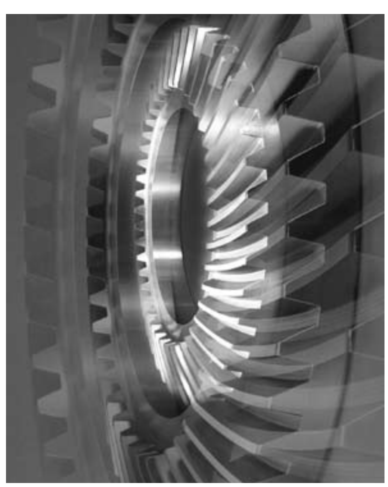
Elementary Information on Gears

The Role Gears are Playing

Gears are some of the most important elements used in machinery. There are few mechanical devices that do not have the need to transmit power and motion between rotating shafts. Gears not only do this most satisfactorily, but can do so with uniform motion and reliability. In addition, they span the entire range of applications from large to small. To summarize:

1. Gears offer positive transmission of power.
2. Gears range in size from small miniature instrument installations, that measure in only several millimeters in diameter, to huge powerful gears in turbine drives that are several meters in ameter.
3. Gears can provide position transmission with very high angular or linear accuracy, such as used in servomechanisms and precision instruments.
4. Gears can couple power and motion between shafts whose axes are parallel, intersecting or skew.
5. Gear designs are standardized in accordance with size and shape which provides for widespread interchangeability.

This introduction is written as an aid for the designer who is a beginner or only superficially knowledgeable about gearing. It provides fundamental, theoretical and practical information. When you select KHK products for your applications please utilize it along with KHK3009 catalog.



KOHARA GEAR INDUSTRY CO., LTD.

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1 Gear Types and Terminology

1.1 Type of gears

In accordance with the orientation of axes, there are three categories of gears:

1. Parallel axes gears
2. Intersecting axes gears
3. Nonparallel and nonintersecting axes gears

Spur and helical gears are the parallel axes gears. Bevel gears are the intersecting axes gears. Screw or crossed helical gears and worm gears handle the third category. Table 1.1 Lists the gear types per axes orientation.

Table 1.1 Types of gears and their categories

Categories of gears	Types of gears	Efficiency(%)
Parallel axes gears	Spur gear	98.0 ~ 99.5
	Spur rack	
	Internal gear	
	Helical gear	
	Helical rack	
	Double helical gear	
Intersecting axes gears	Straight bevel gear	98.0 ~ 99.0
	Spiral bevel gear	
	Zerol bevel gear	
Nonparallel and nonintersecting axes gears	Worm gear	30.0 ~ 90.0
	Screw gear	70.0 ~ 95.0

Also, included in table 1.1 Is the theoretical efficiency range of the various gear types. These figures do not include bearing and lubricant losses. Also, they assume ideal mounting in regard to axis orientation and center distance. Inclusion of these realistic considerations will downgrade the efficiency numbers.

(1) Parallel Axes Gears

(a) Spur Gear

This is a cylindrical shaped gear in which the teeth are parallel to the axis. It has the largest applications and, also, it is the easiest to manufacture.

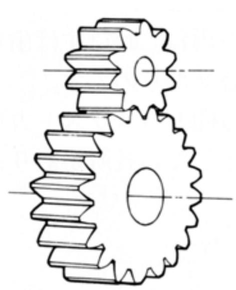


Fig.1.1 Spur gear

(b) Spur Rack

This is a linear shaped gear which can mesh with a spur gear with any number of teeth. The spur rack is a portion of a spur gear with an infinite radius.

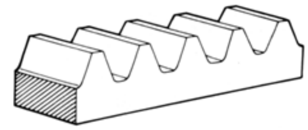


Fig.1.2 Spur rack

(c) Internal Gear

This is a cylindrical shaped gear but with the teeth inside the circular ring. It can mesh with a spur gear. Internal gears are often used in planetary gear systems and also in gear couplings.

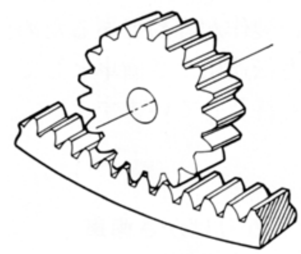


Fig.1.3 Internal gear and spur gear

(d) Helical Gear

This is a cylindrical shaped gear with helicoid teeth. Helical gears can bear more load than spur gears, and work more quietly. They are widely used in industry. A negative is the axial thrust force the helix form causes.

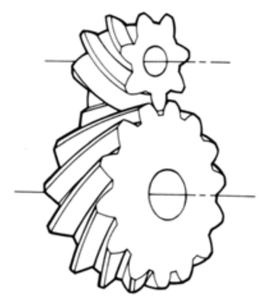


Fig.1.4 Helical gear

(e) Helical Rack

This is a linear shaped gear which meshes with a helical gear. Again, it can be regarded as a portion of a helical gear with infinite radius.

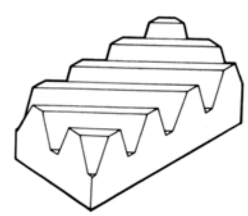


Fig.1.5 Helical rack

(f) Double Helical Gear

This is a gear with both left-hand and right-hand helical teeth. The double helical form balances the inherent thrust forces.

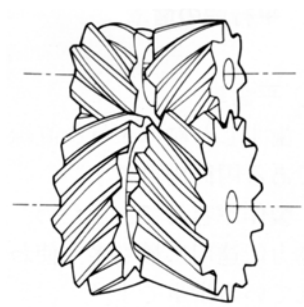


Fig.1.6 Double helical gear

(2) Intersecting Axes Gears

(a) Straight Bevel Gear

This is a gear in which the teeth have tapered conical elements that have the same direction as the pitch cone base line (generatrix). The straight bevel gear is both the simplest to produce and the most widely applied in the bevel gear family.

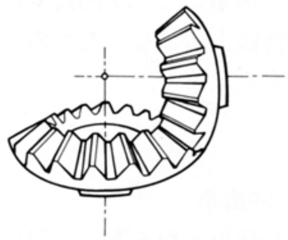


Fig.1.7 Straight bevel gear

(b) Spiral Bevel Gear

This is a bevel gear with a helical angle of spiral teeth. It is much more complex to manufacture, but offers a higher strength and lower noise.

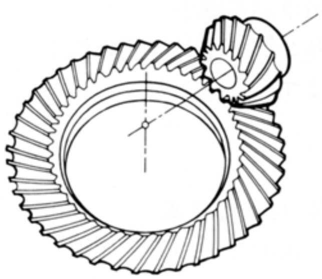


Fig.1.8 Spiral bevel gear

(c) Zerol Bevel Gear

Zerol bevel gear is a special case of spiral bevel gear. It is a spiral bevel with a spiral angle of zero. It has the characteristics of both the straight and spiral bevel gears. The forces acting upon the tooth are the same as for a straight bevel gear.

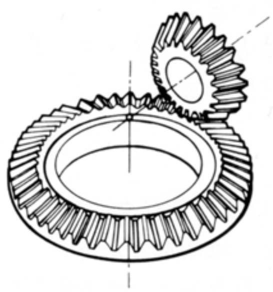


Fig.1.9 Zerol bevel gear

(3) Nonparallell and Nonintersecting Axes Gears

(a) Worm Gear Pair

Worm gear pair is the name for a meshed worm and worm wheel.
The outstanding feature is that it offers a very large gear ratio in a single mesh. It also provides quiet and smooth action. However, transmission efficiency is very poor.

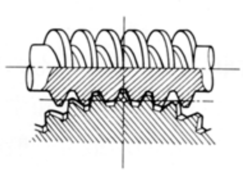


Fig.1.10 Worm gear pair

(b) Screw Gear (Crossed Helical Gear)

A pair of cylindrical gears used to drive non-parallel and non-intersecting shafts where the teeth of one or both members of the pair are of screw form.
Screw gears are used in the combination of screw gear / screw gear, or screw gear / spur gear.
Screw gears assure smooth, quiet operation. However, they are not suitable for transmission of high horsepower.

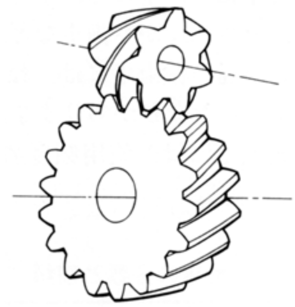


Fig.1.11 Screw gear

(4) Other Special Gears

(a) Face Gear

This is a pseudobevel gear that is limited to 90° intersecting axes. The face gear is a circular disc with a ring of teeth cut in its side face; hence the name face gear

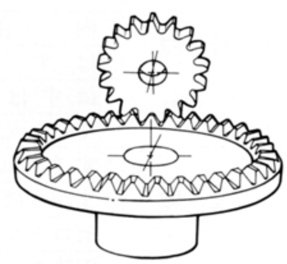


Fig.1.12 Face gear

(b) Enveloping Worm Gear Pair

This worm gear pair uses a special worm shape in that it partially envelops the worm wheel as viewed in the direction of the worm wheel axis. Its big advantage over the standard worm is much higher load capacity. However, the worm wheel is very complicated to design and produce.

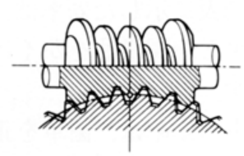


Fig.1.13 Enveloping worm gear pair

(c) Hypoid Gear

This is a deviation from a bevel gear that originated as a special development for the automobile industry. This permitted the drive to the rear axle to be nonintersecting, and thus allowed the auto body to be lowered. It looks very much like the spiral bevel gear. However, it is complicated to design and is the most difficult to produce on a bevel gear generator.

