Introduction to the use of SMath Studio
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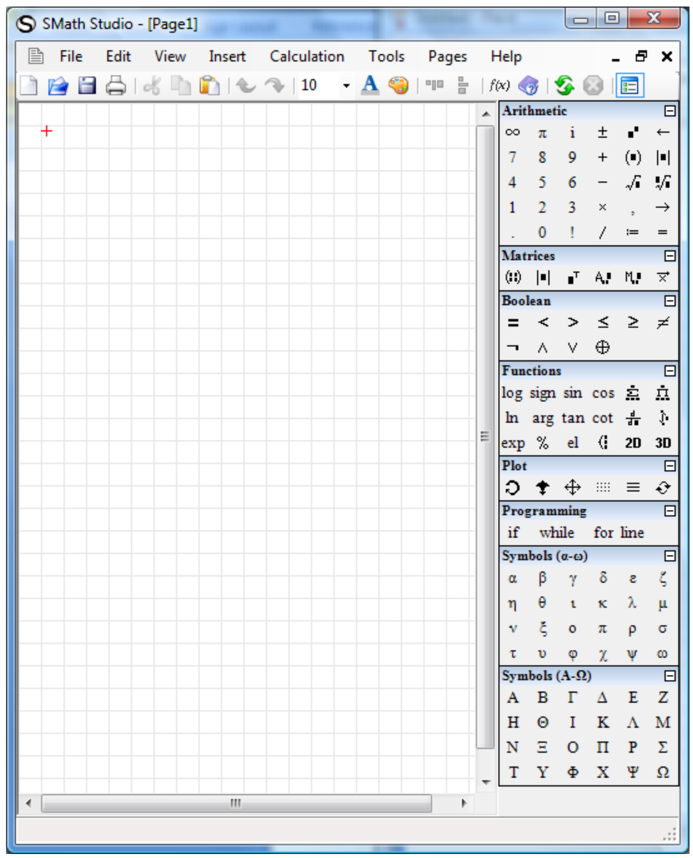
Where to find *SMath Studio*?
SMath Studio 0.85 is freeware that produces notebook type of mathematical calculations. You can download *SMath Studio* at
http://en.smath.info/forum/default.aspx?g=posts&t=202

Getting started
You will find the icon for *SMath Studio* in the folder *SMath\SMath Studio* under your *Program Files* folder in your *Windows* machine, or under your *Program Files (x86)* folder if you have a *64-bit* version of *Windows*. The program is called *SMathStudio_Desktop.exe*. You may want to create a shortcut to the program to place in your desktop or in your *Quick Launch*. Double click on the program icon or on the shortcut to open the *SMath Studio* interface.

The *SMath Studio* interface
To get started, find the *SMath Studio 13* icon in your *Start>Programs* button in Windows. The *SMath Studio* interface is shown here.

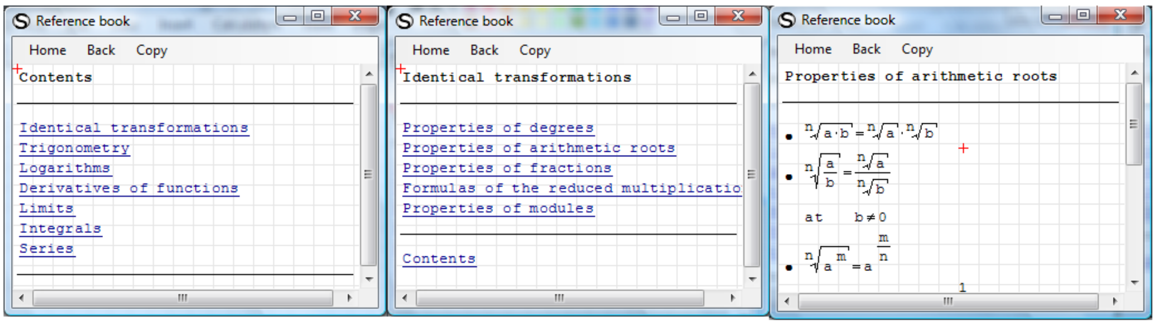


The interface shows a main window with menus, a toolbar, and a number of palettes on the right-hand side of the interface. The palettes contain mathematical, graphical, and programming functions that can be placed in the main window with the purpose of calculating mathematical expression, producing graphs, or building small programs.

The *SMath Studio* menus
The *SMath Studio* interface contains menus entitled *File, Edit, View, Insert, Calculation, Tools, Pages,* and *Help*. Besides these 8 menus, there is also a menu indicated by a page icon located to the left of the menu bar. We'll refer to this menu as the *Control* menu. Explore the different menus to become acquainted with the various options available in them. The *Control*, *File*, and *Edit* menus operation is very similar to those of other *Windows* applications, therefore, the item therein contained would be familiar to *Windows* users. Other menus are presented below.

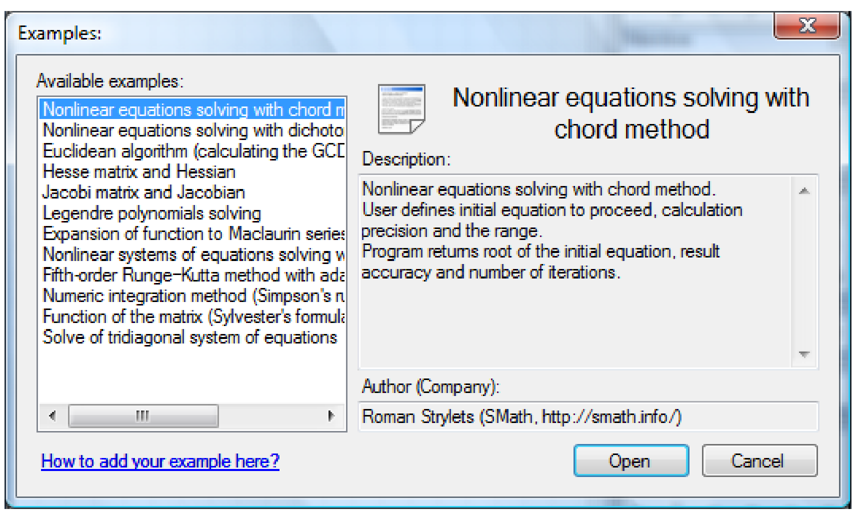
The *Help* menu

The *Help* menu contains the option *Reference book…* which represents a modest attempt by the author to address basic mathematical operations and definitions. The contents of the *Reference book* are shown in the figure below. To find the contents of a particular section of the *Reference book*, click on that section. For example, to see the contents of the section entitled *Limits*, click on the corresponding link to produce the screen in the middle. If you click on the link entitled *Properties of limits*, you get the screen shown to the right in the figure below. This provides some basic properties of limits that can be useful to review that concept from Calculus.

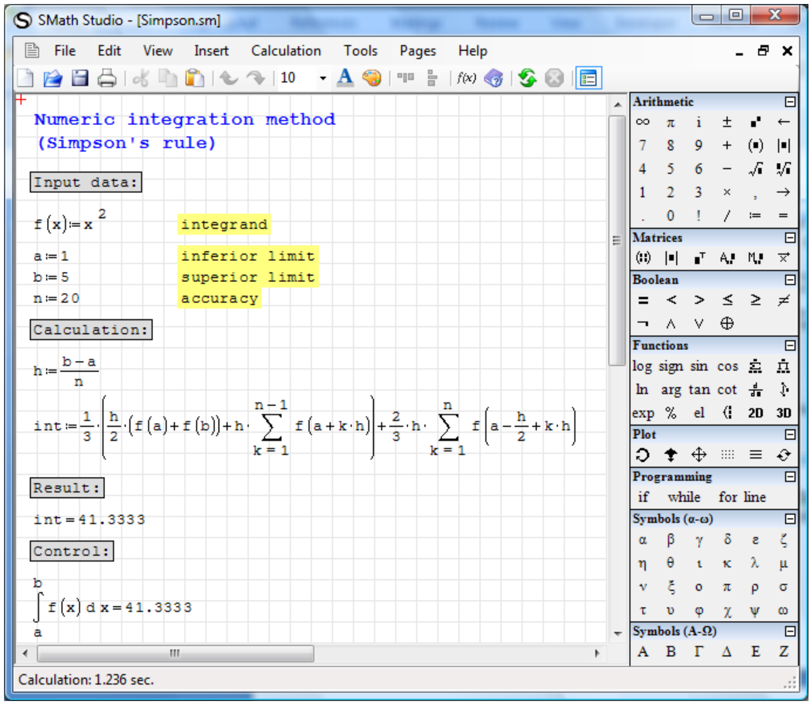


You can go back one page in the *Reference book* by pressing the option *Back*, or return to the Contents page by pressing the option *Home*. The menu option *Copy* lets you copy equations from the *Reference book* that you can then paste in the main interface. To copy a specific equation, first highlight the equation you want to copy. We will illustrate this operation in another section of this document.

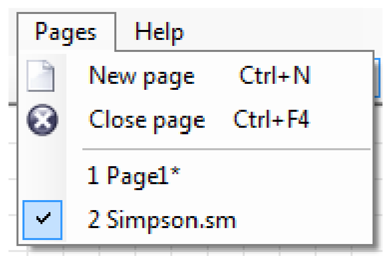
The *Help* menu also contains the option *Examples*, which opens the following menu of examples:



Click on a given example to select it, and press the [Open] button. The worksheet corresponding to the example *Numeric integration method (Simpson's rule)* is shown below.



The *Pages* menu

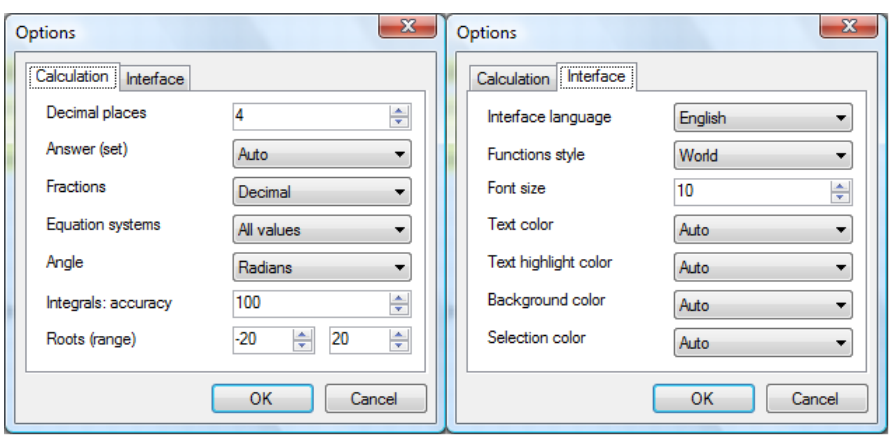


The *Pages* menu shows the pages (worksheets) currently open. After having opened the worksheet for the example shown above, the *Pages* menu will show two pages open, namely, *Page1*,the default page open when we started *SMath Studio*, and *Simpson.sm*, the worksheet we just opened from the *Examples* menu.

The *Pages* menu also shows the options *New page* and *Close page*, whose operation is obvious.

