8 GEAR FORCES

In designing a gear, it is important to analyze the magnitude and direction of the forces acting upon the gear teeth, shafts, bearings, etc. In analyzing these forces, an idealized assumption is made that the tooth forces are acting upon the central part of the tooth flank.

Table 8.1 presents the equations for tangential (circumferential) force F_t (kgf), axial (thrust) force F_x(kgf), and radial force F_r in relation to the transmission force F_n acting upon the central part of the tooh flank.
T and T_1 shown therein represent input torque (kgf·m).

Table 8.1 Forces acting upon a gear

Types of gears		F_t :Tangential force	F_x : Axial force	F_r : Radial force
Spur gear		$F_t = \frac{2000T}{d}$	――――	$F_t \tan\alpha$
Helical gear			$F_t \tan\beta$	$F_t \frac{\tan\alpha_n}{\cos\beta}$
Straight bevel gear		$F_t = \frac{2000T}{d_m}$	$F_t \tan\alpha \sin\delta$	$F_t \tan\alpha \cos\delta$
Spiral bevel gear		d_m is the central reference diameter $d_m = d - b \sin\delta$	When convex surface is working:	
			$\frac{F_t}{\cos\beta_m}(\tan\alpha_n \sin\delta - \sin\beta_m \cos\delta)$	$\frac{F_t}{\cos\beta_m}(\tan\alpha_n \cos\delta + \sin\beta_m \sin\delta)$
			When concave surface is working:	
			$\frac{F_t}{\cos\beta_m}(\tan\alpha_n \sin\delta + \sin\beta_m \cos\delta)$	$\frac{F_t}{\cos\beta_m}(\tan\alpha_n \cos\delta - \sin\beta_m \sin\delta)$
Worm gear pair	Worm (Driver)	$F_t = \frac{2000T_1}{d_1}$	$F_t \frac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}$	$F_t \frac{\sin\alpha_n}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}$
	Worm Wheel (Driven)	$F_t \frac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu \cos\gamma}$	F_t	
Screw gear $\left(\begin{matrix}\Sigma = 90° \\ \beta = 45°\end{matrix}\right)$	Driver gear	$F_t = \frac{2000T_1}{d_1}$	$F_t \frac{\cos\alpha_n \sin\beta - \mu \cos\beta}{\cos\alpha_n \cos\beta + \mu \sin\beta}$	$F_t \frac{\sin\alpha_n}{\cos\alpha_n \cos\beta + \mu \sin\beta}$
	Driven gear	$F_t \frac{\cos\alpha_n \sin\beta - \mu \cos\beta}{\cos\alpha_n \cos\beta + \mu \sin\beta}$	F_t	

8.1 Forces in a Spur Gear Mesh

The Spur Gear's transmission force F_n, which is normal to the tooth surface , as in Figure 8.1, can be resolved into a tangential component, F_t , and a radial component, F_r. Refer to Equation (8.1).

$$\left.\begin{aligned} F_t &= F_n \cos\alpha' \\ F_r &= F_n \sin\alpha' \end{aligned}\right\} \qquad (8.1)$$

There will be no axial force, F_x.

The direction of the forces acting on the gears are shown in Figure 8.2. The tangential component of the drive gear, F_{t1} is equal to the driven gear's tangential component, F_{t2}, but the directions are opposite. Similarly, the same is true of the radial components.

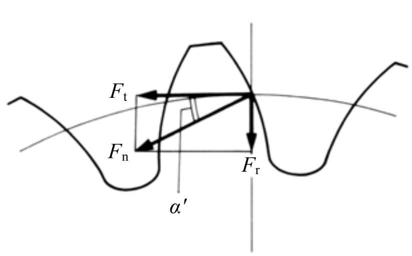

Fig.8.1 Forces acting on a spur gear mesh

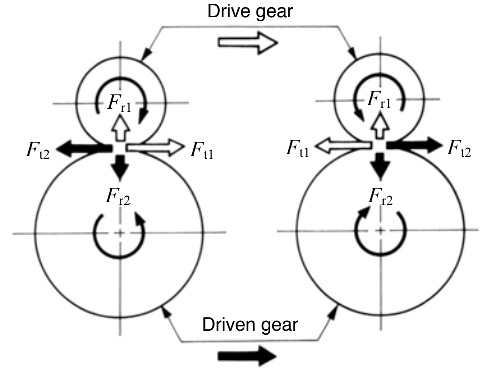

Fig.8.2 Directions of forces acting on a spur gear mesh

8.2 Forces in a Helical Gear Mesh

The helical gear's transmission force, F_n, which is normal to the tooth surface, can be resolved into a tangential component, F_1, and a radial component, F_r, as shown in Figure 8.3.

