ESTIMATIVA A PRIORI DE ERROS DE TRUNCAMENTO

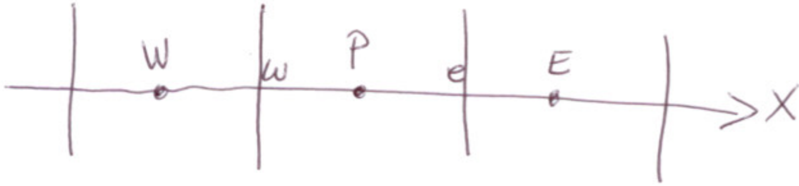
• DIFERENÇAS FINITAS: EXPANSÃO DA SÉRIE DE TAYLOR A PARTIR DO NÓ GENÉRICO P

• VOLUMES FINITOS: EXPANSÃO DA SÉRIE DE TAYLOR A PARTIR DE CADA FACE DO VOLUME DE CONTROLE GENÉRICO P, PARA TERMOS ADVECTIVOS E DIFUSIVOS.

$$\Lambda_x = \sum_{n=0}^{\infty} \Lambda_{x_0}^{(n)} \frac{(x-x_0)^n}{n!}$$ SÉRIE DE TAYLOR

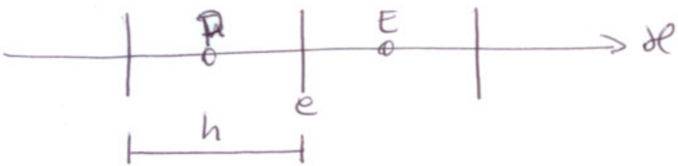
* TERMOS DIFUSIVOS: APROXIMA-SE Λ'_e e Λ'_w

* TERMOS ADVECTIVOS: APROXIMA-SE Λ_e e Λ_w



UDS / A PRIORI / ADVECÇÃO

14 Nov 04



UDS: $\Lambda_e = \Lambda_P$ (aproximado)

$$\Lambda_P = \Lambda_e - \Lambda_e' \frac{h}{2} + \Lambda_e'' \frac{h^2}{8} - \Lambda_e''' \frac{h^3}{48} + \Lambda_e^{iv} \frac{h^4}{384} - \dots$$

$$\mathcal{E}(\lambda_e) = \Lambda_e - \lambda_e$$

$$\Lambda_e = \Lambda_P + \Lambda_e' \frac{h}{2} - \Lambda_e'' \frac{h^2}{8} + \Lambda_e''' \frac{h^3}{48} - \dots \quad \text{(exato)}$$

$$\mathcal{E}(\lambda_e) = \Lambda_e' \frac{h}{2} - \Lambda_e'' \frac{h^2}{8} + \Lambda_e''' \frac{h^3}{48} - \dots$$

$$p_V = 1, 2, 3, \dots \longrightarrow p_L = 1$$

CDS / A PRIORI / ADVECÇÃO E DIFUSÃO 14 Nov 04

CDS: $\lambda_e = \frac{(\Lambda_P + \Lambda_E)}{2}$ [ADVECÇÃO]

$$\Lambda_P = \Lambda_e - \Lambda_e' \frac{h}{2} + \Lambda_e'' \frac{h^2}{8} - \Lambda_e''' \frac{h^3}{48} + \Lambda_e^{iv} \frac{h^4}{384} - \Lambda_e^{v} \frac{h^5}{3840} + \Lambda_e^{vi} \frac{h^6}{46080} - \Lambda_e^{vii} \frac{h^7}{645120}$$

$+$

$$\Lambda_E = \Lambda_e + \Lambda_e' \frac{h}{2} + \Lambda_e'' \frac{h^2}{8} + \Lambda_e''' \frac{h^3}{48} + \Lambda_e^{iv} \frac{h^4}{384} + \Lambda_e^{v} \frac{h^5}{3840} + \Lambda_e^{vi} \frac{h^6}{46080} + \Lambda_e^{vii} \frac{h^7}{645120}$$

$$\Lambda_e = \frac{(\Lambda_P + \Lambda_E)}{2} - \Lambda_e'' \frac{h^2}{8} - \Lambda_e^{iv} \frac{h^4}{384} - \Lambda_e^{vi} \frac{h^6}{46080} - \ldots$$

$$\mathcal{E}(\lambda_e) = \Lambda_e - \lambda_e$$

Vale p/ o $\mathcal{E}$ [T(1/2) média]

$$\mathcal{E}(\lambda_e) = -\Lambda_e'' \frac{h^2}{8} - \Lambda_e^{iv} \frac{h^4}{384} - \Lambda_e^{vi} \frac{h^6}{46080} - \ldots$$ ADVECÇÃO