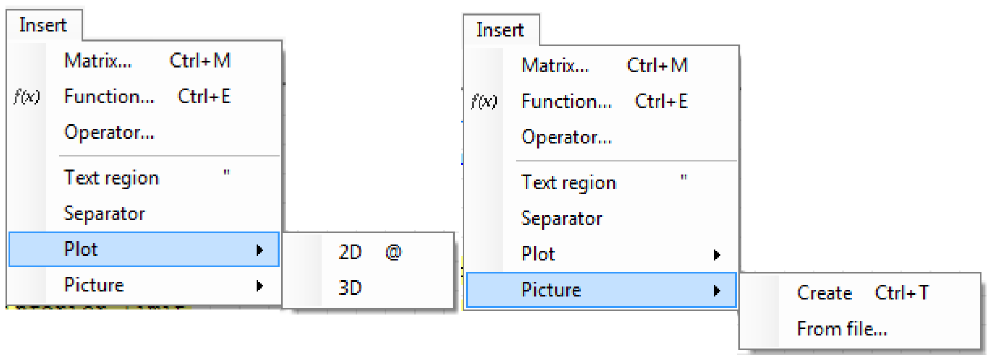
The *Tools* menu

The *Tools* menu has a single item (*Options*) which open the following dialogue form with two tabs. The contents of the two tabs are shown below.

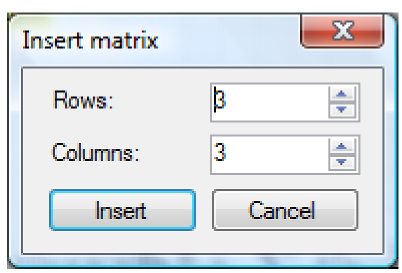


The *Calculation* tab in the *Options* interface lets you modify basic settings for mathematical calculations, whereas the *Interface* option deals with properties of the interface window.

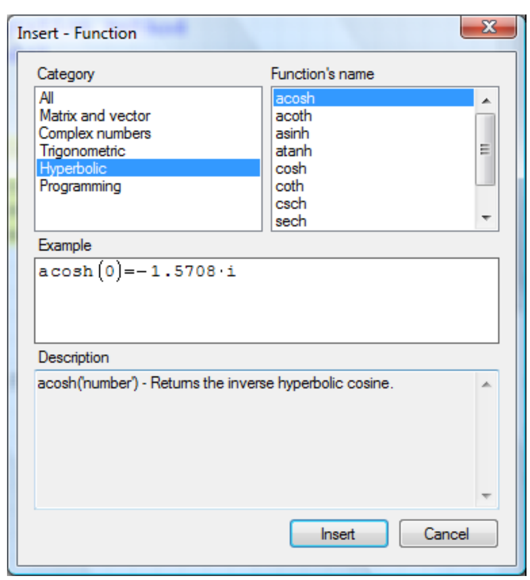
<u>The *Insert* menu</u>
The *Insert* menu allows inserting a variety of items into the worksheet (or page), as indicated in the figure below:



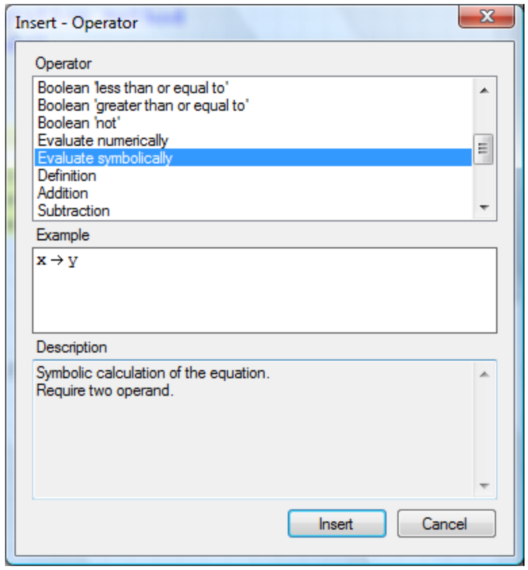
When you select the option *Matrix..* produces an entry form that allows you build a matrix of a pre-determined size:



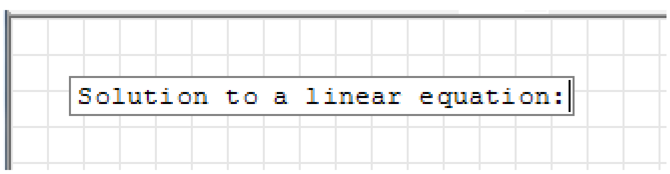
The option *Function…* opens up a menu of mathematical functions, i.e.,



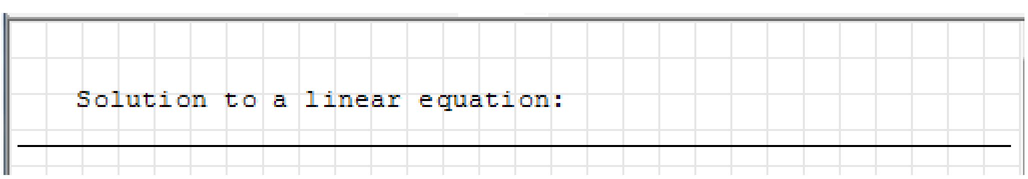
Selecting the option *Operator…* in the *Insert* menu produces a list of Boolean, arithmetic, and other operators, e.g.,



The option *Text region* in the *Insert* menu allows the user to insert text fields in the worksheet with the purpose of documentation. This is equivalent to pressing the double quote (") after clicking in any position in the worksheet. For example:



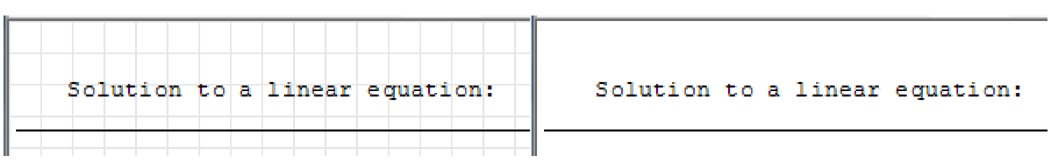
Inserting a *Separator* simply means inserting a horizontal line to separate regions in the worksheet, e.g.,



The insertion of plots and pictures will be illustrated in other sections of this document.

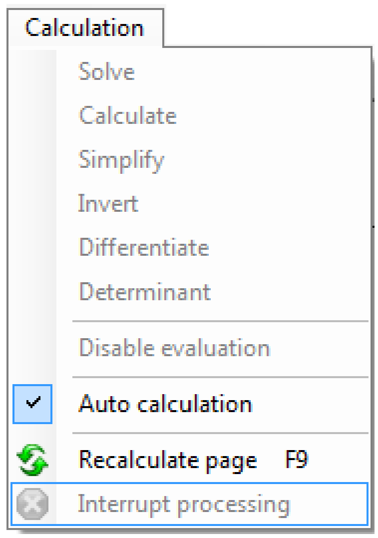
<u>The *View* menu</u>

The *View* menu includes a single option that lets you select or deselect the grid in the interface. Here is the same worksheet with and without a grid:



The *Calculation* menu

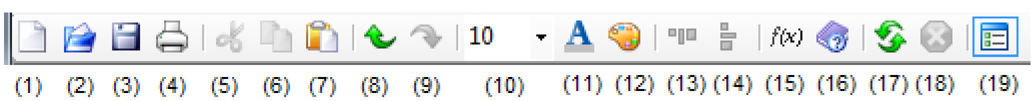
The *Calculation* menu offer options useful when calculating symbolic or numeric expressions. Unless a particular calculation has been selected, these options show as inactive (shadow options) in the menu, e.g.,



The only active options shown above are *Auto calculation* and *Recalculate page*. The meaning of these, and the other options shown above, is obvious.

The *SMath Studio* toolbar

The *SMath Studio* toolbar contains 19 icons briefly described below:



1. New page
2. Open (existing worksheet)
3. Save (current worksheet)
4. Print (current worksheet)
5. Cut
6. Copy
7. Paste
8. Undo (recent action)
9. Redo (recent action)
10. Font size
11. Text color
12. Background color
13. Align horizontally
14. Align vertically
15. Function
16. Reference book (see *Help*)
17. Recalculate page
18. Interrupt process
19. Show/hide side panel

Items (1) through (4) manipulate new or existing worksheets. Items (5) through (7) are editing functions. Items (8) and (9) will un-do and re-do the most recent action. Items (10) through (12) adjust font or background properties. Items (13) and (14) re-align selected cells. Item (15) opens the *Function* menu, as shown earlier. Item (16) is also available under the *Help* menu. Items (17) and (18) were shown earlier in the *Calculation* menu. Item (19) shows or hides the palettes on the right-hand side of the page.

Using *SMath Studio* as a calculator
SMath Studio (or, simply, *SMath*) can be used as a calculator. For example, open a new pate, and click on the main window in an area near the top left corner. A small red cross will indicate the location where you want to enter a calculation. Type the following:

1+2/3+4/5 =

The result is the following expression:

$$1+\frac{2}{3+\frac{4}{5}}=1.5263$$

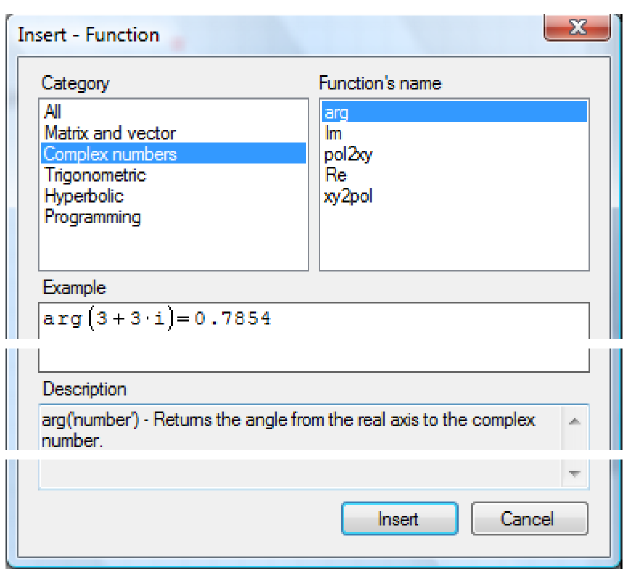
Notice the way that *SMath* interprets the two fractions. Try the following example also: click somewhere else in your worksheet and type:

5 + 2/3 [→]+ (4/5) =

The result is now:

$$5+\frac{2}{3}+\frac{4}{5}=6.4667$$

Functions in *SMath*
Calculations in *SMath* may involve mathematical functions such as *sin*, *cos*, *exp*, etc. You can insert any function by using the menu option *Insert > Function…* or by pressing item (15) [*f*(*x*)] in the toolbar. As indicated earlier, collections of functions under the headings *All*, *Matrix and vector*, *Complex numbers*, *Trigonometric*, *Hyperbolic*, and *Programming* are available. The figure below shows the options for complex number functions:

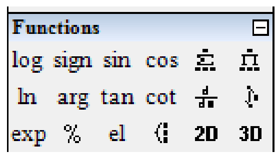


In this example, function *arg* is selected. The argument (*arg*) of a complex number is the angle that a vector representing the complex number in the complex plane forms with the real (*x*) axis. As indicated in the figure above, as you select a particular function the *Insert – Function* form provides an *Example* as well as a brief *Description* of the function.