$$\left.\begin{aligned} F_1 &= F_n \cos\alpha_n \\ F_r &= F_n \sin\alpha_n \end{aligned}\right\} \quad (8.2)$$

The tangential component, F_1, can be further resolved into circular subcomponent, F_t, and axial thrust subcomponent, F_x.

$$\left.\begin{aligned} F_t &= F_1 \cos\beta \\ F_x &= F_1 \sin\beta \end{aligned}\right\} \quad (8.3)$$

Substituting and manipulating the above equations result in:

$$\left.\begin{aligned} F_x &= F_t \tan\beta \\ F_r &= F_t \frac{\tan\alpha_n}{\cos\beta} \end{aligned}\right\} \quad (8.4)$$

The directions of forces acting on a helical gear mesh are shown in Figure 8.4.
The axial thrust sub-component from drive gear, F_{x1}, equals the driven gear's, F_{x2}, but their directions are opposite.
Again, this case is the same as tangential components and radial components.

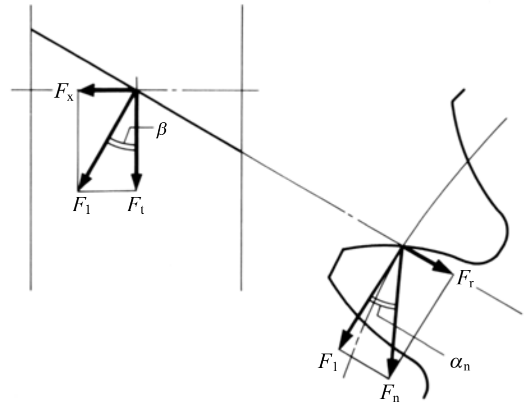

Fig.8.3 Forces acting on a helical gear mesh

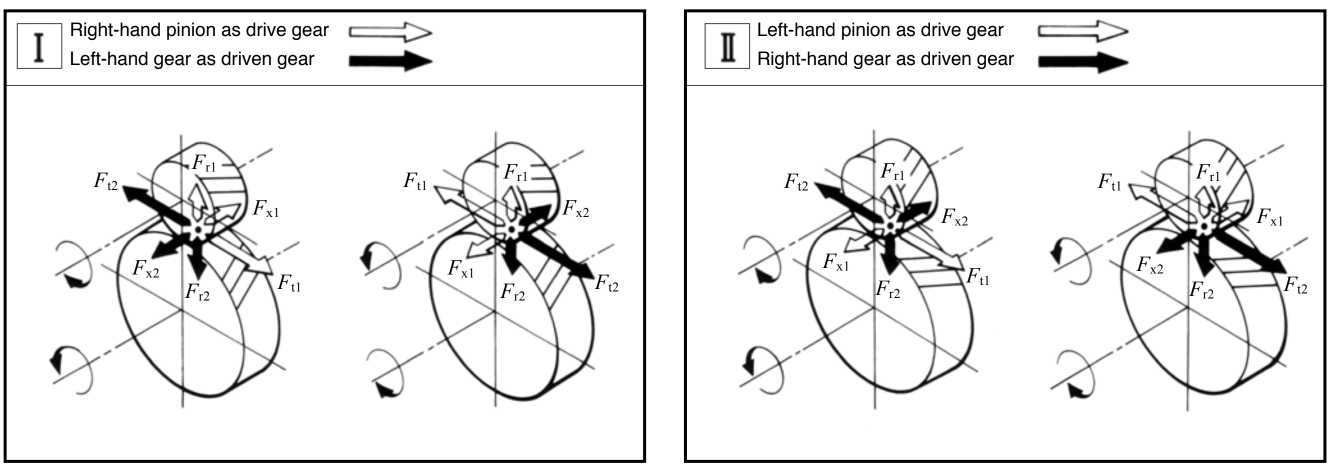

Fig.8.4 Directions of forces acting on a helical gear mesh

8.3 Forces in a Straight Bevel Gear Mesh

The forces acting on a straight bevel gear are shown in Figure 8.5. The force which is normal to the central part of the tooth face, F_n , can be split into tangential component, F_t, and radial component, F_1, in the normal plane of the tooth.

$$\left.\begin{aligned} F_t &= F_n \cos\alpha_n \\ F_1 &= F_n \sin\alpha_n \end{aligned}\right\} \quad (8.5)$$