$p_V = 2, 4, 6, \ldots \rightarrow p_L = 2$

SIMMEC/2000

CDS: $\lambda_e' = \frac{(\Lambda_E - \Lambda_P)}{h}$ [DIFUSÃO]

Das séries de Taylor,

$$\Lambda_E - \Lambda_P = \Lambda_e' h + \Lambda_e''' \frac{h^3}{24} + \Lambda_e^{v} \frac{h^5}{1920} + \Lambda_e^{vii} \frac{h^7}{322560} + \ldots$$

ou

$$\Lambda_e' = \frac{(\Lambda_E - \Lambda_P)}{h} - \Lambda_e''' \frac{h^2}{24} - \Lambda_e^{v} \frac{h^4}{1920} - \Lambda_e^{vii} \frac{h^6}{322560} - \ldots$$

$$\mathcal{E}(\lambda_e') = \Lambda_e' - \lambda_e'$$

$$\mathcal{E}(\lambda_e') = -\Lambda_e''' \frac{h^2}{24} - \Lambda_e^{v} \frac{h^4}{1920} - \Lambda_e^{vii} \frac{h^6}{322560} - \ldots$$ DIFUSÃO

$p_V = 2, 4, 6, \ldots \rightarrow p_L = 2$

SIMMEC/2000

ESQUEMA UDS-2:

$$\phi_e = \phi_P + \frac{(\phi_P - \phi_W)}{2} = \frac{3}{2}\phi_P - \frac{1}{2}\phi_W \qquad (14)$$

de (1):

$$\phi_W = \phi_e - \left.\frac{d\phi}{dx}\right|_e \frac{3\Delta x}{2} + \left.\frac{d^2\phi}{dx^2}\right|_e \frac{9}{8}\Delta x^2 - \left.\frac{d^3\phi}{dx^3}\right|_e \frac{27}{48}\Delta x^3 + \left.\frac{d^4\phi}{dx^4}\right|_e \frac{81}{384}\Delta x^4 + H \qquad (15)$$

Multiplicando (15) por (-1/2):

$$-\frac{\phi_W}{2} = -\frac{\phi_e}{2} + \left.\frac{d\phi}{dx}\right|_e \frac{3}{4}\Delta x - \left.\frac{d^2\phi}{dx^2}\right|_e \frac{9}{16}\Delta x^2 + \left.\frac{d^3\phi}{dx^3}\right|_e \frac{27}{96}\Delta x^3 - \left.\frac{d^4\phi}{dx^4}\right|_e \frac{81}{768}\Delta x^4 + H \qquad (16)$$

Multiplicando (8) por $\frac{3}{2}$:

$$\frac{3}{2}\phi_P = \frac{3}{2}\phi_e - \left.\frac{d\phi}{dx}\right|_e \frac{3}{4}\Delta x + \left.\frac{d^2\phi}{dx^2}\right|_e \frac{3}{16}\Delta x^2 - \left.\frac{d^3\phi}{dx^3}\right|_e \frac{3}{96}\Delta x^3 + \left.\frac{d^4\phi}{dx^4}\right|_e \frac{3}{768}\Delta x^4 + H \qquad (17)$$

Somando (16) com (17):

