}}{\mathrm{k}_{\mathrm{v}}} = \mathrm{C} \left[ \frac{\mathrm{g}(\rho_{\ell} - \rho_{\mathrm{v}})\mathrm{h}_{\mathrm{fg}}^{\prime}\mathrm{D}^{3}}{\nu_{\mathrm{v}}\mathrm{k}_{\mathrm{v}}\left(\mathrm{T}_{\mathrm{s}} - \mathrm{T}_{\mathrm{sat}}\right)} \right]^{1/4}$$
(1)  
$$\overline{\mathrm{h}}_{\mathrm{rad}} = \frac{\varepsilon\sigma\left(\mathrm{T}_{\mathrm{s}}^{4} - \mathrm{T}_{\mathrm{sat}}^{4}\right)}{\mathrm{T}_{\mathrm{s}} - \mathrm{T}_{\mathrm{sat}}}$$
(2)

where C = 0.67 for spheres and  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ . The corrected latent heat is  $h'_{fg} = h_{fg} + 0.8 c_{p,v} (T_s - T_{sat})$ (3) Problem: Quenching of Aluminum Sphere (cont.)

The total heat transfer coefficient is given by Eq. 10.9 as  $\overline{h}^{4/3} = \overline{h}_{conv}^{4/3} + \overline{h}_{rad} \cdot \overline{h}^{1/3}$ 

(4)

Using the foregoing relations, the following results are obtained.

$$\overline{\mathrm{Nu}}_{\mathrm{D}} = \overline{\mathrm{h}}_{\mathrm{cnv}} \left( \mathrm{W} / \mathrm{m}^2 \cdot \mathrm{K} \right) = \overline{\mathrm{h}}_{\mathrm{rad}} \left( \mathrm{W} / \mathrm{m}^2 \cdot \mathrm{K} \right) = \overline{\mathrm{h}} \left( \mathrm{W} / \mathrm{m}^2 \cdot \mathrm{K} \right)$$

$$85.5 = 171 = 12.0 = 180$$

The radiation process contribution is 6.7% of the total heat rate.

(b) For the lumped-capacitance method, from Section 5.3, the energy balance is

$$-\overline{h}A_{s}\left(T_{s}-T_{sat}\right) = \rho_{s}Vc_{s}\frac{dT_{s}}{dt}$$
(5)

where  $\rho_s$  and  $c_s$  are properties of the sphere. Numerically integrating Eq. (5) and evaluating  $\overline{h}$  as a function of  $T_s$ , the following result is obtained for the sphere temperature after 30s.  $T_s(30s) = 300^{\circ}C.$ 

**COMMENTS:** The Biot number associated with the aluminum alloy sphere cooling process for the initial condition is Bi = 0.09. Hence, the lumped-capacitance method is valid.