Again, the radial component, F_1, can be divided into an axial force, F_x, and a radial force, F_r, perpendicular to the axis.

$$\left.\begin{aligned} F_x &= F_1 \sin\delta \\ F_r &= F_1 \cos\delta \end{aligned}\right\} \quad (8.6)$$

And the following can be derived:

$$\left.\begin{aligned} F_x &= F_t \tan\alpha_n \sin\delta \\ F_r &= F_t \tan\alpha_n \cos\delta \end{aligned}\right\} \quad (8.7)$$

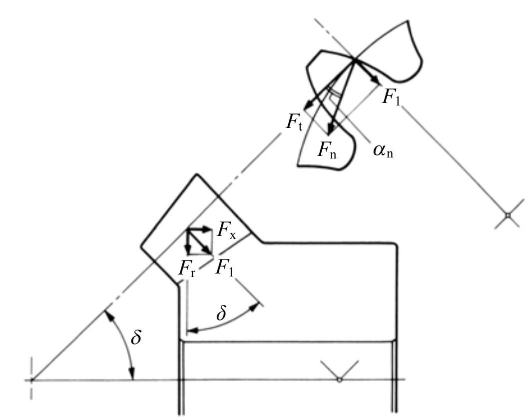

Fig.8.5 Forces acting on a straight bevel gear mesh

Let a pair of straight bevel gears with a shaft angle $\Sigma = 90°$, a pressure angle $\alpha_n = 20°$ and tangential force, F_t, to the central part of tooth face be 100. Axial force, F_x, and radial force, F_r, will be as presented in Table 8.2.

Table 8.2 $\frac{\text{Axial force } F_x}{\text{Radial force } F_r}$ Values

(1) Pinion

Forces on the gear tooth	Gear ratio z_2/z_1						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Axial force / Radial force	25.7 / 25.7	20.2 / 30.3	16.3 / 32.6	13.5 / 33.8	11.5 / 34.5	8.8 / 35.3	7.1 / 35.7

(2) Gear

Forces on the gear tooth	Gear ratio z_2/z_1						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Axial force / Radial force	25.7 / 25.7	30.3 / 20.2	32.6 / 16.3	33.8 / 13.5	34.5 / 11.5	35.3 / 8.8	35.7 / 7.1

Figure 8.6 contains the directions of forces acting on a straight bevel gear mesh. In the meshing of a pair of straight bevel gears with shaft angle $\Sigma = 90°$, the axial force acting on drive gear F_{x1} equals the radial force acting on driven gear F_{r2}. Similarly, the radial force acting on drive gear F_{r1} equals the axial force acting on driven gear F_{x2}. The tangential force F_{t1} equals that of F_{t2}.

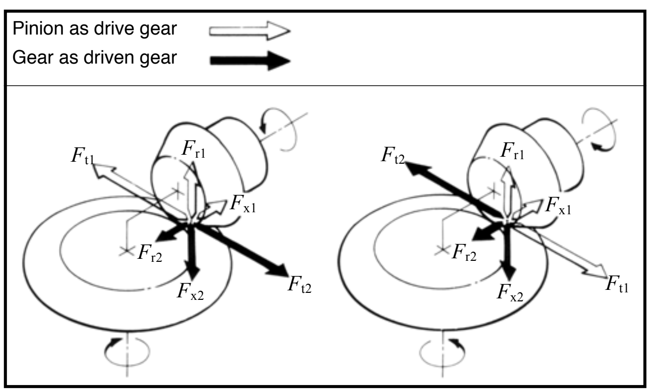

Fig.8.6 Directions of forces acting on a straight bevel gear mesh

All the forces have relations as per Equations (8.8).

$$\left.\begin{aligned} F_{t1} &= F_{t2} \\ F_{r1} &= F_{x2} \\ F_{x1} &= F_{r2} \end{aligned}\right\} \qquad (8.8)$$

8.4 Forces in A Spiral Bevel Gear Mesh

Spiral bevel gear teeth have convex and concave sides. Depending on which surface the force is acting on, the direction and magnitude changes. They differ depending upon which is the driver and which is the driven.