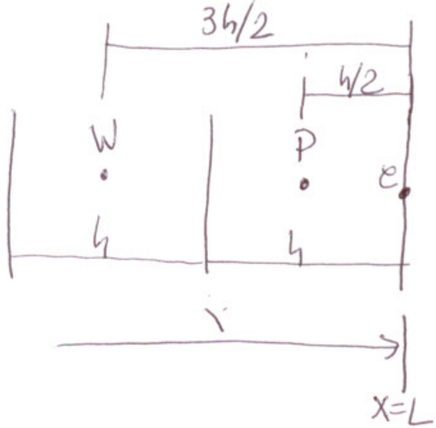
$$\frac{3}{2}\phi_P - \frac{\phi_W}{2} = \phi_e - \left.\frac{d^2\phi}{dx^2}\right|_e \frac{3}{8}\Delta x^2 + \left.\frac{d^3\phi}{dx^3}\right|_e \frac{\Delta x^3}{4} - \left.\frac{d^4\phi}{dx^4}\right|_e \frac{13}{128}\Delta x^4 + H$$

ou

$$\phi_e = \frac{3}{2}\phi_P - \frac{1}{2}\phi_W \underbrace{+ \left.\frac{d^2\phi}{dx^2}\right|_e \frac{3}{8}\Delta x^2 - \left.\frac{d^3\phi}{dx^3}\right|_e \frac{\Delta x^3}{4} + \left.\frac{d^4\phi}{dx^4}\right|_e \frac{13}{128}\Delta x^4 + H}_{\varepsilon} \qquad (18)$$

Comparando (14) com (18):

15 Ago 08



$$T_e'\Big|_{x=L}$$

$$T_P = T_e - T_e'\frac{h}{2} + T_e''\frac{h^2}{8} - T_e'''\frac{h^3}{48} + T_e^{iv}\frac{h^4}{384} - T_e^{v}\frac{h^5}{3840} \qquad X -9$$

$$\begin{cases} T_W = T_e - T_e'\frac{3}{2}h + T_e''\frac{9}{8}h^2 - T_e'''\frac{27}{48}h^3 + T_e^{iv}\frac{81}{384}h^4 - T_e^{v}\frac{243}{3840}h^5 \\ -9T_P = -9T_e + T_e'\frac{9}{2}h - T_e''\frac{9}{8}h^2 + T_e'''\frac{9}{48}h^3 - T_e^{iv}\frac{9}{384}h^4 + T_e^{v}\frac{9}{3840}h^5 \end{cases}$$

$$T_W - 9T_P = -8T_e + 3T_e'h + 0 - T_e'''\frac{18}{48}h^3 + T_e^{iv}\frac{72}{384}h^4 - T_e^{v}\frac{234}{3840}h^5$$

$$\boxed{T_e' = \frac{(8T_e + T_W - 9T_P)}{3h} + T_e'''\frac{6}{48}h^2 - T_e^{iv}\frac{24}{384}h^3 + T_e^{v}\frac{76}{3840}h^4}$$

UDS-2

WUDS / A PRIORI

14 Nov 04
21

Para $h \to 0$:

$$\alpha = \lim_{h\to 0}\left[\frac{(Pe\,h)^2}{10+2(Pe\,h)^2}\right] \Longrightarrow \alpha \to 0 \quad \text{(CDS) na Advecção}$$

$$\beta = \lim_{h\to 0}\left[\frac{1+0{,}005(Pe\,h)^2}{1+0{,}05(Pe\,h)^2}\right] \Longrightarrow \beta \to 1 \quad \text{(CDS) na Difusão}$$

$\forall\, h, Pe$, WUDS $\to$ $\lambda_e = \underbrace{(1/2+\alpha_e)}_{C_P}\Lambda_P + \underbrace{(1/2-\alpha_e)}_{C_E}\Lambda_E$ [ADVECÇÃO]

Da dedução do CDS:

$$C_P\Lambda_P = C_P\Lambda_e - C_P\Lambda_e'\frac{h}{2} + C_P\Lambda_e''\frac{h^2}{8} - C_P\Lambda_e'''\frac{h^3}{48} + C_P\Lambda_e^{iv}\frac{h^4}{384} - C_P\Lambda_e^{v}\frac{h^5}{3\,840} + C_P\Lambda_e^{vi}\frac{h^6}{46\,080} - \dots$$