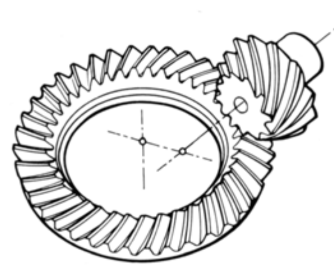


Fig.1.14 Hypoid gear

1.2 Symbols and Terminology

Table 1.2 through 1.6 indicate the symbols and the terminology used in this catalog. JIS B 0121:1999 and JIS B0102:1999 cancel and replace former JIS B0121 (symbols) and JIS B0102 (vocabulary) respectively. This revision has been made to conform to International Standard Organization (ISO) Standard.

Table 1.2 Linear dimensions and circular dimensions

Terms	Symbols
Centre distance	a
Reference pitch	p
Transverse pitch	p_t
Normal pitch	p_n
Axial pitch	p_x
Base pitch	p_b
Transverse base pitch	p_{bt}
Normal base pitch	p_{bn}
Tooth depth	h
Addendum	h_a
Dedendum	h_f
Chordal height	$\bar{h}_a$
Constant chord height	h_c
Working depth	h'
Tooth thickness	s
Normal tooth thickness	s_n
Transverse tooth thickness	s_t
Crest width	s_a
Base thickness	s_b
Chordal tooth thickness	$\bar{s}$
Constant chord	$\bar{s}_c$
Span measurement over k teeth	W
Tooth space	e
Tip and root clearance	c
Circumferential backlash	j_t
Normal backlash	j_n
Radial backlash	j_r
Angular backlash	j_θ
Facewidth	b
Effective facewidth	b'
Lead	p_z
Length of path of contact	g_α
Length of approach path	g_f
Length of recess path	g_a
Overlap length	g_β
Reference diameter	d
Pitch diameter	d'
Tip diameter	d_a
Base diameter	d_b
Root diameter	d_f
Center reference diameter	d_m
Inner tip diameter	d_i
Reference radius	r
Pitch radius	r'
Tip radius	r_a
Base radius	r_b
Root radius	r_f
Radius of curvature of tooth profile	ρ
Cone distance	R
Back cone distance	R_v

Table 1.3 Angular dimensions

Terms	Symbols
Reference pressure angle	α
Working pressure angle	α'
Cutter pressure angle	α_o
Transverse pressure angle	α_t
Normal pressure angle	α_n
Axial pressure angle	α_x
Transverse working pressure angle	α'_t
Tip pressure angle	α_a
Normal working pressure angle	α'_n
Reference cylinder helix angle	β
Pitch cylinder helix angle	β'
Mean spiral angle	β_m
Tip cylinder helix angle	β_a
Base cylinder helix angle	β_b
Reference cylinder lead angle	γ
Pitch cylinder lead angle	γ'
Tip cylinder lead angle	γ_a
Base cylinder lead angle	γ_b
Shaft angle	Σ
Reference cone angle	δ
Pitch angle	δ'
Tip angle	δ_a
Root angle	δ_f
Addendum angle	θ_a
Dedendum angle	θ_f
Transverse angle of transmission	ζ_α
Overlap angle	ζ_β
Total angle of transmission	ζ_γ
Tooth thickness half angle	ψ
Tip tooth thickness half angle	ψ_a
Spacewidth half angle	η
Angular pitch of crown gear	τ
Involute function	$\mathrm{inv}\alpha$

Table 1.4 Size numbers, ratios & speed terms

Terms	Symbols
Number of teeth	z
Equivalent number of teeth	z_v
Number of threads, or number of teeth in pinion	z_1
Gear ratio	u
Transmission ratio	i
Module	m
Transverse module	m_t
Normal module	m_n
Axial module	m_x
Diametral pitch	P
Transverse contact ratio	ε_α
Overlap ratio	ε_β
Total contact ratio	ε_γ
Angular speed	ω
Tangential speed	v
Rotational speed	n
Profile shift coefficient	x
Normal profile shift coefficient	x_n
Transverse profile shift coefficient	x_t
Center distance modification coefficient	y

Table 1.5 Others

Terms	Symbols
Tangential force	F_t
Axial force	F_x
Radial force	F_r
Pin diameter	d_p
Ideal pin diameter	d'_p
Measurement over rollers (pin)	M
Pressure angle at pin center	ϕ
Coefficient of friction	μ
Circular thickness factor	K

Table 1.6 Accuracy/Error terms

Terms	Symbols
Single pitch deviation	f_{pt}
Pitch deviation	f_v or f_{pu}
Total cumulative pitch deviation	F_p
Total profile deviation	F_a
Runout	F_r
Total helix deviation	F_b

A numerical subscript is used to distinguish "pinion" from "gear" (Example: z_1, z_2), "worm" from "worm wheel", "drive gear" from "driven gear", and so forth.

Table 1.7 indicates the Greek alphabet, the internatioal phonetic alphabet.

Table 1.7 The Greek alphabet

Upper case letters	Lower case letters	Spelling
Α	α	Alpha
Β	β	Beta
Γ	γ	Gamma
Δ	δ	Delta
Ε	ε	Epsilon
Ζ	ζ	Zeta
Η	η	Eta
Θ	θ	Theta
Ι	ι	lota
Κ	κ	Kappa
Λ	λ	Lambda
Μ	μ	Mu
Ν	ν	Nu
Ξ	ξ	Xi
Ο	o	Omicron
Π	π	Pi
Ρ	ρ	Rho
Σ	σ	Sigma
Τ	τ	Tau
Υ	υ	Upsilon
Φ	ϕ	Phi
Χ	χ	Chi
Ψ	ψ	Psi
Ω	ω	Omega

2 Gear Trains

The objective of gears is to provide a desired motion, either rotation or linear. This is accomplished through either a simple gear pair or a more involved and complex system of several gear meshes. Also, related to this is the desired speed, direction of rotation and the shaft arrangement.

2.1 Single-Stage Gear Train

A meshed gear is the basic form of a single-stage gear train. It consists of z_1 and z_2 numbers of teeth on the driver and driven gears, and their respective rotations, n_1 & n_2.

The transmission ratio is then:

$$\text{Transmission ratio} = \frac{z_2}{z_1} = \frac{n_1}{n_2} \tag{2.1}$$

Gear trains can be classified into three types:

Transmission ratio < 1 , increasing : $n_1 < n_2$

Transmission ratio = 1 , equal speeds: $n_1 = n_2$

Transmission ratio > 1 , reducing : $n_1 > n_2$

Figure 2.1 illustrates four basic forms. For the very common cases of spur and bevel gear meshes, Figures 2.1(A) and (B), the direction of rotation of driver and driven gears are reversed. In the case of an internal gear mesh, Figure 2.1(C), both gears have the same direction of rotation. In the case of a worm mesh, Figure 2.1(D), the rotation direction of z_2 is determined by its helix hand.

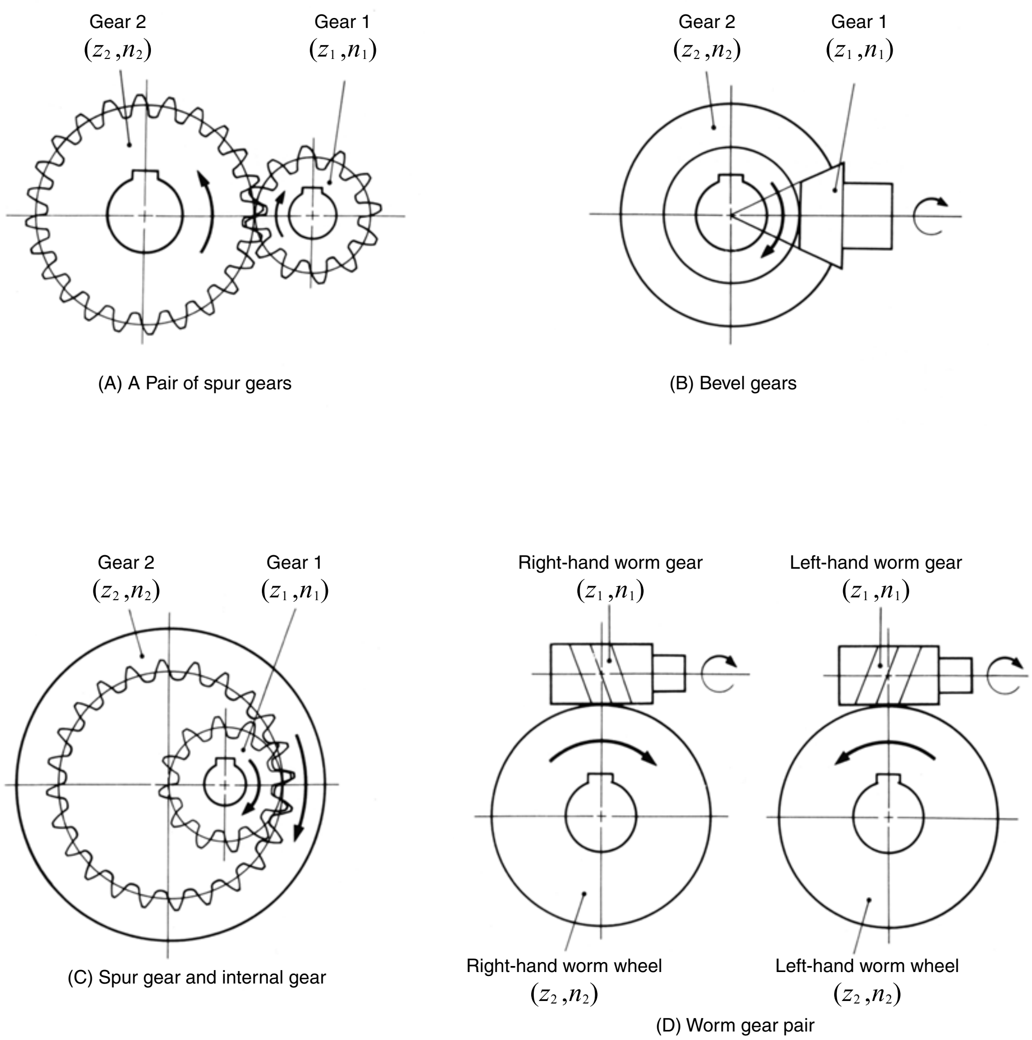


Fig. 2.1 Single-stage gear trains

In addition to these four basic forms, the combination of a rack and pinion can be considered a specific type. The displacement of a rack, l, for rotation θ of the mating pinion is:

$$l = \frac{z_1\theta}{360} \times \pi m \qquad (2.2)$$

where: πm is the reference pitch
z_1 is the number of teeth of the pinion.