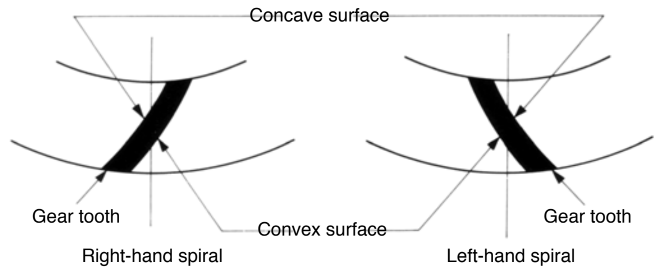

Fig.8.7 Convex surface and concave surface of a spiral bevel gear

Figure 8.7 presents the profile orientations of right-hand and left-hand spiral teeth. If the profile of the driving gear is convex, then the profile of the driven gear must be concave. Table 8.3 presents the convex/concave relationships.

Table 8.3 Concave and convex sides of a spiral bevel gear
Right-hand gear as drive gear

Rotational direction of drive gear	Meshing tooth face	
	Right-hand drive gear	Left-hand driven gear
Clockwise	Convex	Concave
Counterclockwise	Concave	Convex

Left-hand gear as drive gear

Rotational direction of drive gear	Meshing tooth face	
	Left-hand drive gear	Right-hand driven gear
Clockwise	Concave	Convex
Counterclockwise	Convex	Concave

(1) Forces on Convex Side Profile

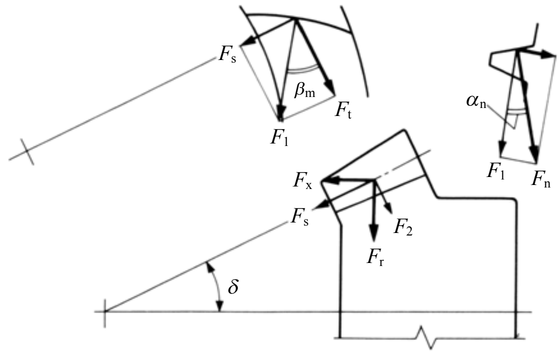

Fig.8.8 When meshing on the convex side of tooth face

The transmission force, F_n, can be resolved into components F_1 and F_2. (See Figure 8.8).

$$\left.\begin{aligned} F_1 &= F_n \cos\alpha_n \\ F_2 &= F_n \sin\alpha_n \end{aligned}\right\} \qquad (8.9)$$

Then F_1 can be resolved into components F_t and F_s:

$$\left.\begin{aligned} F_t &= F_1 \cos\beta_m \\ F_s &= F_1 \sin\beta_m \end{aligned}\right\} \qquad (8.10)$$

On the axial surface, F_2 and F_s can be resolved into axial and radial subcomponents.

$$\left.\begin{aligned} F_x &= F_2 \sin\delta - F_s \cos\delta \\ F_r &= F_2 \cos\delta + F_s \sin\delta \end{aligned}\right\} \qquad (8.11)$$

By substitution and manipulation, we obtain:

$$\left.\begin{aligned} F_x &= \frac{F_t}{\cos\beta_m}\,(\tan\alpha_n \sin\delta - \sin\beta_m \cos\delta) \\ F_r &= \frac{F_t}{\cos\beta_m}\,(\tan\alpha_n \cos\delta + \sin\beta_m \sin\delta) \end{aligned}\right\} \qquad (8.12)$$

(2) Forces on a Concave Side Profile

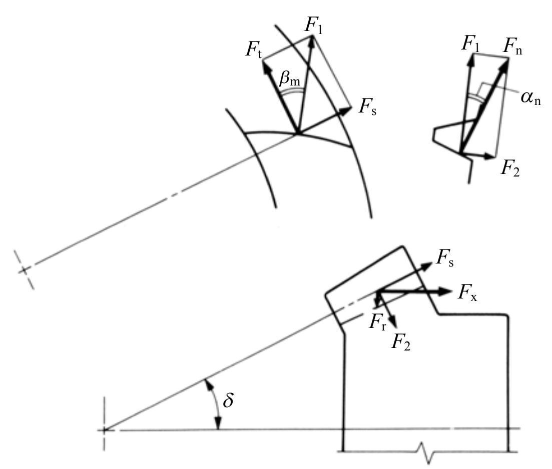

Fig.8.9 When meshing on the concave side of tooth face

On the surface which is normal to the tooth profile at the central portion of the tooth, the transmission force F_n can be split into F_1 and F_2. See Figure 8.9:

$$\left.\begin{aligned} F_1 &= F_n \cos\alpha_n \\ F_2 &= F_n \sin\alpha_n \end{aligned}\right\} \qquad (8.13)$$

And F_1 can be separated into components F_t and F_s on the pitch surface:

$$\left.\begin{aligned} F_t &= F_1 \cos\beta_m \\ F_s &= F_1 \sin\beta_m \end{aligned}\right\} \qquad (8.14)$$