$+$

$$C_E\Lambda_E = C_E\Lambda_e + C_E\Lambda_e'\frac{h}{2} + C_E\Lambda_e''\frac{h^2}{8} + C_E\Lambda_e'''\frac{h^3}{48} + C_E\Lambda_e^{iv}\frac{h^4}{384} + C_E\Lambda_e^{v}\frac{h^5}{3\,840} + C_E\Lambda_e^{vi}\frac{h^6}{46\,080} + \dots$$

$$C_P\Lambda_P + C_E\Lambda_E = \underbrace{(C_P+C_E)}_{1}\Lambda_e + \underbrace{(C_E-C_P)}_{-2\alpha_e}\Lambda_e'\frac{h}{2} + \underbrace{(C_P+C_E)}_{1}\Lambda_e''\frac{h^2}{8} + \underbrace{(C_E-C_P)}_{-2\alpha_e}\Lambda_e'''\frac{h^3}{48} + \underbrace{(C_P+C_E)}_{1}\Lambda_e^{iv}\frac{h^4}{384} + \underbrace{(C_E-C_P)}_{-2\alpha_e}\Lambda_e^{v}\frac{h^5}{3\,840} + \underbrace{(C_E+C_P)}_{1}\Lambda_e^{vi}\frac{h^6}{46\,080} + \dots$$

$$C_P + C_E = \frac{1}{2} + \cancel{\alpha_e} + \frac{1}{2} - \cancel{\alpha_e} = 1$$

$$C_E - C_P = \frac{1}{2} - \alpha_e - \frac{1}{2} - \alpha_e = -2\alpha_e$$

$$\Lambda_e = C_P\Lambda_P + C_E\Lambda_E + \alpha_e\Lambda_e' h - \Lambda_e''\frac{h^2}{8} + \alpha_e\Lambda_e'''\frac{h^3}{24} - \Lambda_e^{iv}\frac{h^4}{384} + \alpha_e\Lambda_e^{v}\frac{h^5}{1920} - \Lambda_e^{vi}\frac{h^6}{46\,080} + \dots$$

$$\varepsilon(\lambda_e) = \Lambda_e - \lambda_e$$

$$\varepsilon(\lambda_e) = +\alpha_e\Lambda_e' h - \Lambda_e''\frac{h^2}{8} + \alpha_e\Lambda_e'''\frac{h^3}{24} - \Lambda_e^{iv}\frac{h^4}{384} + \alpha_e\Lambda_e^{v}\frac{h^5}{1920} - \Lambda_e^{vi}\frac{h^6}{46\,080} + \dots$$

$p_V = 1, 2, 3, \dots \to p_L = 1$ para $\alpha \neq 0$ ADVECÇÃO

$p_V = 2, 4, 6, \dots \to p_L = 2$ para $\alpha = 0$, que é o caso do WUDS para $h \to 0$

WUDS/A PRIORI 21 Nov 04

$$\forall h, Pe,\ \text{WUDS} \rightarrow \lambda'_e = \beta_e \frac{(\Lambda_E - \Lambda_P)}{h} \qquad [\text{DIFUSÃO}]$$

Da dedução do CDS,

$\cancel{\Lambda_E - \Lambda_P \times \beta_e}$ $\Lambda_P \times (-\beta_e)$ e $\Lambda_E \times (\beta_e)$

$\cancel{\beta_e(\Lambda_E - \Lambda_P) = \beta_e \Lambda'_e h}$

$$-\beta_e \Lambda_P = -\beta_e \Lambda_e + \beta_e \Lambda'_e \frac{h}{2} - \beta_e \Lambda''_e \frac{h^2}{8} + \beta_e \Lambda'''_e \frac{h^3}{48} - \beta_e \Lambda^{iv}_e \frac{h^4}{384} + \beta_e \Lambda^{v}_e \frac{h^5}{3840} - \beta_e \Lambda^{vi}_e \frac{h^6}{46\,080} + \dots \quad \text{OK}$$

$+$

$$\beta_e \Lambda_E = \beta_e \Lambda_e + \beta_e \Lambda'_e \frac{h}{2} + \beta_e \Lambda''_e \frac{h^2}{8} + \beta_e \Lambda'''_e \frac{h^3}{48} + \beta_e \Lambda^{iv}_e \frac{h^4}{384} + \beta_e \Lambda^{v}_e \frac{h^5}{3840} + \beta_e \Lambda^{vi}_e \frac{h^6}{46\,080} + \dots \quad \text{OK}$$

$$\beta_e(\Lambda_E - \Lambda_P) = \beta_e \Lambda'_e h + \beta_e \Lambda'''_e \frac{h^3}{24} + \beta_e \Lambda^{v}_e \frac{h^5}{1920} + \beta_e \Lambda^{vii}_e \frac{h^7}{322\,560} + \dots \quad \text{OK}$$