2.2 Double-Stage Gear Train

A double-stage gear train uses two single-stages in a series. Figure 2.2 represents the basic form of an external gear double-stage gear train.

Let the first gear in the first stage be the driver. Then the transmission ratio of the double-stage gear train is:

$$\text{Transmission Ratio} = \frac{z_2}{z_1} \times \frac{z_4}{z_3} = \frac{n_1}{n_2} \times \frac{n_3}{n_4} \qquad (2.3)$$

In this arrangement, $n_2 = n_3$

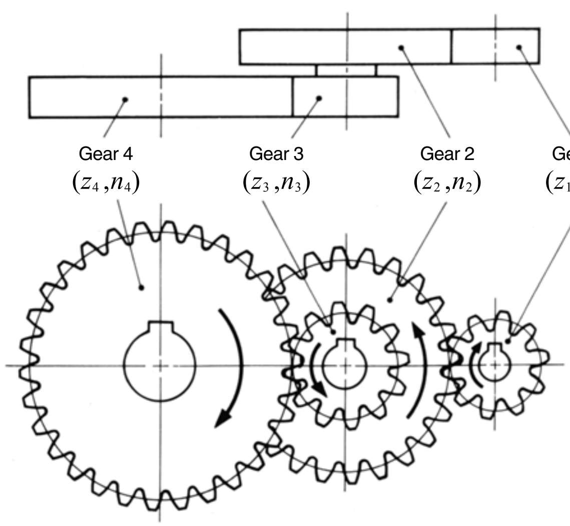


Fig.2.2 Double-stage gear train

In the double-stage gear train, Figure 2.2, gear 1 rotates in the same direction as gear 4. If gears 2 and 3 have the same number of teeth, then the train simplifies as in Figure 2.3. In this arrangement, gear 2 is known as an idler, which has no effect on the transmission ratio. The transmission ratio is then:

$$\text{Transmission Ratio} = \frac{z_2}{z_1} \times \frac{z_3}{z_2} = \frac{z_3}{z_1} \qquad (2.4)$$

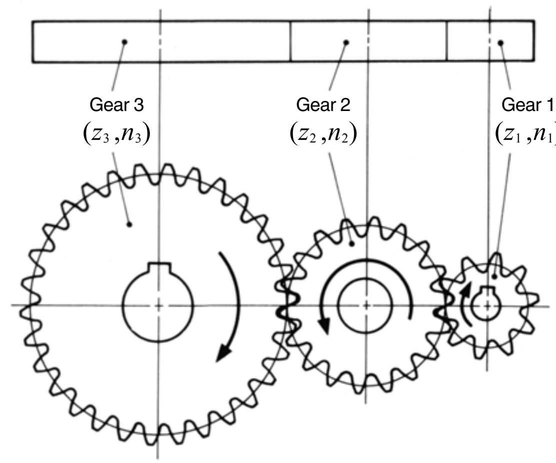


Fig.2.3 Single-stage gear train with an idler

3 Involute Gearing

The invoute profile is the one most commonly used today for gear-tooth forms that are used to transmit power. The beauty of involute gearing is its ease of manufacture and its smooth meshing despite the misalignment of center distance in some degree.

3.1 Module Sizes and Standards

Module m represents the size of involute gear tooth. The unit of module is mm. Module is converted to pitch p , by the factor π .

$$p = \pi m \tag{3.1}$$

Table 3.1 is extracted from JIS B 1701-1973 which defines the tooth profile and dimensions of involute gears. It divides the standard module into three series. Figure 3.1 shows the comparative size of various rack teeth.

Table 3.1 Standard values of module unit: mm

Series 1	Series 2	Series 3	Series 1	Series 2	Series 3
				3.5	
0.1					
	0.15				3.75
			4		
0.2					
	0.25			4.5	
			5		
0.3					
	0.35			5.5	
			6		
0.4					
	0.45				6.5
				7	
0.5					
	0.55		8		
				9	
0.6					
		0.65	10		
	0.7			11	
	0.75		12		
				14	
0.8					
	0.9		16		
				18	
1					
			20		
1.25					
				22	
1.5					
	1.75		25		
				28	
2					
	2.25		32		
				36	
2.5					
	2.75		40		
				45	
3					
		3.25	50		

NOTE: The preferred choices are in the series order beginning with 1.

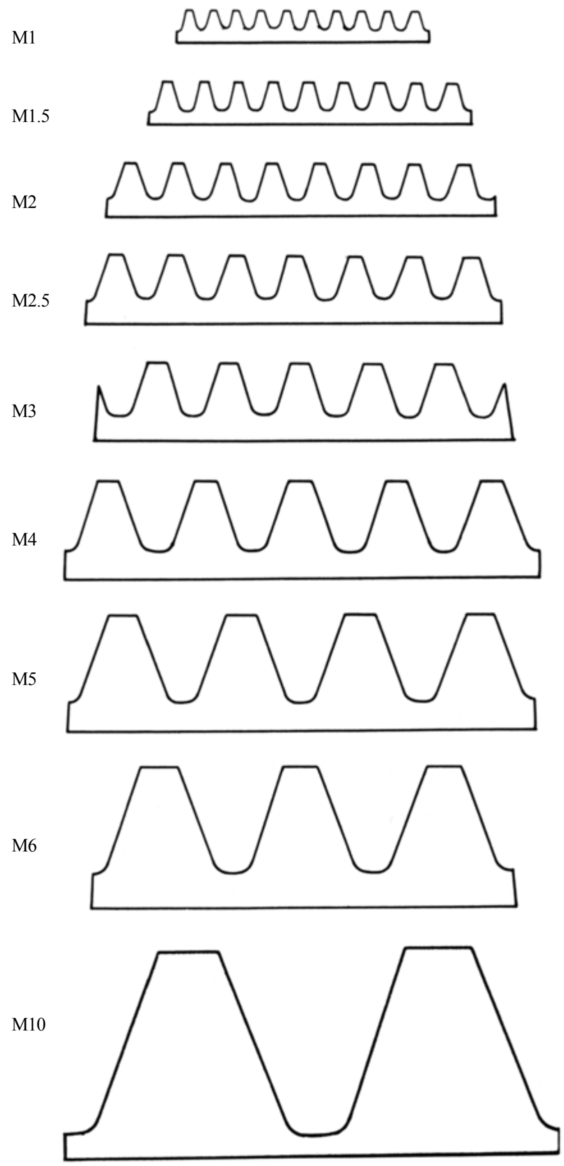


Fig.3.1 Comparative size of various rack teeth

Diametral Pitch $P(D.P.)$, the unit to denote the size of the gear-tooth, is used in the USA, the UK, etc. The transformation from Diametral Pitch $P(D.P.)$ to module m is accomplished by the following equation:

$$m = 25.4 / P$$

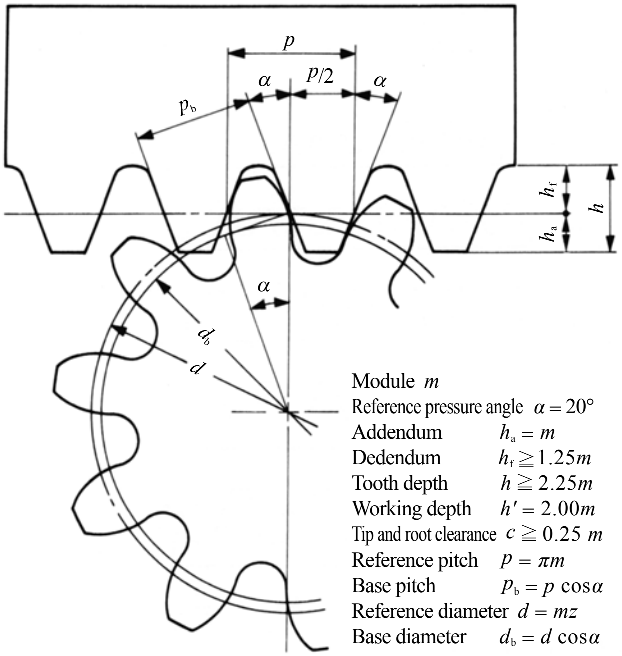


Fig.3.2 The tooth profile and dimension of standard rack

Pitch, p, is also used to represent tooth size when a special desired spacing is wanted, such as to get an integral feed in a mechanism. In this case, a pitch is chosen that is an integer or a special fractional value. This is often the choice in designing position control systems.