Typically, you will start an expression in the worksheet and, at the proper location, insert the function that you need. For example, calculate the expression that uses the hyperbolic function *asinh*:

$$\boxed{4+2\cdot \operatorname{asinh}(3+2\cdot i)=7.9668+1.1413\cdot i}$$

Some functions are available for insertion in the *Functions* palette:



To enter any of those functions simply place a cursor in the desired position and click on the name of the function, e.g., type:

$$\ln\left(\frac{1}{\sin(3.2)}\right)=2.8409+3.1416\cdot i$$

Some of the functions in the *Functions* palette include operations typical from Calculus, such as summation, products, derivatives, and integrals, e.g.,

$$\sum_{k=1}^{5}\left(\frac{1}{k^{2}}\right)=1.4636 \qquad \prod_{x=1}^{10} x=3.6288\cdot 10^{6}$$

$$\frac{d}{dx}\left(x^{2}+1\right)\to 2\cdot x \qquad \int_{1}^{\sqrt{2}}\frac{1}{1+x^{2}}\,dx\to\frac{1699}{10000}$$

In these expressions we used symbols that are available in the *Functions* palette as well as other symbols available elsewhere. These are:

- Infinity (∞) - available in the *Arithmetic* palette
- Symbolic evaluation (→) – available in the *Arithmetic*

Numeric versus symbolic evaluation
An expression that results in a number is evaluated using the numerical evaluation symbol (=), whereas, an expression that results in a symbolic output needs the symbolic evaluation symbol (→). Both symbols are available in the *Arithmetic* palette. The numerical evaluation symbol (=) can be typed directly from the keyboard (it's just the *equal* sign). The symbolic evaluation symbol (→) can also be entered by typing *Ctrl+period* (*Ctrl*+.).

The examples above show the summation and product producing numeric results, while the derivative and the definite integral produce symbolic results. Notice the difference between numeric and symbolic results in the following integral:

$$\int_1^3 \frac{1}{t}\,dt = 1.0986 \qquad \int_1^3 \frac{1}{t}\,dt \rightarrow \frac{5493}{5000}$$

The main difference between numeric and symbolic results in the current version of *SMath Studio* is on whether a result is shown in its decimal form (numeric) or as a fraction (symbolic) and other symbolic results such as square roots, etc. A proper symbolic result in the integral above would have been the expression *ln(3)*, however, *SMath Studio* does not handle such type of symbolic calculations when the result involves a function definition.

For an open-source, free program that would handle such type of symbolic calculations please refer to this web site: http://www.neng.usu.edu/cee/faculty/gurro/Maxima.html.

Note: Use function *eval* to convert from symbolic to numeric results, e.g., *eval*(*sqrt*(3)):

An alternative for entering functions in expressions is simply to type the name of the function. For example, click in another area of the main *SMath Studio* interface, and type the following expression:

2.5*sin(2.5)+1.2*cos(2+3/(1+4*1.2^2))

The results of this operation will be shown as:

$$2.5 \cdot \sin(2.5) + 1.2 \cdot \cos\left(2 + \frac{3}{\left(1 + 4 \cdot 1.2^2\right)}\right) = 0.5767$$

The *Insert - Function* menu in *SMath Studio*
As indicated earlier, the *Insert - Function* menu in *SMath Studio* can be obtained by pressing the [*f*(*x*)] button in the tool bar. This menu includes function groups labeled *All, Matrix and vector, Complex numbers, Trigonometric, Hyperbolic*, and *Programming*.

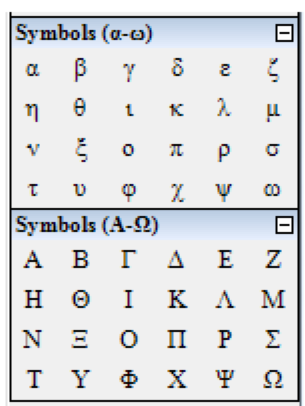
The *Complex numbers, Trigonometric, Hyperbolic*, and *Programming* function groups include a relatively small number of easily recognizable functions. The *Matrix and vector* group includes a total of 30 functions useful for the manipulation of and calculations with vectors and matrices. As indicated before, examples and brief explanations of the functions are available in the *Insert-Function* menu.

The *All* menu includes all existing functions in *SMath Studio*. Herein we group some of those functions according to their applications:

- Functions for input/output and file manipulation: *dfile, rfile, wfile*
- Functions specific to *SMath Studio*: *eval, range, sys*
- Functions for real numbers: *abs, exp, Gamma, ln, log, log10, mod, nthroot, numden, perc, round, sign, sqrt*
- Functions for algebraic manipulation: *expand*
- Functions for solving equations: *polyroots, solve*
- Functions for Calculus applications: *diff, int, product, sum*
- Functions for interpolation: *ainterp, cinterp, linterp*
- Statistics functions: *random*

Note: Function *expand* actually produces factorization of algebraic expressions. Expansion of algebraic expressions is performed automatically in *SMath Studio*, e.g.,

Entering Greek letter in *SMath Studio*
Greek characters can be entered by using the *Symbols* palettes in the interface. These palettes, showing lower-case and upper-case Greek letters, are shown in the figure below.



An alternative way to enter Greek letters is to type a letter in the English alphabet followed by *Ctrl-g*. This will generate a corresponding Greek letter. For example, typing *g Ctrl-g* produces the Greek letter γ (gamma). The corresponding upper case character would be entered as *G Ctrl-g*, resulting in the letter Γ (upper-case gamma).

The table below shows the letters of the Greek alphabet and its closest equivalent English letters.

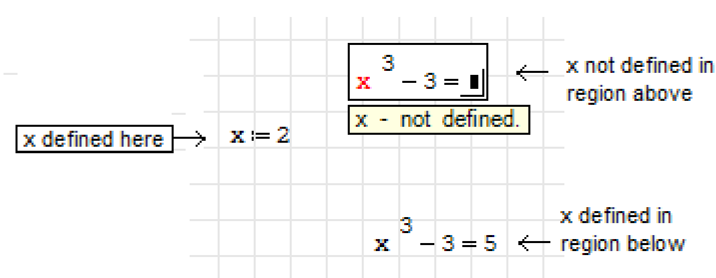
Lower case	Upper case	Letter name	English equivalent	Lower case	Upper case	Letter name	English equivalent
α	A	Alpha	a	ν	N	Nu	n
β	B	Beta	b	ξ	Ξ	Xi	x
γ	Γ	Gamma	g	o	O	Omicron	o
δ	Δ	Delta	d	π	Π	Pi	p
ε	E	Epsilon	e	ρ	P	Rho	r
ζ	Z	Zeta	z	σ	Σ	Sigma	s
η	H	Eta	h	τ	T	Tau	t
θ	Θ	Theta	th	υ	Υ	Upsilon	u
ι	I	Iota	i	ϕ	Φ	Phi	ph
κ	K	Kappa	k	χ	X	Chi	ch
λ	Λ	Lambda	l	ψ	Ψ	Psi	ps
μ	M	Mu	m	ω	Ω	Omega	o

Clicking on any of the letters in the *Symbols* palette will copy that letter to any entry point or text in the worksheet. To illustrate this fact we perform symbolic and numeric calculations with trigonometric functions that features the value π:

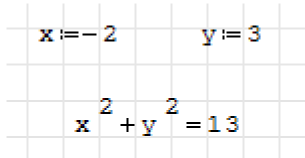
$$\sin\left(\frac{\pi}{6}\right)+\cos\left(\frac{\pi}{6}\right)\to\frac{1+\sqrt{3}}{2} \qquad \sin\left(\frac{\pi}{6}\right)+\cos\left(\frac{\pi}{6}\right)=1.366$$

Notice that SMath Studio assigns the proper value to the symbol π in the calculation.

Defining variables in *SMath Studio*
To define a variable in *SMath Studio* use the *assignment operator* (:=). To enter this operator simply press the *colon* key in your keyboard (:). Click anywhere in the worksheet and make the following variable assignment: $x := 2$, by typing: x : 2. The value of *2* is now stored in the name *x*. The value $x = 2$ will be replaced into any expression containing the name *x* that is located below or to the right of the assignment statement. To check this fact, type an equal sign in a location above the assignment statement $x{:}=2$ and fill the placeholder to the left with the expression x^2-3. The result is inconclusive, as shown below. Then, repeat this operation in a location below or to the right of the assignment statement $x{:}=2$. The result now is 5, as illustrated below.

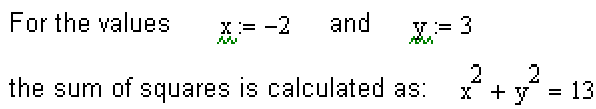


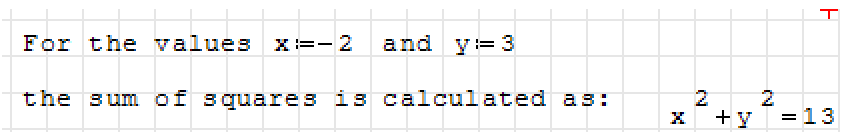
Once you have made variable assignments, you can use the assigned variables to calculate expressions. As an example, assign the values *x: =-2* and *y:=3*, then calculate the following expression: x^2+y^2. The result may look somewhat like this:



Documenting the worksheet

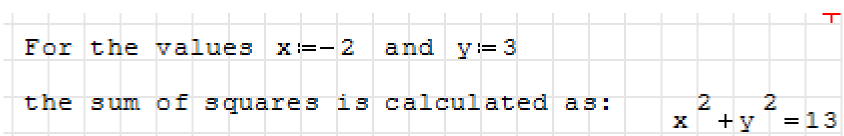
In order to document your worksheet, you can add text to it by clicking on any place in the worksheet and typing " followed by the text. (Alternatively, you can use the option *Insert>Text Region* in the worksheet menu). For example, the last exercise above, can be documented as follows:



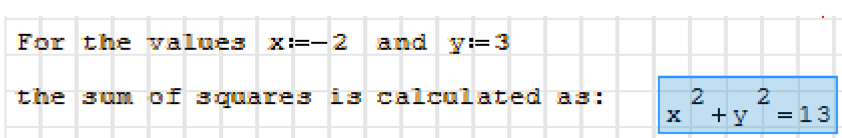


Selecting worksheet fields for editing and repositioning

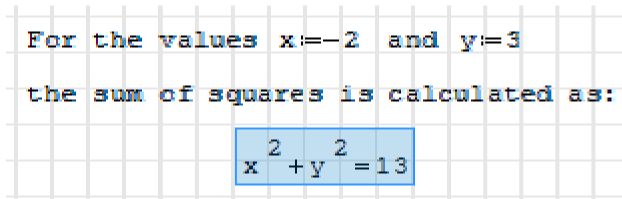
If you click on any of the text lines or the operations shown above, the corresponding field will be shown enclosed in a rectangle. This indicates that the field can be edited for text or math calculations. An example is shown below:



A text or calculation field can also be selected by clicking in a region near the field and dragging the mouse towards the field. In this case, the field is enclosed by a frame with blue boundaries and a blue background, e.g.,



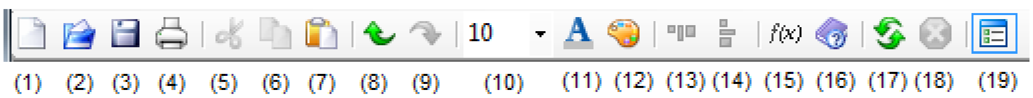
When selected in this way, a field can be dragged and positioned somewhere else in the worksheet. For example, the result shown above could be re-organized as follows:



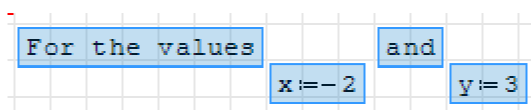
A field selected by dragging can be erased from the worksheet by pressing the [*delete*] or the [*backspace*] key. The selected field can also be copied and pasted by using *Ctrl-C* and *Ctrl-V*, respectively. By dragging on top of two or more fields, multiple fields can be selected simultaneously for editing.

Horizontal and vertical alignment of cells

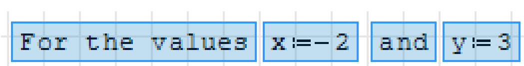
Items (13) and (14) in the *SMath Studio* toolbar (see figure below) provide for horizontal and vertical alignment of cells in the worksheet.