So far, the equations are identical to the convex case. However, differences exist in the signs for equation terms. On the axial surface, F_2 and F_s can be resolved into axial and radial subcomponents. Note the sign differences.

$$\left.\begin{aligned} F_x &= F_2 \sin\delta + F_s \cos\delta \\ F_r &= F_2 \cos\delta - F_s \sin\delta \end{aligned}\right\} \qquad (8.15)$$

The above can be manipulated to yield:

$$\left.\begin{aligned} F_x &= \frac{F_t}{\cos\beta_m}\,(\tan\alpha_n \sin\delta + \sin\beta_m \cos\delta) \\ F_r &= \frac{F_t}{\cos\beta_m}\,(\tan\alpha_n \cos\delta - \sin\beta_m \sin\delta) \end{aligned}\right\} \qquad (8.16)$$

Let a pair of spiral bevel gears have a shaft angle $\Sigma = 90°$, a pressure angle $\alpha_n = 20°$, and a spiral angle $\beta_m = 35°$. If the tangential force, F_t to the central portion of the tooth face is 100, the axial force, F_x, and radial force, F_r, have the relationship shown in Table 8.4.

Table 8.4 Values of $\dfrac{\text{Axial force, } F_x}{\text{Radial force, } F_r}$

(1) Pinion

Meshing tooth face	Gear ratio z_2/z_1						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Concave side of tooth	80.9	82.9	82.5	81.5	80.5	78.7	77.4
	−18.1	−1.9	8.4	15.2	20.0	26.1	29.8
Convex side of tooth	−18.1	−33.6	−42.8	−48.5	−52.4	−57.2	−59.9
	80.9	75.8	71.1	67.3	64.3	60.1	57.3

(2) Gear

Meshing tooth face	Gear ratio z_2/z_1						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Concave side of tooth	80.9	75.8	71.1	67.3	64.3	60.1	57.3
	−18.1	−33.6	−42.8	−48.5	−52.4	−57.2	−59.9
Convex side of tooth	−18.1	−1.9	8.4	15.2	20.0	26.1	29.8
	80.9	82.9	82.5	81.5	80.5	78.7	77.4

The value of axial force, F_x, of a spira bevel gear, from Table 8.4, could become negative. At that point, there are forces tending to push the two gears together. If there is any axial play in the bearing, it may lead to the undesirable condition of the mesh having no backlash. Therefore, it is important to pay particular attention to axial plays.

From Table 8.4(2), we understand that axial turning point of axial force, F_x, changes from positive to negative in the range of gear ratio from 1.5 to 2.0 when a gear carries force on the convex suide. The precise turning point of axial force, F_x, is at the gear ratio z_2/z_1 =1.57357.

Figure 8.10 describes the forces for a pair of spiral bevel gears with shaft angle $\Sigma = 90°$, pressure angle $\alpha_n = 20°$, spiral angle $\beta_m = 35°$ and the gear ratio z_2/z_1, ranging from 1 to 1.57357. Figure 8.11 expresses the forces of another pair of spiral bevel gears taken with the gear ratio z_2/z_1 equal to or larger than 1.57357.

$\Sigma = 90°$, $\alpha_n = 20°$, $\beta_m = 35°$, $u < 1.57357$

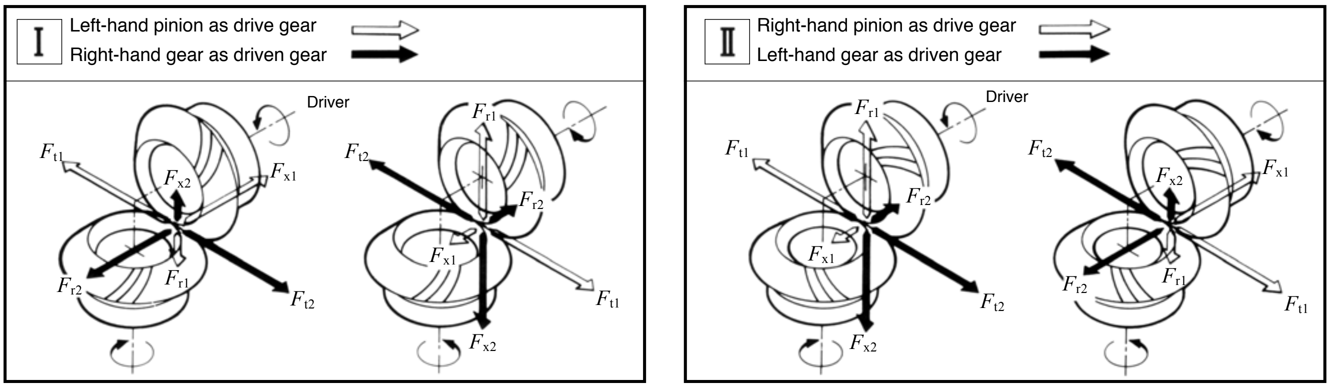

Fig.8.10 The direction of forces carried by spiral bevel gears (1)