Most involute gear teeth have the standard whole depth and a standard pressure angle $\alpha = 20°$. Figure 3.2 shows the tooth profile of a full depth standard rack tooth and mating gear. It has an addendum of $h_a = 1m$ and dedendum $h_f \geq 1.25m$. If tooth depth is shorter than full depth teeth it is called a "stub" tooth; and if deeper than full depth teeth it is a "high" depth tooth.

The most widely used stub tooth has an addendum $h_a = 0.8m$ and dedendum $h_f = 1m$. Stub teeth have more strength than a full depth gear, but contact ratio is reduced. On the other hand, a high tooth can increase contact ratio.

In the standard involute gear, pitch (p) times the number of teeth becomes the length of reference circle:

$$d\pi = \pi mz \tag{3.2}$$

Reference diameter (d) is then:

$$d = mz \tag{3.3}$$

3.2 The Involute Curve

Figure 3.3 shows an element of involute curve. The definition of involute curve is the curve traced by a point on a straight line which rolls without slipping on the circle. The circle is called the base circle of the involutes. We can see, from Figure 3.3, the length of base circle arc *ac* equals the length of straight line *bc*.

$$\tan\alpha = \frac{bc}{oc} = \frac{r_b\theta}{r_b} = \theta \text{ (radian)} \tag{3.4}$$

The θ in Figure 3.3 can be expressed as $\text{inv}\alpha + \alpha$, then Formula (3.4) will become:

$$\text{inv}\alpha = \tan\alpha - \alpha \tag{3.5}$$

Function of α, or $\text{inv}\alpha$, is known as involute function. Involute function is very important in gear design. Involute function values can be obtained from appropriate tables.

With the center of the base circle O at the origin of a coordinate system, the involute curve can be expressed by values of x and y as follows:

$$\left.\begin{aligned} x &= r\cos(\text{inv}\alpha) \\ &= \frac{r_b}{\cos\alpha}\cos(\text{inv}\alpha) \\ y &= r\sin(\text{inv}\alpha) \\ &= \frac{r_b}{\cos\alpha}\sin(\text{inv}\alpha) \end{aligned}\right\} \tag{3.6}$$

where, $r = r_b / \cos\alpha$

The drawings of involute tooth-form can be easily created with this equation.

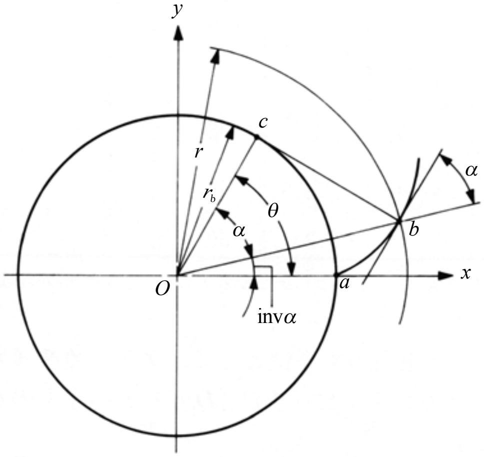


Fig.3.3 The involute curve

3.3 Meshing of Involute Gear

Figure 3.4 shows a pair of standard gears meshing together. The contact point of the two involutes, as Figure 3.4 shows, slides along the common tangent of the two base circles as rotation occurs. The common tangent is called the line of contact, or line of action.

A pair of gears can only mesh correctly if the pitches and the pressure angle are the same. That the pressure angles must be identical becomes obvious from the following equation for base pitch:

$$p_b = \pi m \cos\alpha \qquad (3.7)$$

Thus, if the pressure angles are different, the base pitches cannot be identical.

The contact length ab shown Figure 3.4 is described as "Length of path of contact.

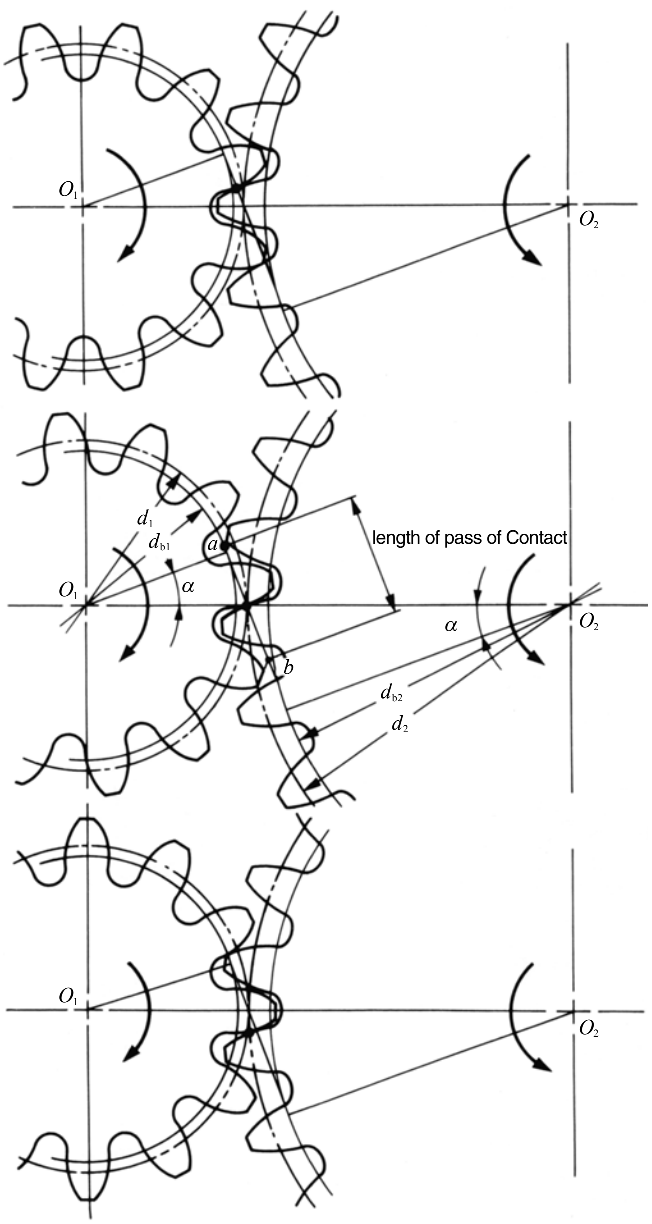


Fig.3.4 The meshing of involute gear

The contact ratio can be expressed by the following equation:

$$\text{Transverse Contact ratio } \varepsilon_\alpha = \frac{\text{Length of path of contact } ab}{\text{Base pitch } p_b} \qquad (3.8)$$

It is good practice to maintain a transverse contact ratio of 1.2 or greater.

Under no circumstaces should the ratio drop below 1.1. Module m and the pressure angle α are the key items in the meshing of gears.

3.4 The Generating of a Spur Gear

Involute gears can be readily generated by rack type cutters. The hob is in effect a rack cutter. Gear generation is also accomplished with gear type cutters using a shaper or planer machine.

Figure 3.5 illustrates how an involute gear tooth profile is generated. It shows how the pitch line of a rack cutter rolling on a pitch circle generates a spur gear.

Gear shapers with pinion cutters can also be used to generate involute gears. Gear shapers can not only generate external gears but also generate internal gears.

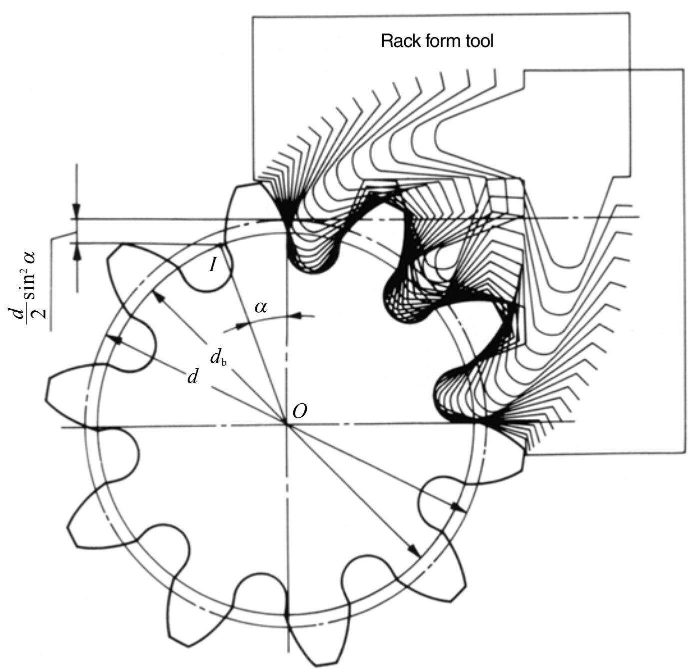


Fig. 3.5 The generating of a standard spur gear
($\alpha = 20°$, $z = 10$, $x = 0$)

3.5 Undercutting

Undercutting is the phenomenon that some of tooth dedendum is cut by the edge of a generating tool. In case gears with small number of teeth is generated as is seen in Figure 3.5, undercut occurs when the cutting is made deeper than interfering point I. The condition for no undercutting in a standard spur gear is given by the expression:

$$m \leqq \frac{mz}{2} \sin^2\alpha \qquad (3.9)$$

and the minimum number of teeth is:

$$z = \frac{2}{\sin^2\alpha} \qquad (3.10)$$

For pressure angle 20 degrees, the minimum number of teeth free of undercutting is 17. However, the gears with 16 teeth or under can be usable if its strength or contact ratio pose any ill effect.