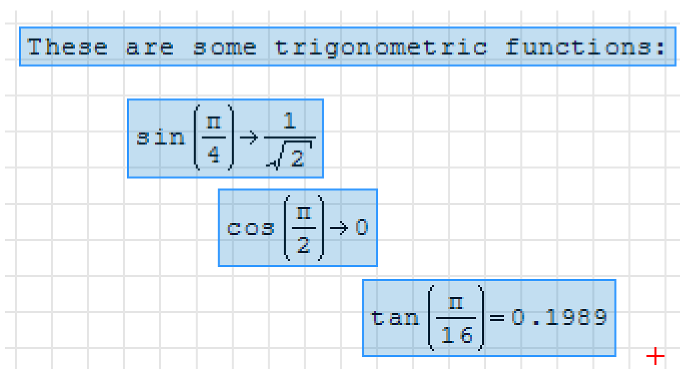
Consider the following text and calculation cells that are misaligned horizontally:



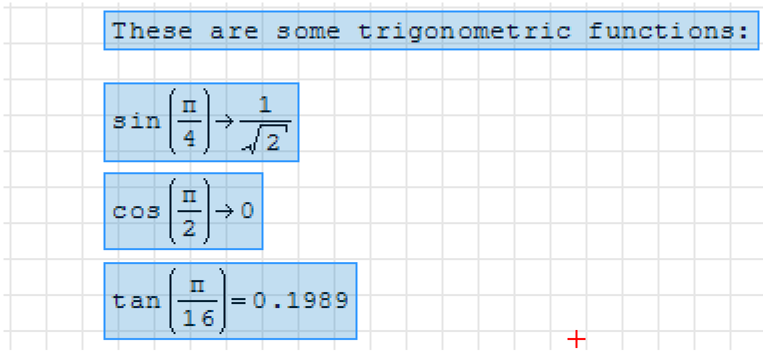
The four cells were selected by dragging the mouse, while holding the left button, over the cells. Next, press button (13) in the toolbar to get the following result:



Now, for an exercise on vertical alignment, consider the following vertically misaligned cells:



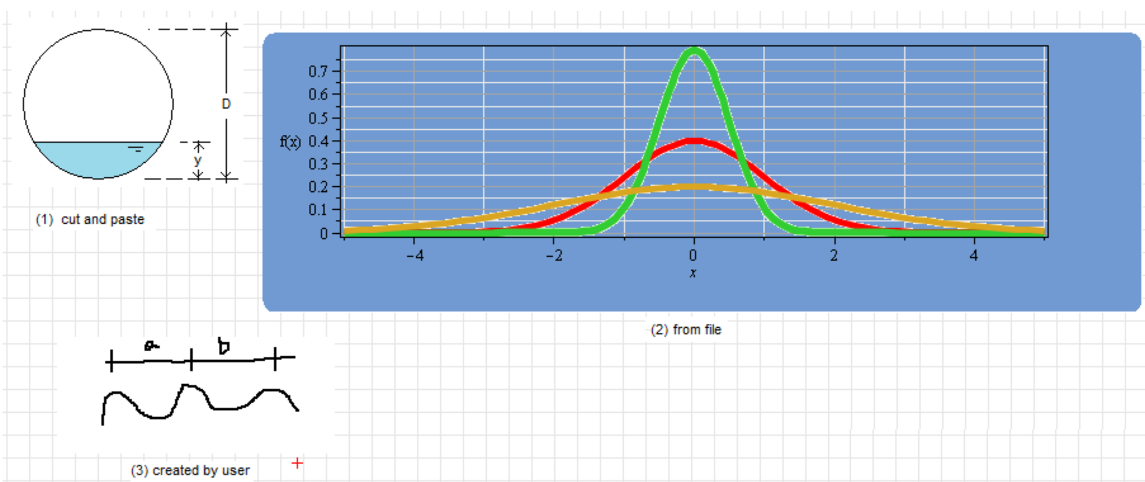
After pressing button (14) in the toolbar we get the following result:



Adding images to the worksheet
Images scanned into the computer or produced by other graphic software can be incorporated into your worksheet in different ways.

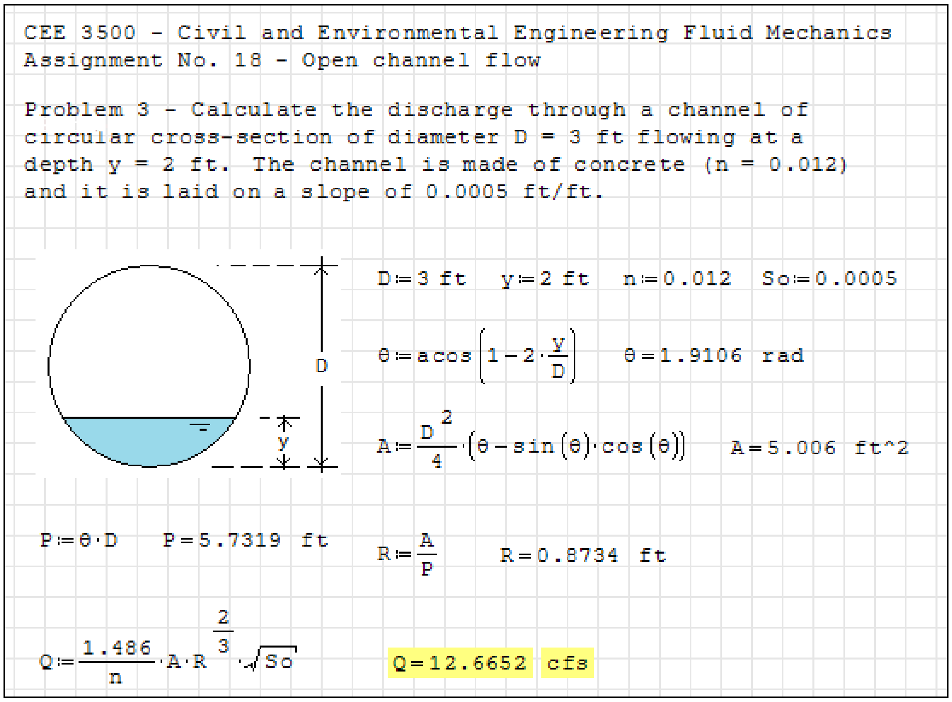
1 – Copying a figure from other software, clicking and using *Paste* (or *Ctrl-v*).
2 – Using *Insert* > *Picture* > *From file* …, to insert a picture contained in a file.
3 – Using *Insert* > *Picture* > *Create* …, to insert a picture frame where you can draw your own picture. Use the mouse as a pencil. *SMath Studio* does not provide for other drawing tools

Examples of the three cases are illustrated below. The only images allowed to insert from a file are *bitmap* images. Through the cut-and-paste procedure, you can paste images from any format.



With text and calculation cells, plus image cells imported into or created within *SMath Studio*, you can produce technical reports as would be required for class assignments or for an engineering project.

Application – Writing an assignment in *SMath* Studio
The figure in next page shows a class assignment in fluid mechanics/hydraulics prepared in *SMath Studio*.



CEE 3500 - Civil and Environmental Engineering Fluid Mechanics
Assignment No. 18 - Open channel flow

Problem 3 - Calculate the discharge through a channel of circular cross-section of diameter D = 3 ft flowing at a depth y = 2 ft. The channel is made of concrete (n = 0.012) and it is laid on a slope of 0.0005 ft/ft.

$D := 3$ ft $\quad y := 2$ ft $\quad n := 0.012 \quad So := 0.0005$

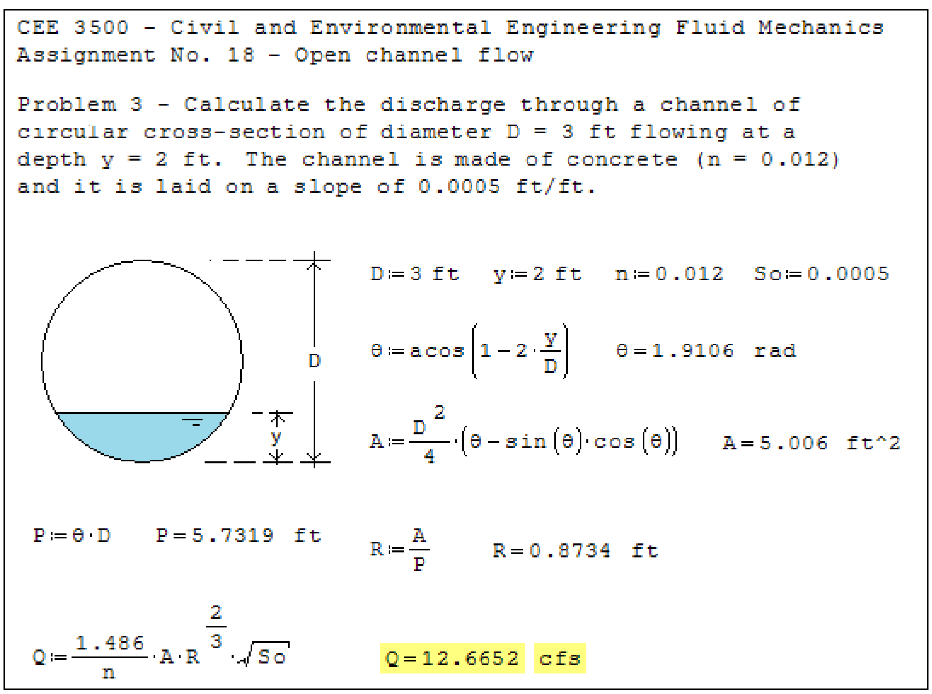
$\theta := \mathrm{acos}\left(1 - 2\cdot\frac{y}{D}\right) \qquad \theta = 1.9106$ rad

$A := \frac{D^2}{4}\cdot(\theta - \sin(\theta)\cdot\cos(\theta)) \qquad A = 5.006$ ft^2

$P := \theta\cdot D \qquad P = 5.7319$ ft $\qquad R := \frac{A}{P} \qquad R = 0.8734$ ft

$Q := \frac{1.486}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot\sqrt{So} \qquad Q = 12.6652$ cfs

If you prefer, you can click off the grid using the *View* menu to produce the following:



CEE 3500 - Civil and Environmental Engineering Fluid Mechanics
Assignment No. 18 - Open channel flow

Problem 3 - Calculate the discharge through a channel of circular cross-section of diameter D = 3 ft flowing at a depth y = 2 ft. The channel is made of concrete (n = 0.012) and it is laid on a slope of 0.0005 ft/ft.

D
y

$D := 3$ ft $\quad y := 2$ ft $\quad n := 0.012 \quad So := 0.0005$

$\theta := \mathrm{acos}\left(1 - 2\cdot\frac{y}{D}\right) \qquad \theta = 1.9106$ rad

$A := \frac{D^2}{4}\cdot(\theta - \sin(\theta)\cdot\cos(\theta)) \qquad A = 5.006$ ft^2

$P := \theta\cdot D \qquad P = 5.7319$ ft $\qquad R := \frac{A}{P} \qquad R = 0.8734$ ft

$Q := \frac{1.486}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot\sqrt{So} \qquad Q = 12.6652$ cfs

Notes:
(1) To enter multiple lines in a text box, use *Shift-Enter* at the end of each line.
(2) *SMath Studio* doesn't yet have the ability to handle units. The units you see in these examples were inserted as text fields.