$\Sigma = 90°$, $\alpha_n = 20°$, $\beta_m = 35°$, $u \geq 1.57357$

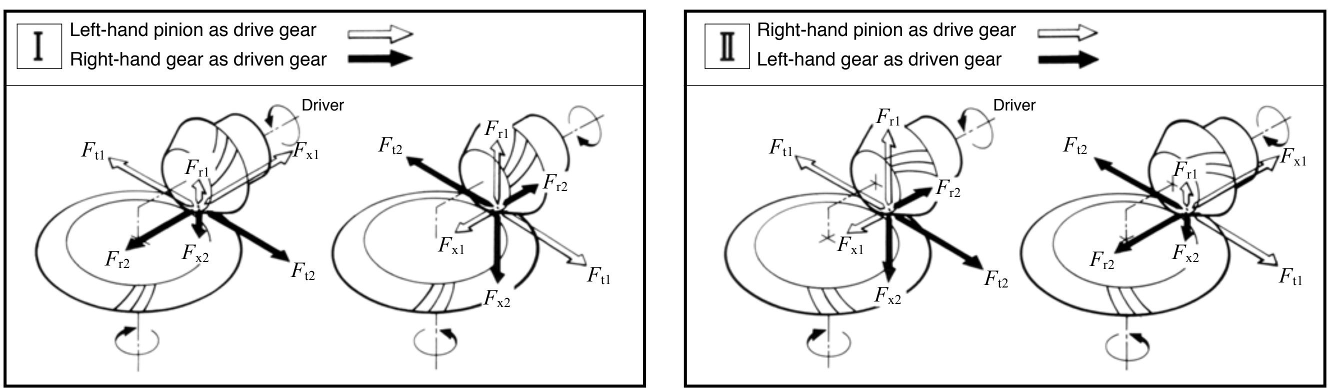

Fig.8.11 The direction of forces carried by spiral bevelgears (2)

8.5 Forces in a Worm Gear Pair Mesh

(1) Worm as the Driver

For the case of a worm as the driver, Figure 8.12, the transmission force, F_n, which is normal to the tooth surface at the pitch circle can be resolved into components F_1 and F_{r1}.

$$F_1 = F_n \cos\alpha_n$$
$$F_{r1} = F_n \sin\alpha_n \qquad (8.17)$$

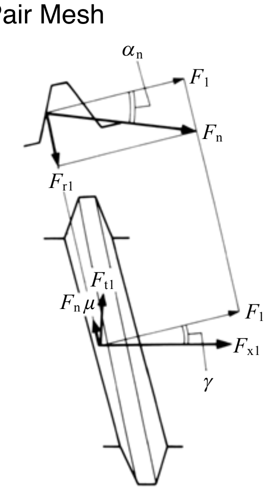

Fig..12 Forces acting on the tooth surface of a worm

At the pitch surface of the worm, there is, in addition to the tangential component, F_1, a friction sliding force on the tooth surface, $F_n\mu$. These two forces can be resolved into the circular and axial directions as:

$$\left.\begin{aligned} F_{t1} &= F_1 \sin\gamma + F_n\mu \cos\gamma \\ F_{x1} &= F_1 \cos\gamma - F_n\mu \sin\gamma \end{aligned}\right\} \qquad (8.18)$$

and by substitution, the result is:

$$\left.\begin{aligned} F_{t1} &= F_n(\cos\alpha_n \sin\gamma + \mu \cos\gamma) \\ F_{x1} &= F_n(\cos\alpha_n \cos\gamma - \mu \sin\gamma) \\ F_{r1} &= F_n \sin\alpha_n \end{aligned}\right\} \qquad (8.19)$$

Figure 8.13 presents the direction of forces in a worm gear pair mesh with a shaft angle $\Sigma = 90°$. These forces relate as follows:

$$\left.\begin{aligned} F_{x1} &= F_{t2} \\ F_{t1} &= F_{x2} \\ F_{r1} &= F_{r2} \end{aligned}\right\} \quad (8.20)$$

In a worm gear pair mesh with a shaft angle $\Sigma = 90°$, the axial force acting on drive gear F_{x1} equals the tangential force acting on driven gear F_{t2}. Similarly, the tangential force acting on drive gear F_{t1} equals the axial force acting on driven gear F_{x2}. The radial force F_{r1} equals that of F_{r2}.