~~Substituindo $\Lambda_E - \Lambda_P$ em $\lambda'_e = \beta_e \frac{(\Lambda_E - \Lambda_P)}{h}$~~

$$\Lambda'_e = \frac{(\Lambda_E - \Lambda_P)}{h} - \Lambda'''_e \frac{h^2}{24} - \Lambda^{v}_e \frac{h^4}{1920} - \Lambda^{vii}_e \frac{h^6}{322\,560} - \dots \quad \equiv \text{CDS puro}$$

$$\varepsilon(\lambda'_e) = \Lambda'_e - \lambda'_e$$

$$\varepsilon(\lambda'_e) = \frac{(\Lambda_E - \Lambda_P)}{h} - \beta_e \frac{(\Lambda_E - \Lambda_P)}{h} - \dots$$

$$\varepsilon(\lambda'_e) = (1 - \beta_e) \frac{(\Lambda_E - \Lambda_P)}{h} - \Lambda'''_e \frac{h^2}{24} - \Lambda^{v}_e \frac{h^4}{1920} - \Lambda^{vii}_e \frac{h^6}{322\,560} - \dots \qquad \text{DIFUSÃO}$$

$p_V = -1, 2, 4, 6, \dots \rightarrow p_L = -1$ para $\beta \neq 1$

$p_V = 2, 4, 6, \dots \rightarrow p_L = 2$ para $\beta = 1$, que é o caso do WUDS para $h \rightarrow 0$

QUICK / A PRIORI / ADVECÇÃO 15 Nov 04

$$\text{QUICK:}\quad \lambda_e = \frac{6}{8}\Lambda_P + \frac{3}{8}\Lambda_E - \frac{1}{8}\Lambda_W$$

$$\frac{6}{8}\Lambda_P = \frac{6}{8}\Lambda_e - \frac{6}{8}\Lambda_e^{i}\frac{h}{2} + \frac{6}{8}\Lambda_e^{ii}\frac{h^2}{8} - \frac{6}{8}\Lambda_e^{iii}\frac{h^3}{48} + \frac{6}{8}\Lambda_e^{iv}\frac{h^4}{384} - \frac{6}{8}\Lambda_e^{v}\frac{h^5}{3840} + \frac{6}{8}\Lambda_e^{vi}\frac{h^6}{46080} - \frac{6}{8}\Lambda_e^{vii}\frac{h^7}{645120} + \dots$$

$$\frac{3}{8}\Lambda_E = \frac{3}{8}\Lambda_e + \frac{3}{8}\Lambda_e^{i}\frac{h}{2} + \frac{3}{8}\Lambda_e^{ii}\frac{h^2}{8} + \frac{3}{8}\Lambda_e^{iii}\frac{h^3}{48} + \frac{3}{8}\Lambda_e^{iv}\frac{h^4}{384} + \frac{3}{8}\Lambda_e^{v}\frac{h^5}{3840} + \frac{3}{8}\Lambda_e^{vi}\frac{h^6}{46080} + \frac{3}{8}\Lambda_e^{vii}\frac{h^7}{645120} + \dots$$

$$-\frac{1}{8}\Lambda_W = -\frac{1}{8}\Lambda_e + \frac{1}{8}\Lambda_e^{i}\frac{3h}{2} - \frac{1}{8}\Lambda_e^{ii}\frac{9}{8}h^2 + \frac{1}{8}\Lambda_e^{iii}\frac{27}{48}h^3 - \frac{1}{8}\Lambda_e^{iv}\frac{81}{384}h^4 + \frac{1}{8}\Lambda_e^{v}\frac{243}{3840}h^5 - \frac{1}{8}\Lambda_e^{vi}\frac{729}{46080}h^6 + \frac{1}{8}\Lambda_e^{vii}\frac{2187}{645120}h^7 - \dots$$

$$\frac{6}{8}\Lambda_P + \frac{3}{8}\Lambda_E - \frac{1}{8}\Lambda_W = \Lambda_e + \Lambda_e^{iii}\frac{h^3}{16} - \Lambda_e^{iv}\frac{3}{128}h^4 + \Lambda_e^{v}\frac{h^5}{128} - \Lambda_e^{vi}\frac{h^6}{512} + \dots$$