3.6 Profile Shifting

As Figure 3.5 shows, a gear with 20 degrees of pressure angle and 10 teeth will have a huge undercut volume. To prevent undercut, a positive correction must be introduced. A positive correction, as in Figure 3.6, can prevent undercut. Undercutting will get worse if a negative correction is applied. See Figure 3.7. The extra feed of gear cutter (xm) in Figures 3.6 and 3.7 is the amount of shift or correction. And x is the profile shift coefficient.

The condition to prevent undercut in a spur gear is:

$$m - xm \leqq \frac{zm}{2} \sin^2\alpha \qquad (3.11)$$

The number of teeth without undercut will be:

$$z = \frac{2(1-x)}{\sin^2\alpha} \qquad (3.12)$$

The profile shift coefficient without undercut is:

$$x = 1 - \frac{z}{2} \sin^2\alpha \qquad (3.13)$$

Profile shift is not merely used to prevent undercut. It can be used to adjust center distance between two gears.
If a positive correction is applied, such as to prevent undercut in a pinion, the tooth tip is sharpened.

Table 3.2 presents the calculation of top land thickness (Crest width).

Table 3.2 The calculations of top land thickness (Crest width)

No.	Item	Symbol	Formula	Example
1	Tip pressure angle	α_a	$\cos^{-1}\frac{d_b}{d_a}$	$m = 2$ $\alpha = 20°$ $z = 16$ $x = +0.3$ $d = 32$ $d_b = 30.07016$ $d_a = 37.2$
2	Tip tooth thickness half angle	ψ_a	$\frac{\pi}{2z} + \frac{2x\tan\alpha}{z} + (\mathrm{inv}\alpha - \mathrm{inv}\alpha_a)$ (radian)	$\alpha_a = 36.06616°$ $\mathrm{inv}\,\alpha_a = 0.098835$ $\mathrm{inv}\,\alpha = 0.014904$ $\psi_a = 1.59815°$
3	Crest width	s_a	$\psi_a \cdot d_a$	(0.027893 radian) $s_a = 1.03762$

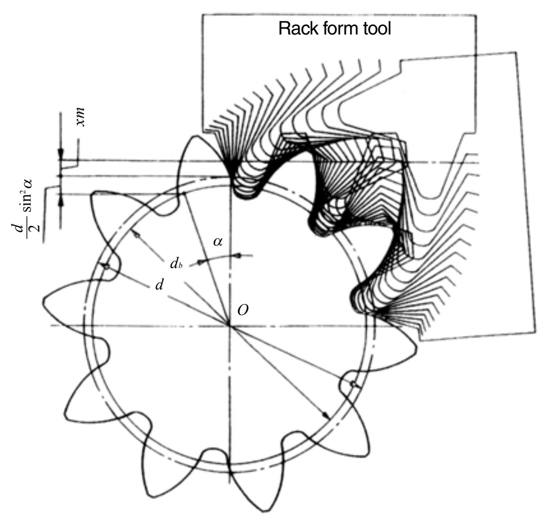


Fig.3.6 Generating of positive shifted spur gear
($\alpha = 20°, z = 10, x = +0.5$)

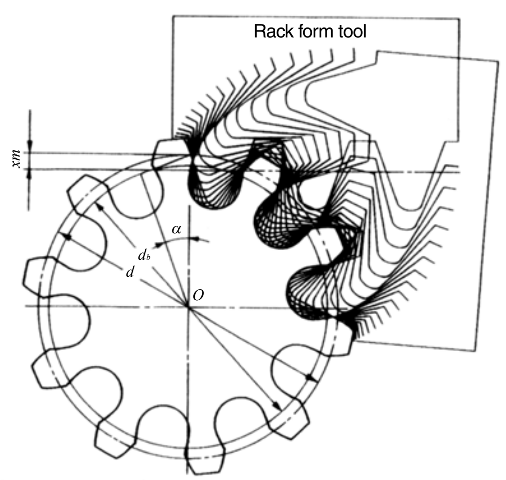


Fig.3.7 The generating of negative shifted spur gear
($\alpha = 20°, z = 10, x = -0.5$)

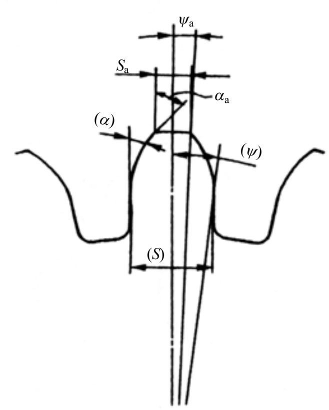


Fig. 3.8 Top land thickness
(Crest width)

4 Calculation of Gear Dimensions

The following should be taken into consideration in due order at the early stage of designing:

To calculate the required strength —— to determine the specifications, the materials to be used, and the degree of accuracy.
To calculate the dimensions —— in order to provide the necessary data for the gear shaping.
To calclate the tooth thickness —— in order to provide the necessary data for cutting and grinding.
To calculate the necessary amount of backlash
To calculate the forces to be acting on the gear —— to provide the necessary information useful for selecting the proper shafts and bearings.
To consider what kind of lubrication is necessary and appropriate.

The explanation is given, hereafter, as to items necessary for the design of gears. The calculation of the dimentions comes first. The dimentions are to be calculated in accordance with the fundamental specifications of each type of gears. The processes of turning etc. are to be carried out on the basis of that data.

4.1 Spur Gears

(1) Standard Spur Gear

Figure 4.1 shows the meshing of standard spur gears. The meshing of standard spur gears means reference circles of two gears contact and roll with each other. The calculation formulas are in Table 4.1.

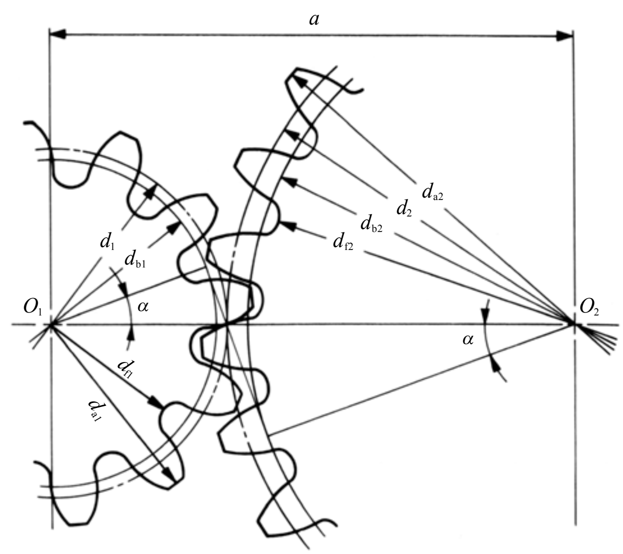


Fig.4.1 The meshing of standard spur gears
($\alpha = 20°$, $z_1 = 12$, $z_2 = 24$, $x_1 = x_2 = 0$)

Table 4.1 The calculation of standard spur gears

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Module	m		3	
2	Reference pressure angle	α		20°	
3	Number of teeth	z		12	24
4	Center distance	a	$\frac{(z_1 + z_2)\,m}{2}$ NOTE	54.000	
5	Reference diameter	d	zm	36.000	72.000
6	Base diameter	d_b	$d \cos\alpha$	33.829	67.658
7	Addendum	h_a	$1.00m$	3.000	3.000
8	Tooth depth	h	$2.25m$	6.750	6.750
9	Tip diameter	d_a	$d + 2m$	42.000	78.000
10	Root diameter	d_f	$d - 2.5m$	28.500	64.500

NOTE : The subscripts 1 and 2 of z_1 and z_2 denote pinion and gear.

All calculated values in Table 4.1 are based upon given module (m) and number of teeth (z_1 and z_2). If instead module (m), center distance (a) and speed ratio (i) are given, then the number of teeth , z_1 and z_2, would be calculated with the formulas as shown in Table 4.2.

Table 4.2 The calculation of number of teeth

No.	Item	Symbol	Formula		Example	
1	Module	m			3	
2	Center distance	a			54.000	
3	Transmission ratio	i			0.8	
4	Sum of No. of teeth	$z_1 + z_2$	$\frac{2a}{m}$		36	
5	Number of teeth	z	$\frac{z_1 + z_2}{i+1}$	$\frac{i(z_1+z_2)}{i+1}$	16	20

Note that the number of teeth probably will not be integer values by calculation with the formulas in Table 4.2. In that case, it will be necessary to resort to profile shifting or to employ helical gears to obtain as near transmission ratio as possible.

(2) Profile Shifted Spur Gear

Figure 4.2 shows the meshing of a pair of profile shifted gears. The key items in profile shifted gears are the operating (working) pitch diameters (d') and the working (operating) pressure angle (α').

These values are obtainable from the modified center distance and the following formulas:

$$\left.\begin{aligned} d'_1 &= 2a\,\frac{z_1}{z_1+z_2} \\ d'_2 &= 2a\,\frac{z_2}{z_1+z_2} \\ \alpha' &= \cos^{-1}\left(\frac{d_{b1}+d_{b2}}{2a}\right) \end{aligned}\right\} \quad (4.1)$$

In the meshing of profile shifted gears, it is the operating pitch circle that are in contact and roll on each other that portrays gear action.

Table 4.3 presents the calculation where the profile shiht coefficient has been set at x_1 and x_2 at the beginning. This calculation is based on the idea that the amount of the tip and root clearance should be 0.25 m.