The equations concerning worm and worm wheel forces contain the coefficient μ. The coefficient of friction has a great effect on the transmission of a worm gear pair. Equation (8-21) presents the efficiency when the worm is the driver.

$$\left.\begin{aligned} \eta_R &= \frac{T_2}{T_1 i} = \frac{F_{t2}}{F_{t1}} \tan\gamma \\ &= \frac{\cos\alpha_n \cos\gamma - \mu \sin\gamma}{\cos\alpha_n \sin\gamma + \mu \cos\gamma} \tan\gamma \end{aligned}\right\} \quad (8.21)$$

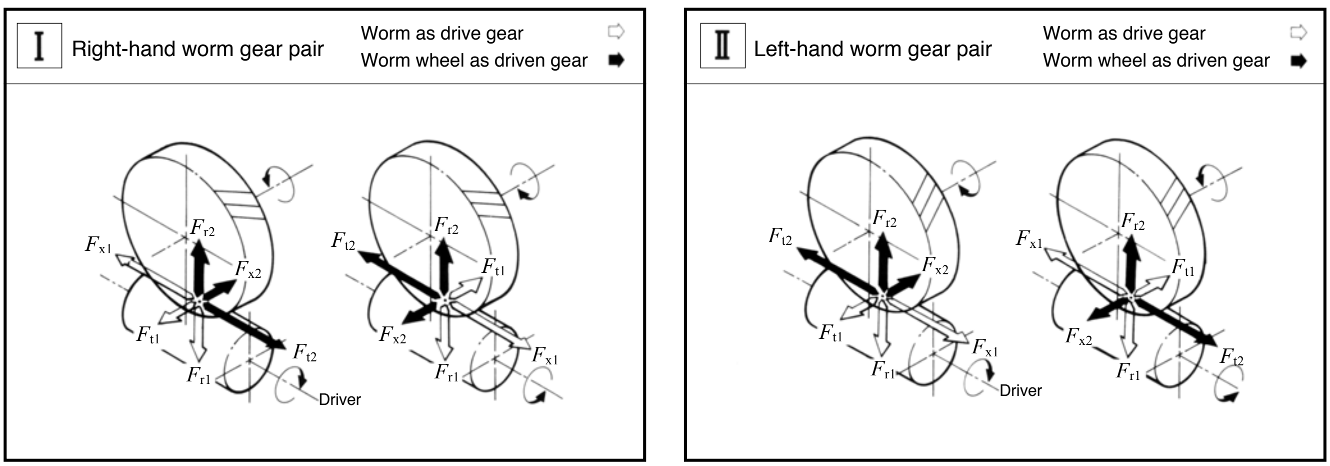

Figure 8.13 Direction of forces in a worm gear pair mesh

(2) Worm Wheel as the Driver

For the case of a worm wheel as the driver, the forces are as in Figure 8.14 and per Equations (8.22).

$$\left.\begin{aligned} F_{t2} &= F_n(\cos\alpha_n \cos\gamma + \mu \sin\gamma) \\ F_{x2} &= F_n(\cos\alpha_n \sin\gamma - \mu \cos\gamma) \\ F_{r2} &= F_n \sin\alpha_n \end{aligned}\right\} \quad (8.22)$$

When the worm and worm wheel are at 90° shaft angle, Equations (8.20) apply. Then, when the worm wheel is the driver, the transmission efficiency η_I is expressed as per Equation (8.23).

$$\left.\begin{aligned} \eta_I &= \frac{T_1 i}{T_2} = \frac{F_{t1}}{F_{t2} \tan\gamma} \\ &= \frac{\cos\alpha_n \sin\gamma - \mu\cos\gamma}{\cos\alpha_n \cos\gamma + \mu\sin\gamma} \frac{1}{\tan\gamma} \end{aligned}\right\} \quad (8.23)$$

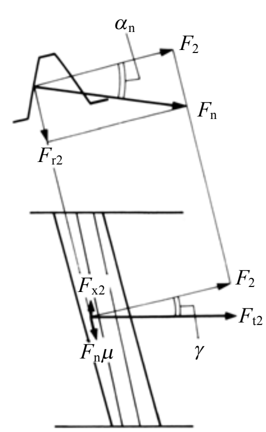

Fig.8.14 Forces in a worm gear pair mesh

8.6 Forces in a Screw Gear Mesh

The forces in a screw gear mesh are similar to those in a worm gear pair mesh. For screw gears that have a shaft angle $\Sigma = 90°$, merely replace the worm's lead angle γ, in Equation (8.22), with the screw gear's helix angle β_1.