ou

$$\Lambda_e = \frac{6}{8}\Lambda_P + \frac{3}{8}\Lambda_E - \frac{1}{8}\Lambda_W - \Lambda_e^{iii}\frac{h^3}{16} + \Lambda_e^{iv}\frac{3}{128}h^4 - \Lambda_e^{v}\frac{h^5}{128} + \dots$$

$$\varepsilon(\lambda_e) = \Lambda_e - \lambda_e$$

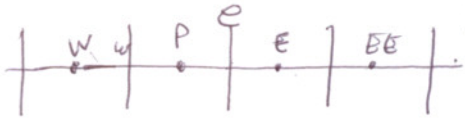
$$\varepsilon(\lambda_e) = -\Lambda_e^{iii}\frac{h^3}{16} + \Lambda_e^{iv}\frac{3}{128}h^4 - \Lambda_e^{v}\frac{h^5}{128} + \dots$$

ADVECÇÃO

$$p_V = 3, 4, 5, \dots \rightarrow p_L = 3$$

30 set 10

CDS-4 PARA T_e'



$$\left.\begin{aligned} T_P = T_e & - T_e'\frac{h}{2} + T_e''\frac{h^2}{8} - T_e'''\frac{h^3}{48} + T_e^{iv}\frac{h^4}{384} - T_e^{v}\frac{h^5}{3840} + T_e^{vi}\frac{h^6}{46080} \\ & - T_e^{vii}\frac{h^7}{645120} + T_e^{viii}\frac{h^8}{10.321.920} - \frac{T_e^{ix}h^9}{185.794.560} + \ldots \end{aligned}\right\} \times(-27)$$

$$\left.\begin{aligned} T_E = T_e & + T_e'\frac{h}{2} + T_e''\frac{h^2}{8} + T_e'''\frac{h^3}{48} + T_e^{iv}\frac{h^4}{384} + T_e^{v}\frac{h^5}{3840} + T_e^{vi}\frac{h^6}{46080} \\ & + T_e^{vii}\frac{h^7}{645120} + \frac{T_e^{viii}h^8}{10.321.920} + \frac{T_e^{ix}h^9}{185.794.560} + \ldots \end{aligned}\right\} \times(27)$$

$$\left.\begin{aligned} T_W = T_e & - T_e'\frac{3h}{2} + T_e''\frac{9h^2}{8} - T_e'''\frac{27h^3}{48} + T_e^{iv}\frac{81}{384}h^4 - T_e^{v}\frac{243}{3840}h^5 \\ & + T_e^{vi}\frac{729h^6}{46080} - T_e^{vii}\frac{2187h^7}{645120} + \frac{T_e^{viii}6561h^8}{10.321.920} - \frac{T_e^{ix}h^9 19.683}{185.794.560} + \ldots \end{aligned}\right\} \times 1$$

$$\left.\begin{aligned} T_{EE} = T_e & + T_e'\frac{3}{2}h + T_e''\frac{9}{8}h^2 + T_e'''\frac{27}{48}h^3 + T_e^{iv}\frac{81}{384}h^4 + T_e^{v}\frac{243}{3840}h^5 \\ & + T_e^{vi}\frac{729h^6}{46080} + T_e^{vii}\frac{2187h^7}{645120} + T_e^{viii}\frac{6561h^8}{10.321.920} + T_e^{ix}\frac{19.683h^9}{185.794.560} \end{aligned}\right\} \times(-1)$$

$$-27T_P + 27T_E + T_W - T_{EE} = 24T_e'h - \frac{9}{80}T_e^{v}h^5 - \frac{15}{2240}T_e^{vii}h^7 - \frac{91T_e^{ix}}{430080}h^9 - \ldots$$

ou

$$T_e' = \underbrace{\frac{(27T_E + T_W - 27T_P - T_{EE})}{24h}}_{\text{APROXIMAÇÃO}} \underbrace{+ \frac{3T_e^{v}}{640}h^4 + \frac{1\,T_e^{vii}}{3584}h^6 + \frac{91T_e^{ix}h^8}{10321920} + \ldots}_{\varepsilon}$$

$$p_V = 4, 6, 8, \ldots \longrightarrow p_L = 4$$