User-defined functions
You can define your own functions by using the expression

function_name(*argument*(*s*)): *function_expression*

Examples of definitions and evaluations of functions are shown below:

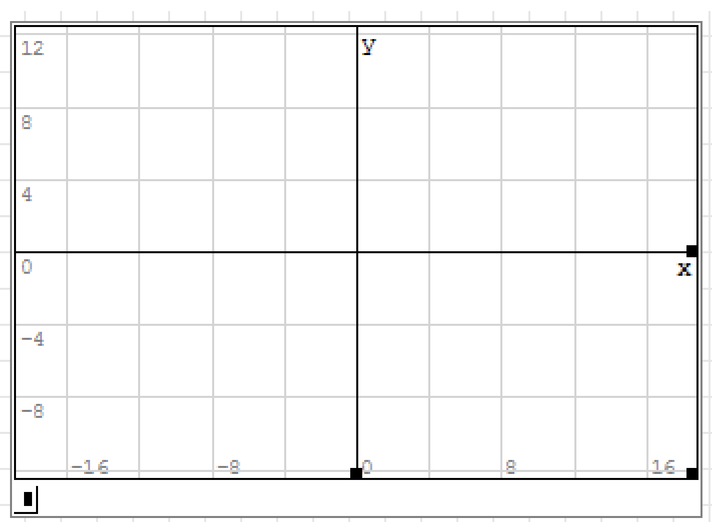
$$f(x) := \frac{x^2+1}{x+2} \qquad f(2)=1.25 \qquad f(3)=2$$

$$g(x, y) := x^2 + y^2 \qquad g(2, 3)=13 \qquad g(5, -2)=29$$

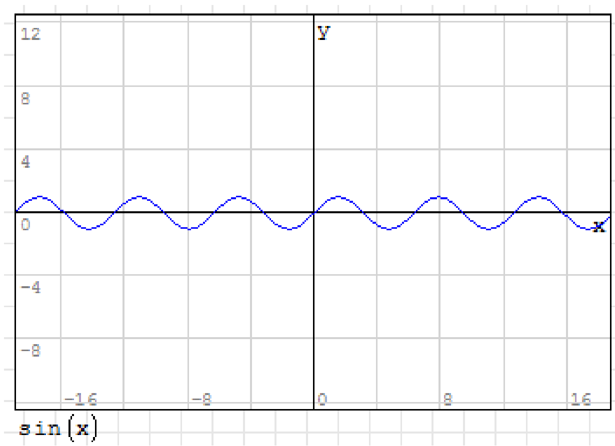
$$h(x) := \sum_{k=1}^{x} \left(k^2\right) \qquad h(2)=5 \qquad h(3)=14$$

$$r(x) := \int_0^x \frac{1}{\sqrt{1+t}}\, dt \qquad r(2)=1.4641 \qquad r(3)=2$$

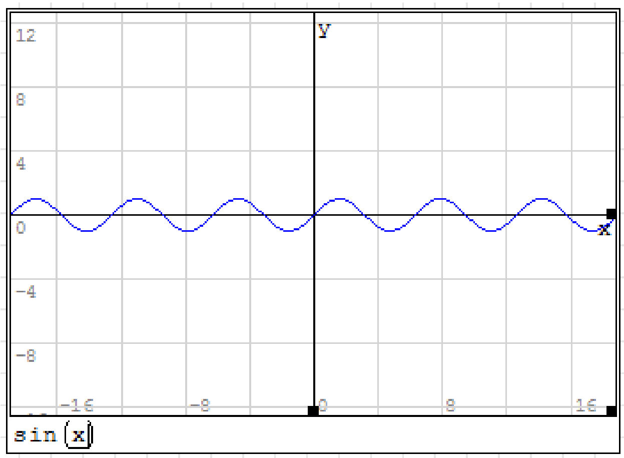
Plots of functions in 2D
SMath Studio allows the user to insert two-dimensional (2D) plots by using the *2D* option in the *Plot* palette, or the menu option *Insert > Plot > 2D*. This action will open a 2-D graph window, as illustrated below:



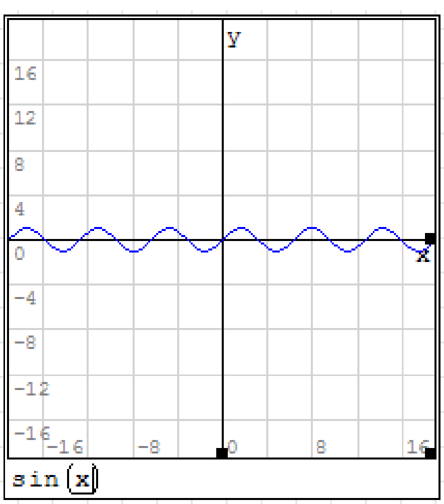
In the place holder on the lower left corner of the graph window you can enter the name of a function to plot, e.g.,

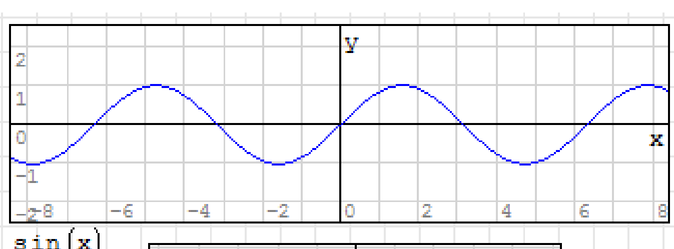


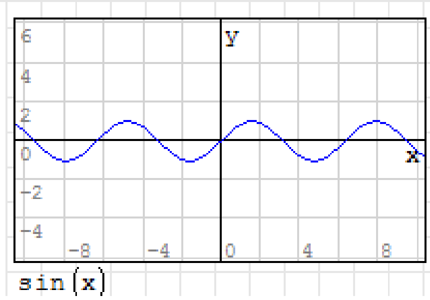
If you click on the graph window, you can drag one of the window handles shown below (right-hand side, bottom, or lower right corner) to modify the size of the window.



Some resizing examples are illustrated below:

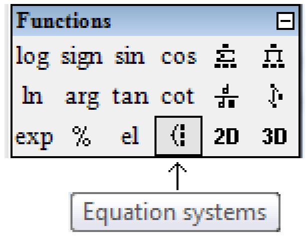






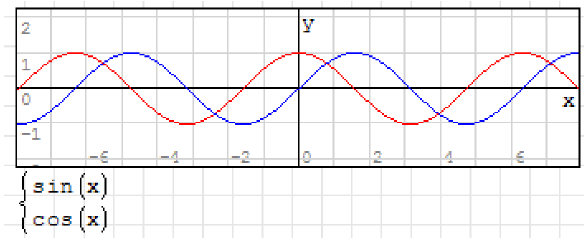
Plotting more than one function

To plot one than one function, use the *Equation systems* option in the *Functions* palette, namely,



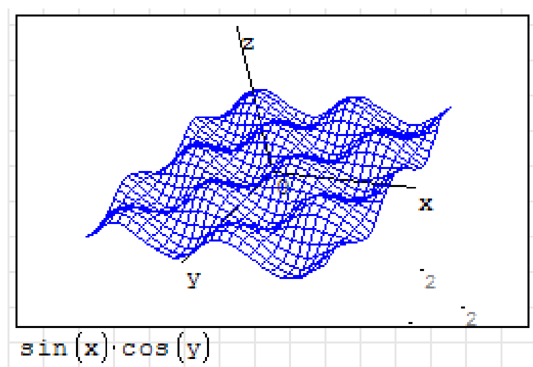
,

to insert more than one function in the function placeholder for the graph window. Here is an example:

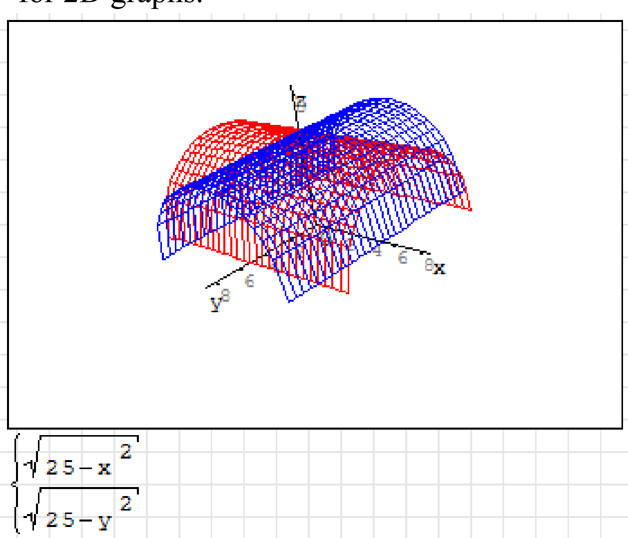


Plots of functions in 3D

Use the *3D* option in the *Plots* palette, or the menu option *Insert* > *Plots* > *3D*. Type the equation *f*(*x*,*y*) in the place holder located in the lower left corner of the graph window. You can drag the mouse pointer in the graph window to rotate the graph for a better view of the mesh surface, as illustrated here:



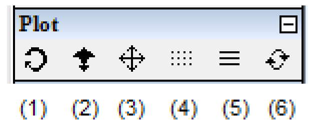
Resizing of the graph is also possible as done for 2D graphs.



As we did with 2D plots, you can plot more than one 3D plot as illustrated in the graph to the left.

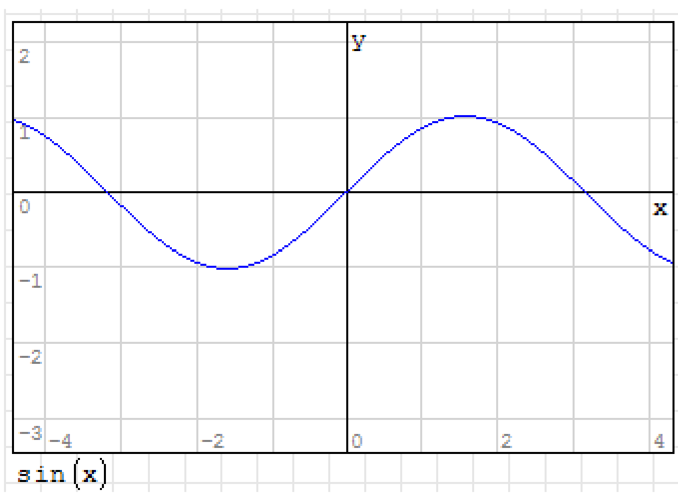
Additional manipulation of graphs

Additional manipulation of graphs is possible using the *Plot* palette. The *Plot* palette includes the following buttons:



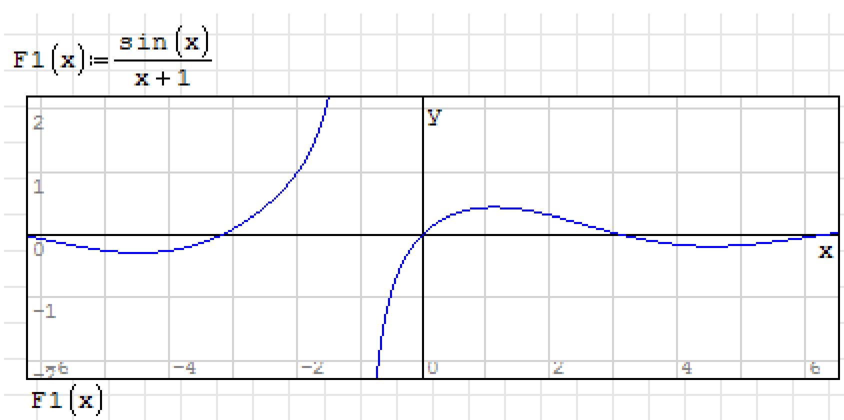
(1) Rotate – for 3D graphs, default setting
(2) Scale – With *Ctrl* or *Shift*, while dragging mouse over the graph, for proportional scaling – play around with this command to understand its operation
(3) Move – move the graph around in existing graph window
(4) Graph by points – show graph made of discrete points
(5) Graph by lines – show graphs made of continuous lines
(6) Refresh – refresh graph window to its default settings

Try these options on your own to learn more about the different *Plot* palette actions. The example below shows a proportional scaling using item (3) above, while holding down the *Shift* key.