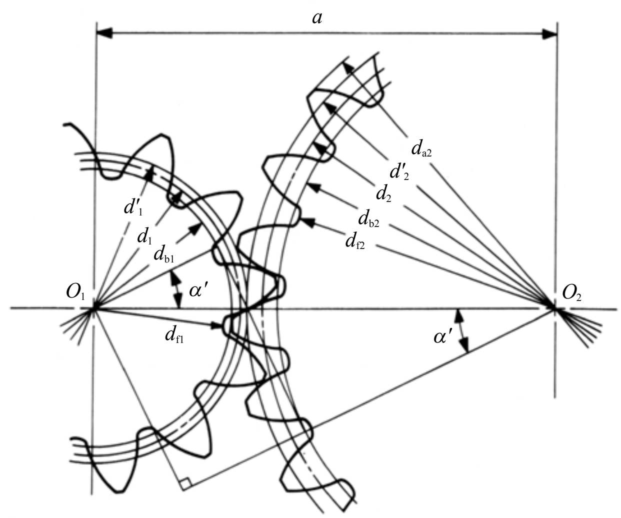


Fig. 4.2 The meshing of profile shifted gears
($\alpha = 20°$, $z_1 = 12$, $z_2 = 24$, $x_1 = +0.6$, $x_2 = +0.36$)

Table 4.3 The calculation of profile shifted spur gear (1)

No.	Item	Symbol	Formula	Example Pinion (1)	Example Gear (2)
1	Module	m		3	
2	Reference pressure angle	α		20°	
3	Number of teeth	z		12	24
4	Profile shift coefficient	x		0.6	0.36
5	Involute function α'	inv α'	$2\tan\alpha\left(\frac{x_1+x_2}{z_1+z_2}\right)+\mathrm{inv}\alpha$	0.034316	
6	Working pressure angle	α'	Find from Involute Function Table	26.0886°	
7	Center distance modification coefficient	y	$\frac{z_1+z_2}{2}\left(\frac{\cos\alpha}{\cos\alpha'}-1\right)$	0.83329	
8	Center distance	a	$\left(\frac{z_1+z_2}{2}+y\right)m$	56.4999	
9	Reference diameter	d	zm	36.000	72.000
10	Base diameter	d_b	$d\cos\alpha$	33.8289	67.6579
11	Working pitch diameter	d'	$\frac{d_b}{\cos\alpha'}$	37.667	75.333
12	Addendum	h_{a1} h_{a2}	$(1+y-x_2)\,m$ $(1+y-x_1)\,m$	4.420	3.700
13	Tooth depth	h	$\{2.25+y-(x_1+x_2)\}m$	6.370	
14	Tip diameter	d_a	$d+2h_a$	44.840	79.400
15	Root diameter	d_f	d_a-2h	32.100	66.660

A standard spur gear is, according to Table 4.3, a profile shifted gear with 0 coefficient of shift; that is , $x_1 = x_2 = 0$.

Table 4.4 is the inverse formula of items from 4 to 8 of Table 4.3.

Table 4.4 The calculation of profile shifted spur gear (2)

No.	Item	Symbol	Formula	Example	
1	Center distance	a		56.4999	
2	Center distance modification coefficient	y	$\frac{a}{m} - \frac{z_1 + z_2}{2}$	0.8333	
3	Working pressure angle	α'	$\cos^{-1}\left(\frac{\cos\alpha}{\frac{2y}{z_1 + z_2} + 1}\right)$	26.0886°	
4	Sum of profile shift coefficient	$x_1 + x_2$	$\frac{(z_1 + z_2)(\mathrm{inv}\alpha' - \mathrm{inv}\alpha)}{2\tan\alpha}$	0.9600	
5	Profile shift coefficient	x		0.6000	0.3600

There are several theories concerning how to distribute the sum of profile shift coefficient ($x_1 + x_2$) into pinion (x_1) and gear (x_2) separately. BSS (British) and DIN (German) standards are the most often used. In the example above, the 12 tooth pinion was given sufficient correction to prevent undercut, and the residual profile shift was given to the mating gear.

(3) Rack and Spur Gear

Table 4.5 presents the method for calculating the mesh of a rack and spur gear.

Figure 4.3(1) shows the the meshing of standard gear and a rack. In this meshing, the reference sircle of the gear touches the pitch lin of the rack.

Figure 4.3(2) shows a profile shifted spur gear, with positive correction xm, meshed with a rack. The spur gear has a larger pitch radius than standard, by the amount xm. Also, the pitch line of the rack has shifted outward by the amount xm.

Table 4.5 presents the calculation of a meshed profile shifted spur gear and rack. If the profile shift coefficient x_1 is 0, then it is the case of a standard gear meshed with the rack.

Table 4.5 The calculation of dimensions of a profile shifted spur gear and a rack

No.	Item	Symbol	Formula	Example	
				Spur gear	Rack
1	Module	m		3	
2	Reference pressure angle	α		20°	
3	Number of teeth	z		12	—
4	Profile shift coefficient	x		0.6	
5	Height of pitch line	H		—	32.000
6	Working pressure angle	α'		20°	
7	Mounting distance	a	$\frac{zm}{2}+H+xm$	51.800	
8	Reference diameter	d	zm	36.000	—
9	Base diameter	d_b	$d\cos\alpha$	33.829	
10	Working pitch diameter	d'	$\frac{d_b}{\cos\alpha'}$	36.000	
11	Addendum	h_a	$m(1+x)$	4.800	3.000
12	Tooth depth	h	$2.25m$	6.750	
13	Tip diameter	d_a	$d+2h_a$	45.600	—
14	Root diameter	d_f	d_a-2h	32.100	

One rotation of the spur gear will displace the rack l one circumferential length of the gear's reference circle, per the formula:

$$l=\pi mz \qquad (4.2)$$

The rack displacement, l, is not changed in any way by the profile shifting. Equation (4.2) remains applicable for any amount of profile shift.

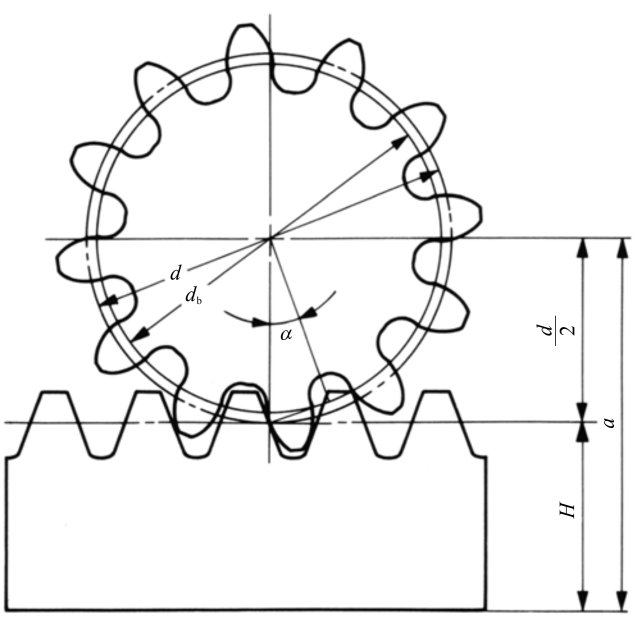


Fig.4.3(1) The meshing of standard spur gear and rack
(α = 20°, z_1 = 12, x_1 = 0)

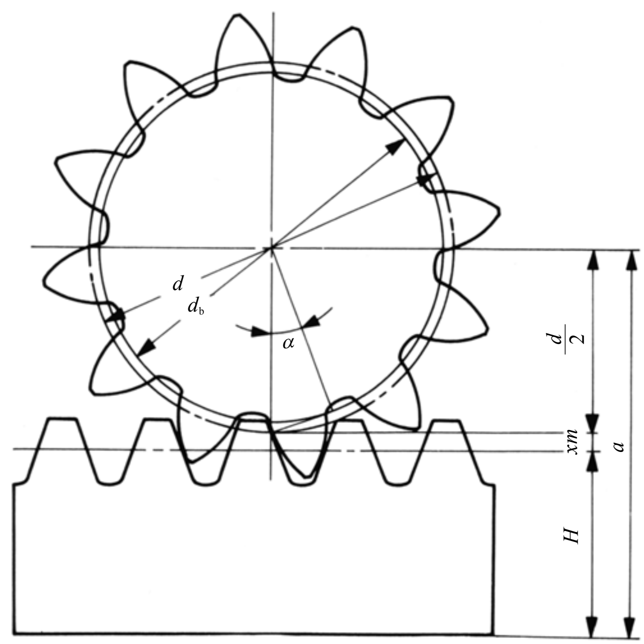


Fig.4.3(2) The meshing of profile shifted spur gear and rack
(α = 20°, z_1 = 12, x_1 = + 0.6)

4.2 Internal Gears

(1) Internal Gear Calculations

Figure 4.4 presents the mesh of an internal gear and external gear. Of vital importance is the working pitch diameters (d') and working pressure angle (α'). They can be derived from center distance (a') and Equations (4.3).

$$\left.\begin{aligned} d'_1 &= 2a\,\frac{z_1}{z_2-z_1} \\ d'_2 &= 2a\,\frac{z_2}{z_2-z_1} \\ \alpha' &= \cos^{-1}\left(\frac{d_{b2}-d_{b1}}{2a}\right) \end{aligned}\right\} \qquad (4.3)$$

Table 4.6 shows the calculation steps. It will become a standard gear calculation if $x_1 = x_2 = 0$.