In the general case when the shaft angle is not 90° , as in Figure 8.15, the driver screw gear has the same forces as for a worm mesh. These are expressed in Equations (8.24).

$$\left.\begin{aligned} F_{t1} &= F_n(\cos\alpha_n\cos\beta_1 + \mu\sin\beta_1)\\ F_{x1} &= F_n(\cos\alpha_n\sin\beta_1 - \mu\cos\beta_1)\\ F_{r1} &= F_n\sin\alpha_n \end{aligned}\right\} \quad (8.24)$$

Forces acting on the driven gear can be calculated per Equations (8.25).

$$\left.\begin{aligned} F_{t2} &= F_{x1}\sin\Sigma + F_{t1}\cos\Sigma\\ F_{x2} &= F_{t1}\sin\Sigma - F_{x1}\cos\Sigma\\ F_{r2} &= F_{r1} \end{aligned}\right\} \quad (8.25)$$

If the Σ term in Equation (8.25) is 90°, it becomes identical to Equation (8.20).
Figure 8.16 presents the direction of forces in a screw gear mesh when the shaft angle $\Sigma = 90°$, and $\beta_1 = \beta_2 = 45°$.

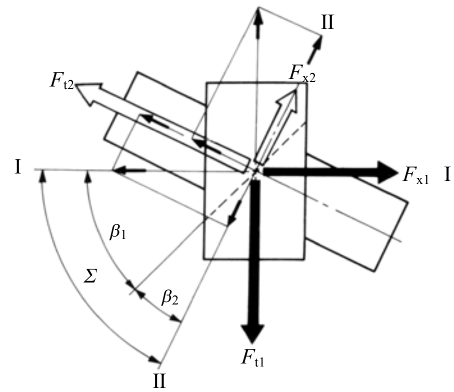

Fig.8.15 The forces in a screw gear mesh

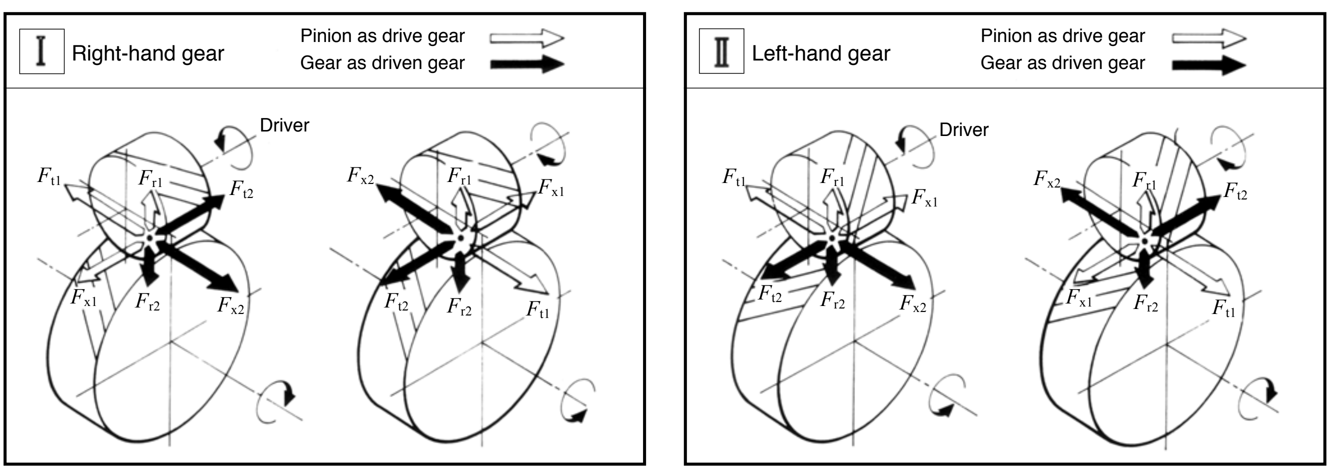

Fig.8.16 Direction of forces in a screw gear mesh