Plotting your own functions

Define a function as indicated earlier, and replace the function name in the graph window place holder. Here is an example:

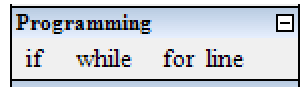


Defining functions with the *if* function

Some functions require more than one expression for their definition. For example,

$$F(x) = \begin{cases} x+1, & x < 2 \\ x^2, & x \geq 2 \end{cases}.$$

This function can be defined using the *if* function in the *Programming* palette. When selecting this option, the following programming structure, with placeholders, is produced:




```
F(x):=if ∎
        ∎
      else
        ∎
```

To program the function shown above, we can use:

```
F(x):=if x<2
          x+1
        else
           2
          x
```

If the definition of the function requires more than two expressions, it will be necessary to use nested *if* programming structures. Consider the following example:

Function defined as:

$$F(x) = \begin{cases} x+1, & x < 2 \\ (x+1)^2, & 2 \leq x \leq 4 \\ (x+1)^3, & x > 4 \end{cases}$$

Function programmed as:

```
F(x):=if x<2
          x+1
      else
          if x<4
               ((x+1)^2)
          else
               ((x+1)^3)
```

A plot of this function is shown below:

More details on programming in *SMath Suite* will be provided in a later section.

Functions defined by calculus operations

The *Functions* palette include four operations that are proper from calculus: summation, product, derivative, and integrals. These functions can be incorporated in the definition of functions as illustrated in the following examples:

$$f(x) := \sum_{k=1}^{x} \left(\frac{1}{k}\right) \qquad f(2) = 1.5 \qquad f(2.5) = \blacksquare$$

Limits must be integer.

$$g(x) := \prod_{k=1}^{x} \left(\frac{k}{10}\right) \qquad g(2) = 0.02 \qquad g(2.5) = \blacksquare$$

Limits must be integer.

$$h(x, t) := \frac{d}{dt}\left(x^2 + \frac{t \cdot \ln(t)}{x}\right) \qquad h(x, t) \rightarrow \frac{1 + \ln(t)}{x}$$

$$r(x) := \int_1^x \left[(1 + t)^2\right] dt \qquad r(2) = 6.3333 \qquad r(2.5) = 11.625$$

Notice that the functions $f(x)$ and $g(x)$, whose argument defines the upper limit of the summation and product, are only defined for integer values.

Solving single equations with *solve*

Function *solve* can be used to solve single equations. The function can be called using either of the following calls:

solve(*equation*, *variable*)

or

solve(*equation*, *variable*, *lower limit*, *upper limit*)

Examples using both symbolic and numeric results are shown below:

$$\mathrm{solve}(x^3 - 18 = 0, x) \rightarrow \frac{26207}{10000} \qquad \mathrm{solve}(x^3 - 18 = 0, x) = 2.6207$$

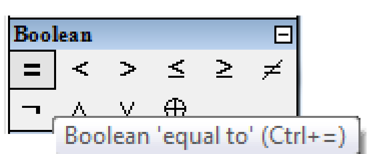
$$\mathrm{solve}(x^2 - 18 = 0, x, -5, 5) \rightarrow \begin{pmatrix} -\frac{21213}{5000} \\ \frac{21213}{5000} \end{pmatrix}$$

$$\mathrm{solve}(x^2 - 18 = 0, x, -5, 5) = \begin{pmatrix} -4.2426 \\ 4.2426 \end{pmatrix} \qquad F(x) := x \cdot \ln(x) - 5$$

$$\mathrm{solve}(F(x) = 0, x, -10, 10) \rightarrow \frac{37687}{10000} \qquad \mathrm{solve}(F(x) = 0, x, -10, 10) = 3.7687$$

Function *solve* provides only real roots. Polynomial equations can be solved using *polyroots*, as described below.

Note: to enter the Boolean equal sign (bold =) use *Ctrl+=*, or the symbol in the *Boolean palette.*



Application: Solving for the flow depth in an open channel flow
The following example shows how to use function *solve* to calculate the normal depth of flow in an open channel flow of circular cross-section. The problem statement and solution commentaries were all entered using *SMath Studio*. Also, a figure was created using *Window's Paint* and pasted into the *SMath Studio* worksheet.

Problem form open-channel hydraulics:

An open channel with a circular cross-section of diameter D = 3 ft is laid on a slope So = 0.0005 and carries a discharge Q = 10 cfs. If the channel is made of concrete, with a Manning's coefficient n = 0.012, find the normal depth of flow, y= ?

Solution: Use Manning's equation: $Q = \frac{1.486}{n} \cdot \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \cdot \sqrt{So}$

Q = discharge, n = Manning's coefficient, A = area, P = wetted perimeter, So = bed slope.

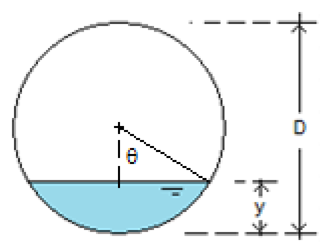
Functions for A and P are defined below. These functions use the auxiliary variable θ(y), that represents the half center angle in the flow.

$D := 3\ ft \qquad Q := 10\ cfs \qquad So := 0.0005 \qquad n := 0.012$

$\theta(y) := acos\left(1 - 2 \cdot \frac{y}{D}\right)$

$A(y) := \frac{D^2}{4} \cdot \left(\theta(y) - sin(\theta(y)) \cdot cos(\theta(y))\right)$

$P(y) := \theta(y) \cdot D$

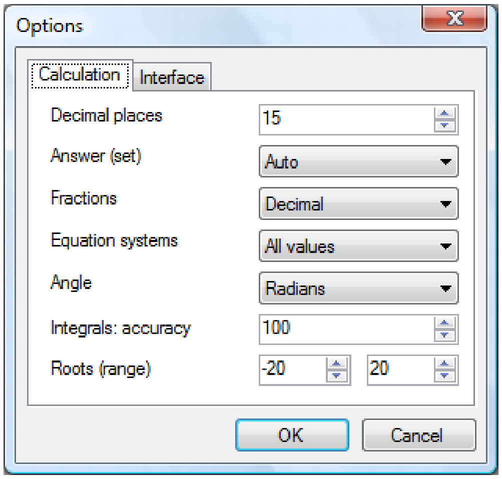


$$solve\left(Q = \frac{1.486}{n} \cdot \frac{A(y)^{\frac{5}{3}}}{P(y)^{\frac{2}{3}}} \cdot \sqrt{So}\ ,\ y\ ,\ 0\ ,\ 3\right) = \begin{pmatrix} 1.7077 \\ 1.7077 \end{pmatrix}$$

Thus, the solution is y = 1.7077 ft.

Changing the number of decimals in the output

Most of the numeric calculations and equation solutions shown so far in this document use four decimals (default value). Using the *Tools* > *Options* menu item you can change the number of decimals in the output. For example, changing to 15 decimals for output, we solve the equation presented above to obtain the following results:



$$\mathrm{solve}\left(x^2-18=0\ ,\ x\right)=\begin{pmatrix}-4.24264068711929\\ 4.24264068711928\end{pmatrix}$$

For the rest of the examples in this document we will use the default number of decimal figures, i.e., 4 decimals.

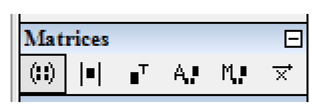
Solving polynomial equations with *polyroots*

Function *polyroots* returns the roots of a polynomial using as argument a column vector with the polynomial coefficients. The coefficients must be entered in the vector in increasing order of the power of the independent variable. For example, the vector corresponding to the polynomial $1 + 2x - 5x^2$ is:

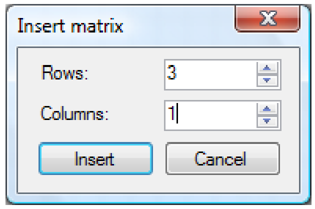
$$v=\begin{bmatrix}1\\2\\-5\end{bmatrix}.$$

Using *SMath Studio*, the roots of this polynomial are calculated as follows:

$$v:=\begin{pmatrix}1\\2\\-5\end{pmatrix}\qquad \mathrm{polyroots}(v)=\begin{pmatrix}-0.2899\\0.6899\end{pmatrix}$$



To enter a column vector, a row vector, or a matrix, use the first icon in the *Matrices* palette. This icon opens a dialogue box where the user can select the number of rows and columns for defining a vector or matrix. By default, the dialogue box uses a 3×3 matrix. For the column vector *v*, shown above, we used, of course, 3 rows and 1 column:



Unlike function *solve*, which provides only real functions, function *polyroots* provides either real or complex roots, or both. For example:

$$w := \begin{pmatrix} 2 \\ -3 \\ 5 \\ 7 \end{pmatrix} \qquad \mathrm{polyroots}(w) = \begin{pmatrix} 0.2647 + 0.3996 \cdot i \\ 0.2647 - 0.3996 \cdot i \\ -1.2436 \end{pmatrix}$$

Application: Solving homogeneous, linear, constant-coefficients ODEs
A *n*-th order, homogeneous, linear ordinary differential equation (ODE) with constant coefficients is represented by the expression:

$$\frac{d^n x}{dt^n} + a_{n\ 1} \frac{d^{n\ 1} x}{dt^{n\ 1}} + \ldots + a_2 \frac{d^2 x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = 0$$

Introducing the notation $D^n = d^n(\ )/dt^n$, the general *n*-th order equation can be written as:

$$D^n x + a_{n\ 1} D x + \ldots + a_2 D^2 x + a_1 D x + a_0 x = 0 \, .$$

This expression can be “factored” in terms of the *D* operators to read:

$$(D^n + a_{n\ 1} D^{n\ 1} + \ldots + a_2 D^2 + a_1 D + a_0) x = 0$$

If we take the expression between parentheses and replace the *D* operators with the variable λ, and make it equal to zero, we would have produced the so-called *characteristic equation* of the ODE, i.e.,

$$\lambda^n + a_{n\ 1} \lambda^{n\ 1} + \ldots + a_2 \lambda^2 + a_1 \lambda + a_0 = 0$$

There are, in principle, *n* solutions of this polynomial equation, namely, λ_1, λ_2, ..., λ_n. Some of these solutions could be complex (or imaginary), or even repeated. In the case that all solutions are real and non-repeating, the general solution to the original homogeneous ODE is given by:

$$x(t) = A_1 e^{(\lambda_1 t)} + A_2 e^{(\lambda_2 t)} + \ldots + A_n e^{(\lambda_n t)} \, .$$

If a root λ_m, repeats *k* times, it will result in *k* terms in the solution given by

$$e^{\lambda_m t} \cdot (A_1 + A_2 t + A_3 t^2 + \cdots + A_k t^{k-1})$$

If there are complex roots, they will be an even number of them, i.e., pairs of complex conjugates. Thus, if one of those pairs of complex conjugate roots is $\lambda_1 = \alpha + \beta$i and $\lambda_2 = \alpha - \beta$i, then the following terms will appear in the solution:

$$A_1 e^{\alpha t}\cos(\beta t) + A_2 e^{\alpha t}\sin(\beta t)$$

Function *polyroots* can be used to solve the characteristic equation, or characteristic polynomial, for an *n*-th order, homogeneous, linear ODE. The roots of the characteristic polynomial can then be used to build the solution. One example is shown below.