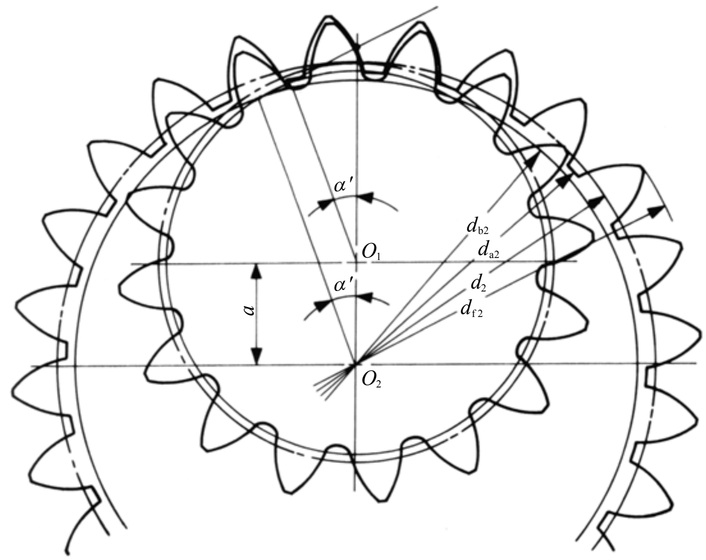


Fig.4.4 The meshing of internal gear and external gear
(α = 20°, z_1 = 16, z_2 = 24, $x_1 = x_2 = +0.5$)

Table 4.6 The calculation of a profile shifted internal gear and externl gear (1)

No.	Item	Symbol	Formula	Example	
				External gear	Internal gear
1	Module	m		3	
2	Reference pressure angle	α		20°	
3	Number of teeth	z		16	24
4	Profile shift coefficient	x		0	+ 0.5
5	Involute function α'	invα'	$2\tan\alpha\left(\frac{x_2-x_1}{z_2-z_1}\right)+\mathrm{inv}\alpha$	0.060401	
6	Working pressure angle	α'	Find from involute Function Table	31.0937°	
7	Center distance modification coefficient	y	$\frac{z_2-z_1}{2}\left(\frac{\cos\alpha}{\cos\alpha'}-1\right)$	0.389426	
8	Center distance	a	$\left(\frac{z_2-z_1}{2}+y\right)m$	13.1683	
9	Reference diameter	d	zm	48.000	72.000
10	Base diameter	d_b	$d\cos\alpha$	45.105	67.658
11	Working pitch diameter	d'	$\frac{d_b}{\cos\alpha'}$	52.673	79.010
12	Addendum	h_{a1} h_{a2}	$(1+x_1)m$ $(1-x_2)m$	3.000	1.500
13	Tooth depth	h	$2.25m$	6.75	
14	Tip diameter	d_{a1} d_{a2}	d_1+2h_{a1} d_2-2h_{a2}	54.000	69.000
15	Root diameter	d_{f1} d_{f2}	$d_{a1}-2h$ $d_{a2}+2h$	40.500	82.500

If the center distance (a) is given, x_1 and x_2 would be obtained from the inverse calculation from item 4 to item 8 of Table 4.6. These inverse formulas are in Table 4.7.

Table 4.7 The calculation of profile shifted internal gear and external gear (2)

No.	Item	Symbol	Formula	Example	
1	Center distance	a		13.1683	
2	Center distance modification coefficient	y	$\frac{a}{m} - \frac{z_2 - z_1}{2}$	0.38943	
3	Working pressure angle	α'	$\cos^{-1}\left(\frac{\cos\alpha}{\frac{2y}{z_2 - z_1} + 1}\right)$	31.0937°	
4	Difference of profile shift coefficient	$x_2 - x_1$	$\frac{(z_2 - z_1)(\mathrm{inv}\alpha' - \mathrm{inv}\alpha)}{2\tan\alpha}$	0.5	
5	Profile shift coefficient	x		0	0.5

Pinion cutters are often used in cutting internal gears and external gears. The actual value of tooth depth and root diameter, after cutting, will be slightly different from the calculation. That is because the cutter has a profile shift coefficient. In order to get a correct tooth profile, the profile shift coefficient of cutter should be taken into consideration.

(2) Interference In Internal Gears

Three different types of interference can occur with internal gears:

(a) Involute Interference,
(b) Trochoid Interference, and
(c) Trimming Interference.

(a) Involute Interference

This occurs between the dedendum of the external gear and the addendum of the internal gear. It is prevalent when the number of teeth of the external gear is small. Involute interference can be avoided by the conditions cited below:

$$\frac{z_1}{z_2} \geqq 1 - \frac{\tan\alpha_{a2}}{\tan\alpha'} \tag{4.4}$$

where α_{a2} is the pressure angle at a tip of the internal gear tooth.

$$\alpha_{a2} = \cos^{-1}\left(\frac{d_{b2}}{d_{a2}}\right) \tag{4.5}$$

α': working pressure angle

$$\alpha' = \cos^{-1}\left\{\frac{(z_2 - z_1)\, m \cos\alpha}{2a}\right\} \tag{4.6}$$

Equiation (4.5) is true only if the tip diameter of the internal gear is bigger than the base circle:

$$d_{a2} \geqq d_{b2} \tag{4.7}$$

For a standard internal gear, where $\alpha = 20°$, Equation (4.7) is valid only if the number of teeth is $z_2 > 34$.

(b) Trochoid Interference

This refers to an interference occurring at the addendum of the external gear and the dedendum of the internal gear during recess tooth action. It tends to happen when the difference between the numbers of teeth of the two gears is small. Equation (4.8) presents the condition for avoiding trochoidal interference.

$$\theta_1 \frac{z_1}{z_2} + \mathrm{inv}\alpha' - \mathrm{inv}\alpha_{a2} \geqq \theta_2 \tag{4.8}$$

Here

$$\left.\begin{aligned} \theta_1 &= \cos^{-1}\left(\frac{r_{a2}^2 - r_{a1}^2 - a^2}{2ar_{a1}}\right) + \mathrm{inv}\alpha_{a1} - \mathrm{inv}\alpha' \\ \theta_2 &= \cos^{-1}\left(\frac{a^2 + r_{a2}^2 - r_{a1}^2}{2ar_{a2}}\right) \end{aligned}\right\} \tag{4.9}$$

where α_{a1} is the pressure angle of the spur gear tooth tip:

$$\alpha_{a1} = \cos^{-1}\left(\frac{d_{b1}}{d_{a1}}\right) \tag{4.10}$$

$$\alpha_{a2} = \cos^{-1}\left(\frac{d_{b2}}{d_{a2}}\right)$$

In the meshing of an external gear and a standard internal gear $\alpha = 20°$, trochoid interference is avoided if the difference of the number of teeth, $z_1 - z_2$, is larger than 9.

(c) Trimming Interference

This occurs in the radial direction in that it prevents pulling the gears apart. Thus, the mesh must be assembled by sliding the gears together with an axial motion. It tends to happen when the numbers of teeth of the two gears are very close. Equation (4.11) indicates how to prevent this type of interference.

$$\theta_1 + \mathrm{inv}\alpha_{a1} - \mathrm{inv}\alpha' \geqq \frac{z_2}{z_1}(\theta_2 + \mathrm{inv}\alpha_{a2} - \mathrm{inv}\alpha') \tag{4.11}$$

$$\left.\begin{aligned} \theta_1 &= \sin^{-1}\sqrt{\frac{1-(\cos\alpha_{a1}/\cos\alpha_{a2})^2}{1-(z_1/z_2)^2}} \\ \theta_2 &= \sin^{-1}\sqrt{\frac{(\cos\alpha_{a2}/\cos\alpha_{a1})^2-1}{(z_2/z_1)^2-1}} \end{aligned}\right\} \tag{4.12}$$

This type of interference can occur in the process of cutting an internal gear with a pinion cutter. Should that happen, there is danger of breaking the tooling.

Table 4.8(1) shows the limit for the pinion cutter to prevent trimming interference when cutting a standard internal gear, with pressure angle $\alpha_0 = 20°$, and no profile shift, i.e., $x_0 = 0$.

Table 4.8(1) The limit to prevent an internal gear From trimming interference $\alpha_0=20°$ $x_0=x_2=0$

z_0	15	16	17	18	19	20	21	22	24	25	27
z_2	34	34	35	36	37	38	39	40	42	43	45

z_0	28	30	31	32	33	34	35	38	40	42
z_2	46	48	49	50	51	52	53	56	58	60

z_0	44	48	50	56	60	64	66	80	96	100
z_2	62	66	68	74	78	82	84	98	114	118

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of the pinion cutter ranges from 15 to 22 (z_0= 15 to 22).

Table 4.8(2) shows the limit for a profile shifted pinion cutter to prevent trimming interference while cutting a standard internal gear. The correction (x_0) is the magnitude of shift which was assumed to be: $x_0 = 0.0075\,z_0 + 0.05$.

Table 4.8(2) The limit to prevent an internal gear from trimming interference $\alpha_0 = 20°$, $x_2=0$

z_0	15	16	17	18	19	20	21	22	24	25	27
x_0	0.1625	0.17	0.1775	0.185	0.1925	0.2	0.2075	0.215	0.23	0.2375	0.2525
z_2	36	38	39	40	41	42	43	45	47	48	50

z_0	28	30	31	32	33	34	35	38	40	42
x_0	0.26	0.275	0.2825	0.29	0.2975	0.305	0.3125	0.335	0.35	0.365
z_2	52	54	55	56	58	59	60	64	66	68

z_0	44	48	50	56	60	64	66	80	96	100
x_0	0.38	0.41	0.425	0.47	0.5	0.53	0.545	0.65	0.77	0.8
z_2	71	76	78	86	90	95	98	115	136	141

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of the pinion cutter ranges from 15 to 19 (z_0 = 15 to 19).