Solve the equation: $5\dfrac{d^3y}{dx^3}+3\dfrac{dy}{dx}+y=0$

Solution: Using the "D" operator, we can write the ODE as:

$$\left(5\cdot D^3+3\cdot D+1\right)\cdot y=0$$

The characteritic equation is:

$$5\cdot\lambda^3+3\cdot\lambda+1=0 \quad \text{or} \quad 1+3\cdot\lambda+0\cdot\lambda^2+5\cdot\lambda^3=0$$

The roots of this characteristic equation are found using:

$$\text{polyroots}\left(\begin{pmatrix}1\\3\\0\\5\end{pmatrix}\right)=\begin{pmatrix}-0.2919\\0.1459+0.8148\cdot i\\0.1459-0.8148\cdot i\end{pmatrix}$$

Thus, the solution to the ODE can be written as:

$$y(x)=A1\cdot\exp(-0.2919\cdot t)+\exp(0.1459\cdot t)\cdot\left(A2\cdot\cos(0.8148\cdot t)+A3\cdot\sin(0.8148\cdot t)\right)$$

To determine the constants A1, A2, and A3, we need to apply certain initial conditions,e.g.,

$$y(0)=1 \quad y'(0)=-2 \quad Y''(0)=3$$

For which we need to find the first and second derivatives of y(x).

The calculation of derivatives is all performed using symbolic calculations, and the results are very long. Also, the current version of *SMath Studio* (version 0.85, 09/2009) doesn't simplify results and keeps the value *exp*(*0*) instead of 1. Helping *SMath Studio* with these shortcomings, and with a lot of patience, you can get to put together the system of linear equations needed to solve for the coefficients A1, A2, and A3, in the solution to this ODE. (The figure below shows only parts of some calculations).

$$\frac{d}{dt}\left(A1\cdot\exp(-0.2919\cdot t)+\exp(0.1459\cdot t)\cdot(A2\cdot\cos(0.8148\cdot t)+A3\cdot\sin(0.8148\cdot t))\right)\rightarrow\frac{-72975000000\cdot A1\cdot\exp\left(-\frac{\;\cdot\;}{10000}\right)+}{}$$

$$\frac{d}{dt}\left(\frac{-72975000000\cdot A1\cdot\exp\left(-\frac{2919\cdot t}{10000}\right)+10000\cdot\left(20370000\cdot\left(-A2\cdot\sin\left(\frac{2037\cdot t}{2500}\right)+A3\cdot\cos\left(\frac{2037\cdot t}{2500}\right)\right)+3647500\cdot\left(A2\cdot\cos\left(\frac{2}{}\right.\right.\right.}{250000000000}\right.$$

Now, I'm going to substitute t = 0 into these results by making: $t := 0$

$$A1\cdot\exp(-0.2919\cdot t)+\exp(0.1459\cdot t)\cdot(A2\cdot\cos(0.8148\cdot t)+A3\cdot\sin(0.8148\cdot t))\rightarrow(A1+A2)\cdot\exp(0)$$

thus, $A1+A2=1$ Eq(1)

$$\frac{-72975000000\cdot A1\cdot\exp\left(-\frac{2919\cdot t}{10000}\right)+10000\cdot\left(20370000\cdot\left(-A2\cdot\sin\left(\frac{2037\cdot t}{2500}\right)+A3\cdot\cos\left(\frac{2037\cdot t}{2500}\right)\right)+3647500\cdot\left(A2\cdot\cos\left(\frac{2037\cdot t}{2500}\right)+A\right.\right.}{250000000000}$$

$$\frac{(-72975000000\cdot A1+10000\cdot(20370000\cdot A3+3647500\cdot A2))\cdot\exp(0)}{250000000000}$$

Using: $-\frac{72975000000}{250000000000}=-0.2919$ $\frac{10000\cdot(20370000)}{250000000000}=0.8148$ $\frac{10000\cdot 3647500}{250000000000}=0.1459$

Thus, $-0.2919\cdot A1+0.1459\cdot A2+0.8148\cdot A3=-2$ Eq(2)

$$\frac{21301402500\cdot A1\cdot\exp\left(-\frac{2919\cdot t}{10000}\right)+\left(10000\cdot\left(-16597476\cdot\left(A2\cdot\cos\left(\frac{2037\cdot t}{2500}\right)+A3\cdot\sin\left(\frac{2037\cdot t}{2500}\right)\right)+2971983\cdot\left(-A2\cdot\sin\left(\frac{2037\cdot t}{2500}\right)+\right.\right.\right.}{}$$

$$\frac{(10000\cdot(-16597476\cdot A2+2971983\cdot A3)+1459\cdot(20370000\cdot A3+3647500\cdot A2)+21301402500\cdot A1)\cdot\exp(0)}{250000000000}$$

$\frac{21301402500}{250000000000}=0.0852$ $\frac{10000\cdot(-16597476)}{250000000000}+\frac{1459\cdot 3647500}{250000000000}=-0.6426$ $\frac{10000\cdot(2971983)}{250000000000}+\frac{20370000}{250000000000}=0.119$

Thus, $0.0852\cdot A1+-0.6426\cdot A2+0.119\cdot A3=3$ Eq(3)

The resulting equations can be summarized as follows:

In summary:

$$A1+A2=1$$
$$-0.2919\cdot A1+0.1459\cdot A2+0.8148\cdot A3=-2$$
$$0.0852\cdot A1+(-0.6426)\cdot A2+0.119\cdot A3=3$$

This system of 3 linear equation can be written as:

$$\begin{pmatrix}1 & 1 & 0\\ -0.2919 & 0.1459 & 0.8148\\ 0.0852 & -0.6426 & 0.119\end{pmatrix}\cdot\begin{pmatrix}A1\\ A2\\ A3\end{pmatrix}=\begin{pmatrix}1\\ -2\\ 3\end{pmatrix}$$

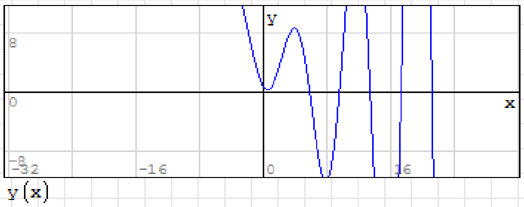
The solution is found by using the inverse matrix, function "invert"

$$Ainv := \text{invert}\left(\begin{pmatrix}1 & 1 & 0\\ -0.2919 & 0.1459 & 0.8148\\ 0.0852 & -0.6426 & 0.119\end{pmatrix}\right) \qquad Ainv=\begin{pmatrix}0.8385 & -0.1845 & 1.263\\ 0.1615 & 0.1845 & -1.263\\ 0.2715 & 1.1282 & 0.6786\end{pmatrix}$$

$$Asol := Ainv\cdot\begin{pmatrix}1\\ -2\\ 3\end{pmatrix} \qquad Asol=\begin{pmatrix}4.9966\\ -3.9966\\ 0.0511\end{pmatrix}$$

Thus, the solution is as shown below. Also, a graph of y(x) is included:

$$y(x) := 4.9966 \cdot \exp(-0.2919 \cdot x) + \exp(0.1459 \cdot x) \cdot ((-3.9966) \cdot \cos(0.8148 \cdot x) + 0.0511 \cdot \sin(0.8148 \cdot x))$$

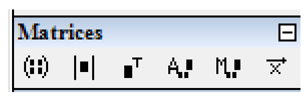


And, since we are dealing with matrices already, we introduce their use in *SMath Studio* next.

Operations with matrices
The functions available under the *Insert* > *Function* menu under the heading *Matrix and vector* can be roughly classified into the following categories:

- Creating matrices: *augment, diag, identity, mat, matrix, stack*
- Extracting rows, columns, elements: *col, el, row, submatrix, vminor, minor*
- Characterizing matrices: *cols, det, length, max, min, norm1, norme, normi, rank, rows, tr*
- Sorting functions: *csort, reverse, rsort, sort*
- Matrix operations: *alg, invert, transpose*

Some of these functions are also available in the *Matrices* palette:



(1) (2) (3) (4) (5) (6)

(1) Matrix 3x3 (Ctrl+M) (*mat*)
(2) Determinant (*det*)
(3) Matrix transpose (Ctrl+1) (*transpose*)
(4) Algebraic addition to matrix (*alg*)
(5) Minor (*minor*)
(6) Cross product

The only function from the *Matrices* palette not available from the *Insert* > *Function* > *Matrix and vector* functions is (6) Cross product, which applies to two vectors of three elements.

It is important to indicate that even though you can form row vectors, many of the matrix and vector functions defined in *SMath Studio* apply only to column vectors. Thus, column vectors are considered true vectors for the purpose of matrix operations in *SMath Studio*. For example, to calculate a cross-product you will need two column vectors of three elements. Dot products, on the other hand, can be performed on column vectors of any length. The dot product of two column vectors is calculated as the matrix product of the transpose of one of the vectors times the second vector. Examples of matrix and vector operations follow. The vector and matrix names used in the examples suggest the dimensions of the vector or matrix. For example, u2x1 suggest the vector $\mathbf{u}_{2\times 1}$, and A3x3 suggests the matrix $\mathbf{A}_{3\times 3}$.

Examples of matrix creation:

$$u2x1 := \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad u3x1 := \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \quad v3x1 := \begin{pmatrix} 0 \\ -1 \\ -5 \end{pmatrix} \quad A3x3 := \begin{pmatrix} -2 & 3 & 2 \\ 0 & 1 & -5 \\ -3 & 5 & 2 \end{pmatrix}$$

Matrix creation examples:

$M3 := \mathrm{augment}(u3x1, A3x3)$ $\quad M3 = \begin{pmatrix} 1 & -2 & 3 & 2 \\ -3 & 0 & 1 & -5 \\ 2 & -3 & 5 & 2 \end{pmatrix}$

$M1 := \mathrm{diag}(u2x1)$ $\quad M1 = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$

$M2 := \mathrm{diag}(v3x1)$ $\quad M2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -5 \end{pmatrix}$

$I3x3 := \mathrm{identity}(3)$ $\quad I3x3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$M0 := \mathrm{matrix}(3, 3)$ $\quad M0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Examples of extracting rows, columns, or elements from matrices:

$vec01 := \mathrm{col}(A3x3, 2)$ $\quad vec01 = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$

$x := A3x3_{2\,2}$ $\quad x = 1$

$vec02 := \mathrm{row}(A3x3, 3)$ $\quad vec02 = \begin{pmatrix} -3 & 5 & 2 \end{pmatrix}$

$MM1 := \mathrm{submatrix}(A3x3, 2, 3, 2, 3)$ $\quad MM1 = \begin{pmatrix} 1 & -5 \\ 5 & 2 \end{pmatrix}$

Function "vminor":
$MM2 := M_{2\,2}(A3x3)$ $\quad MM2 = \begin{pmatrix} -2 & 2 \\ -3 & 2 \end{pmatrix}$

Function "minor":
$MMin := M_{2\,2}(A3x3)$ $\quad MMin = 2$

Examples of matrix characterization:

$$A3x3 := \begin{pmatrix} -2 & 3 & 2 \\ 0 & 1 & -5 \\ -3 & 5 & 2 \end{pmatrix}$$

Characterizing matrices:

$\mathrm{cols}(A3x3) = 3$ $\mathrm{rows}(A3x3) = 3$

$|A3x3| = -3$ $\mathrm{length}(A3x3) = 9$

$\mathrm{length}(A3x3) = 9$ $\max(A3x3) = 5$

$\min(A3x3) = -5$ $\mathrm{norm1}(A3x3) = 9$

$\mathrm{norme}(A3x3) = 9$ $\mathrm{normi}(A3x3) = 10$

$\mathrm{rank}(A3x3) = 3$ $\mathrm{tr}(A3x3) = 1$

Examples of matrix/vector sorting functions:

$$A3x3 := \begin{pmatrix} -2 & 3 & 2 \\ 0 & 1 & -5 \\ -3 & 5 & 2 \end{pmatrix} \qquad u3x1 := \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$$

Sorting:

$$\mathrm{csort}(A3x3, 1) = \begin{pmatrix} -3 & 5 & 2 \\ -2 & 3 & 2 \\ 0 & 1 & -5 \end{pmatrix} \qquad \mathrm{rsort}(A3x3, 2) = \begin{pmatrix} 2 & -2 & 3 \\ -5 & 0 & 1 \\ 2 & -3 & 5 \end{pmatrix}$$

$$\mathrm{reverse}(A3x3) = \begin{pmatrix} -3 & 5 & 2 \\ 0 & 1 & -5 \\ -2 & 3 & 2 \end{pmatrix} \qquad \mathrm{reverse}(u3x1) = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$

$$\mathrm{sort}(u3x1) = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

Examples of matrix operations:

$$A3x3 := \begin{pmatrix} -2 & 3 & 2 \\ 0 & 1 & -5 \\ -3 & 5 & 2 \end{pmatrix} \qquad B3x3 := \begin{pmatrix} 0 & -1 & 2 \\ -2 & 8 & 3 \\ 4 & -4 & -8 \end{pmatrix} \qquad v3x1 := \begin{pmatrix} 0 \\ -1 \\ -5 \end{pmatrix}$$

Matrix operations:

$$\text{invert}(A3x3) = \begin{pmatrix} -9 & -1.3333 & 5.6667 \\ -5 & -0.6667 & 3.3333 \\ -1 & -0.3333 & 0.6667 \end{pmatrix} \qquad A3x3 + B3x3 = \begin{pmatrix} -2 & 2 & 4 \\ -2 & 9 & -2 \\ 1 & 1 & -6 \end{pmatrix}$$

Function "transpose":

$$A3x3^{T} = \begin{pmatrix} -2 & 0 & -3 \\ 3 & 1 & 5 \\ 2 & -5 & 2 \end{pmatrix} \qquad A3x3 \cdot v3x1 = \begin{pmatrix} -13 \\ 24 \\ -15 \end{pmatrix}$$

Examples of vector products:

Vector products:

$$u3x1 := \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \qquad v3x1 := \begin{pmatrix} 0 \\ -1 \\ -5 \end{pmatrix}$$

Dot product: $u3x1^{T} \cdot v3x1 = (-7)$

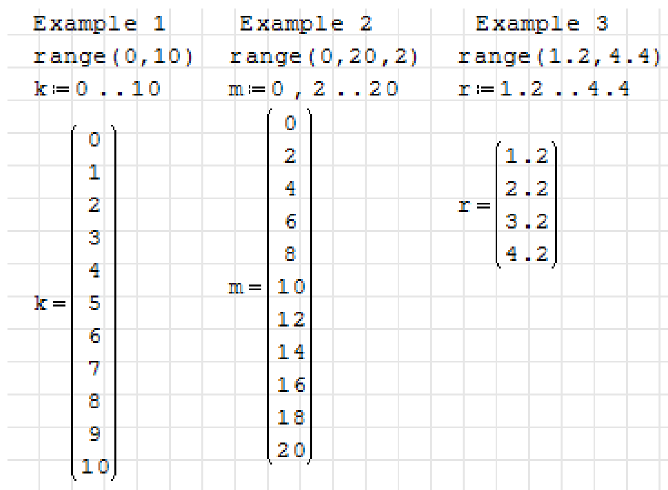
Cross product: $u3x1 \times v3x1 = \begin{pmatrix} 17 \\ 5 \\ -1 \end{pmatrix}$

Ranges and vectors

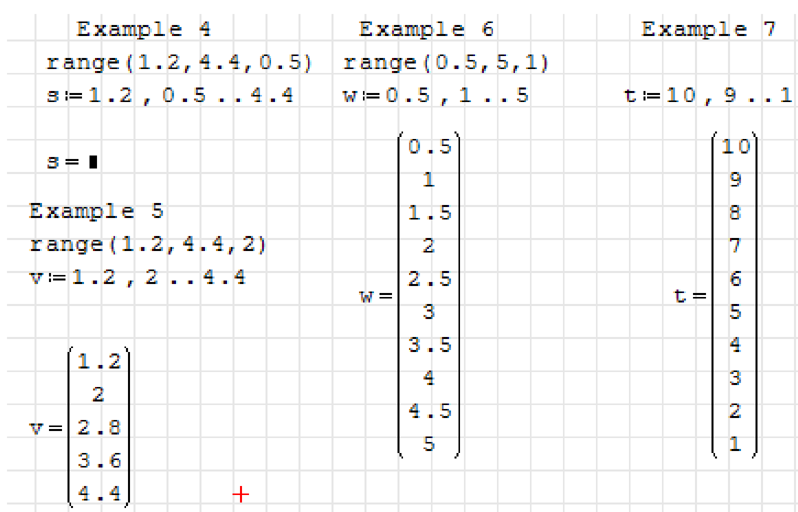
Function *range* is used to generate a vector (column vector) with elements equally spaced. The general form of the *range* function is as follows:

- *range*(*start*, *end*): shows up in the worksheet as *start..end*, and produces a vector whose elements are *start*, *start* + 1, *start* + 2, etc. The last element is the lowest value smaller than *last* by less than one.
- *range*(*start, end, start+increment*): shows up in the worksheet as *start, start+increment* .. *end*, and produces a vectors whose elements are *start, start + increment, start + 2 increment,* etc. The last element is the lowest value smaller than *end* by less than *increment*.

The following examples illustrate the use of function *range*. The examples are numbered 1 through 7. Immediately below the Example number the *range* function call that produces the range is shown. For instance, in *Example 1*, typing the expression *range(0,10)* produces the range *0..10*, assigned to *k*.



- Example 1 shows a vector from *0* to *10* with default increment of *1*.
- Example 2 shows a vector from *0* to *20* with increment of *2*.
- Example 3 shows a vector from *1.2* to *4.4* with default increment of *1*.



- Example 4 shows an impossibility since the value of *start* + *increment* < *start*, but *end* > *start*
- Example 5 shows a vector from *1.2* to *4.4* in increments of *2*
- Example 6 shows a vector from *0.5* to *5* in increments of *0.5*
- Example 7 shows a vector from 10 down to 1 in negative increments of *-1*

Vectors can be combined linearly to produce other vectors, as illustrated here:

$$x := 0..5 \qquad y := 2\cdot x + 3 \qquad z := 0.8\cdot x + 0.7\cdot y$$

$$x = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \qquad y = \begin{pmatrix} 3 \\ 5 \\ 7 \\ 9 \\ 11 \\ 13 \end{pmatrix} \qquad z = \begin{pmatrix} 2.1 \\ 4.3 \\ 6.5 \\ 8.7 \\ 10.9 \\ 13.1 \end{pmatrix}$$

The following examples illustrate three different ways to calculate products of vectors of the same lengths:

1. Product of the vectors produces the scalar (dot) product
2. Product of a vector transpose with another vector produces the same scalar (dot) product as the only element of a 1×1 matrix
3. Product of a vector with the transpose of another vector produces a matrix

$$x\cdot x = 55 \qquad x\cdot y = 155$$

$$x^T\cdot x = (55) \qquad x^T\cdot y = (155)$$

$$x\cdot x^T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 4 & 6 & 8 & 10 \\ 0 & 3 & 6 & 9 & 12 & 15 \\ 0 & 4 & 8 & 12 & 16 & 20 \\ 0 & 5 & 10 & 15 & 20 & 25 \end{pmatrix}$$

Individual elements of a vector can be extracted by using the name of the vector with a sub-index. To enter a sub-index, type the name of the vector and press the left square bracket key ([). Press the right arrow key [→] a couple of times to select the indexed variable. Examples:

$$x := 0..5 \qquad x_3 = 2$$

Vectors of the same length can be put together in a matrix through function *augment*:

$$\text{augment}(x, y, z) = \begin{pmatrix} 0 & 3 & 2.1 \\ 1 & 5 & 4.3 \\ 2 & 7 & 6.5 \\ 3 & 9 & 8.7 \\ 4 & 11 & 10.9 \\ 5 & 13 & 13.1 \end{pmatrix}$$

This can be used to build tables of data.

for loops and parametric plots

A parametric plot is defined by points ($x(t)$, $y(t)$), where t is referred to as the parameter. In this example we generate data (x,y) and (y,x) using *for* loops, and plotting the resulting matrices.

First create column vectors x and y with 21 elements:

$x := \mathrm{matrix}(21, 1) \quad n := \mathrm{length}(x) \quad n = 21 \quad y := x$

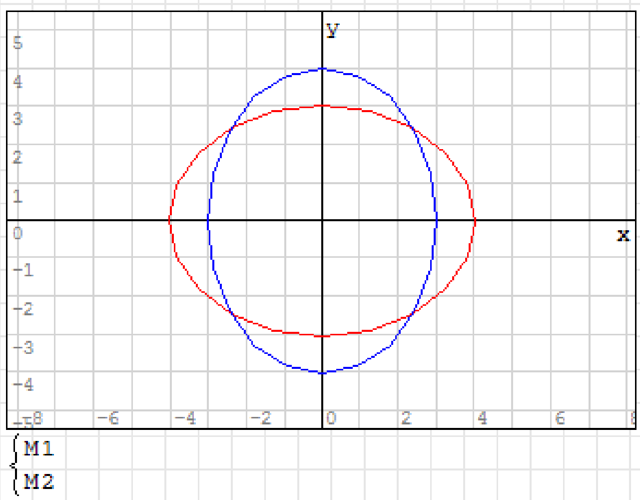
Use a "for" loop to generate parametric data in x,y:

$$\text{for } k \in 1..n \quad \left| \begin{array}{l} x_k := 3\cdot\sin\left(k\cdot\frac{\pi}{10}\right) \\ y_k := 4\cdot\cos\left(k\cdot\frac{\pi}{10}\right) \end{array} \right.$$

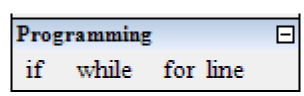
Note: in this program we use the "for" and the "line" commands from the "Programming" palette.

Form matrices with data points (x,y) and (y,x), and produce a plot of the matrices M1 and M2:

$M1 := \mathrm{augment}(x, y) \qquad M2 := \mathrm{augment}(y, x)$



Commands in the *Programming* palette



The *Programming* palette includes four commands, namely, *if*, *while*, *for*, and *line*. We have presented examples of the commands *if*, *for*, and *line*, in this document. Here is an example of the *while* command:

$k := 10 \quad b := 0$ (initial values)

$$\text{while } k \ge 5 \quad \left| \begin{array}{l} b := b + k \\ k := k - 1 \end{array} \right.$$

$k = 4 \quad b = 45$ (final values)