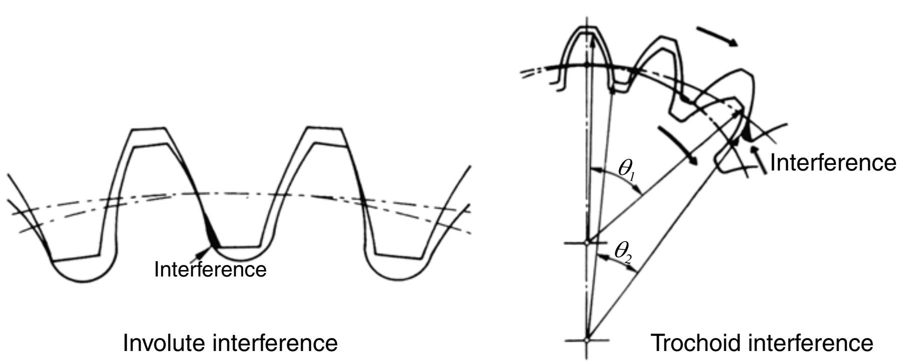


Fig.4.5 Involute interference and trochoid interference

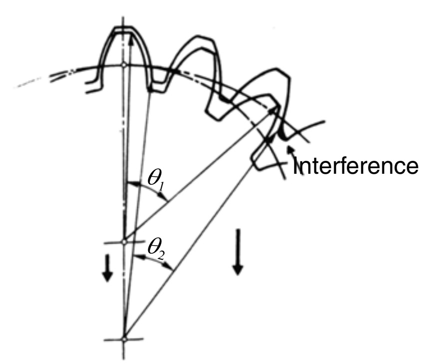


Fig.4.6 Trimming interference

4.3 Helical Gears

A helical gear such as shown in Figure 4.7 is a cylindrical gear in which the teeth flank are helicoid. The helix angle in reference cylinder is β, and the displacement of one rotation is the lead, p_z.

The tooth profile of a helical gear is an involute curve from an axial view, or in the plane perpendicular to the axis. The helical gear has two kinds of tooth profiles – one is based on a normal system, the other is based on an transverse system.

Pitch measured perpendicular to teeth is called normal pitch, p_n. And p_n divided by π is then a normal module, m_n.

$$m_n = \frac{p_n}{\pi} \tag{4.13}$$

The tooth profile of a helical gear with applied normal module, m_n, and normal pressure angle α_n belongs to a normal system.

In the axial view, the pitch on the reference is called the transverse pitch, p_t. And p_t divided by π is the transverse module, m_t.

$$m_t = \frac{p_t}{\pi} \tag{4.14}$$

These transverse module m_t and transverse pressure angle α_t are the basic configuration of transverse system helical gear.
In the normal system, helical gears can be cut by the same gear hob if module m_n and pressure angle α_n are constant, no matter what the value of helix angle β.

It is not that simple in the transverse system. The gear hob design must be altered in accordance with the changing of helix angle β, even when the module m_t and the pressure angle α_t are the same.
Obviously, the manufacturing of helical gears is easier with the normal system than with the transverse system in the plane perpendicular to the axis.

In meshing helical gears, they must have the same helix angle but with opposite hands.

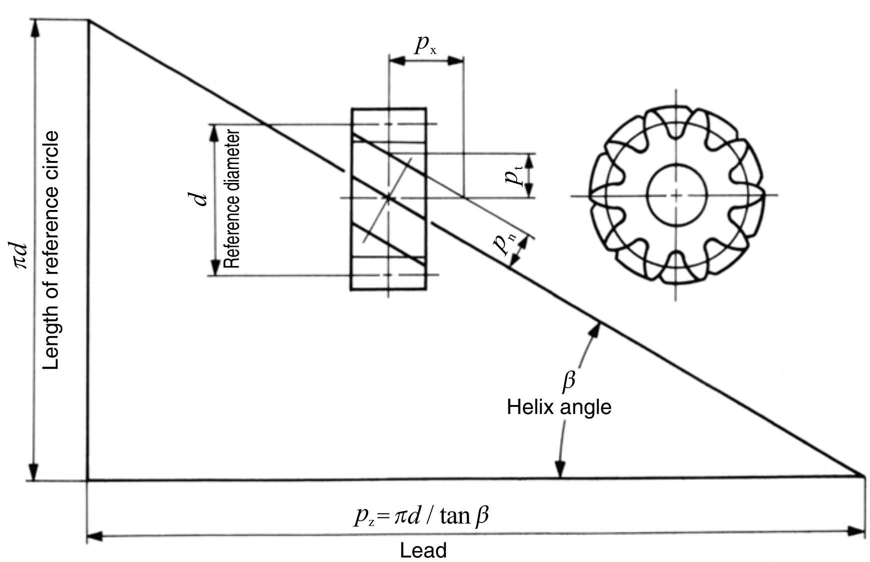


Fig.4.7 Fundamental relationship of a helical gear (Right-hand)

(1) Normal System Helical Gear

In the normal system, the calculation of a profile shifted helical gear, the working pitch diameter d' and transverse working pressure angle α'_t is done per Equations (4.15). That is because meshing of the helical gears in the transverse plane is just like spur gears and the calculation is similar.

$$\left.\begin{aligned} d'_1 &= 2a\,\frac{z_1}{z_1+z_2} \\ d'_2 &= 2a\,\frac{z_2}{z_1+z_2} \\ \alpha'_t &= \cos^{-1}\left(\frac{d_{b1}+d_{b2}}{2a}\right) \end{aligned}\right\} \qquad (4.15)$$

Table 4.9 shows the calculation of profile shifted helical gears in the normal system. If normal profile shift coefficients x_{n1}, x_{n2} are zero, they become standard gears.

Table 4.9 The calculation of a profile shifted helical gear in the normal system (1)

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Normal module	m_n		3	
2	Normal pressure angle	α_n		20°	
3	Reference cylinder helix angle	β		30°	
4	Number of teeth &helical hand	z		12(L)	60(R)
5	Transverse pressure angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)$	22.79588°	
6	Normal profile shift coefficient	x_n		+ 0.09809	0
7	Involute function α'_t	$\mathrm{inv}\,\alpha'_t$	$2\tan\alpha_n\left(\frac{x_{n1}+x_{n2}}{z_1+z_2}\right)+\mathrm{inv}\alpha_t$	0.023405	
8	Transverse woking pressure angle	α'_t	Find from involute Function Table	23.1126°	
9	Center distance modification coefficient	y	$\frac{z_1+z_2}{2\cos\beta}\left(\frac{\cos\alpha_t}{\cos\alpha'_t}-1\right)$	0.09744	
10	Center distance	a	$\left(\frac{z_1+z_2}{2\cos\beta}+y\right)m_n$	125.000	
11	Reference diameter	d	$\frac{zm_n}{\cos\beta}$	41.569	207.846
12	Base diameter	d_b	$d\cos\alpha_t$	38.322	191.611
13	Working pitch diameter	d'	$\frac{d_b}{\cos\alpha'_t}$	41.667	208.333
14	Addendum	h_{a1} h_{a2}	$(1+y-x_{n2})\,m_n$ $(1+y-x_{n1})\,m_n$	3.292	2.998
15	Tooth depth	h	$\{2.25+y-(x_{n1}+x_{n2})\}\,m_n$	6.748	
16	Tip diameter	d_a	$d+2h_a$	48.153	213.842
17	Root diameter	d_f	d_a-2h	34.657	200.346

If center distance, a, is given, the normal profile shift coefficients x_{n1} and x_{n2} can be calculated from Table 4.10. These are the inverse equations from items 4 to 10 of Table 4.9.

Table 4.10 The calculations of a profile shifted helical gear in the normal system (2)

No.	Item	Symbol	Formula	Example	
1	Center distance	a		125	
2	Center distance modification coefficient	y	$\frac{a}{m_n} - \frac{z_1+z_2}{2\cos\beta}$	0.097447	
3	Transverse working pressure angle	α'_t	$\cos^{-1}\left(\frac{\cos\alpha_t}{\frac{2y\cos\beta}{z_1+z_2}+1}\right)$	23.1126°	
4	Sum of profile shift coefficient	$x_{n1}+x_{n2}$	$\frac{(z_1+z_2)(\text{inv}\alpha'_t - \text{inv}\alpha_t)}{2\tan\alpha_n}$	0.09809	
5	Normal profile shift coefficient	x_n		0.09809	0

The transformation from a normal system to a transverse system is accomplished by the following equations:

$$\left.\begin{aligned} x_t &= x_n \cos\beta \\ m_t &= \frac{m_n}{\cos\beta} \\ \alpha_t &= \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right) \end{aligned}\right\} \quad (4.16)$$

(2) Transverse System Helical Gear

Table 4.11 shows the calculation of profile shifted helical gears in a transverse system. They become standard if $x_{t1} = x_{t2} = 0$.

Table 4.11 The calculation of a profile shifted helical gear in the transverse system (1)

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Transverse module	m_t		3	
2	Transverse pressure angle	α_t		20°	
3	Reference cylinder helix angle	β		30°	
4	Number of teeth & helical hand	z		12 (L)	60 (R)
5	Transverse profile shift coefficient	x_t		0.34462	0
6	Involute function α'_t	$\mathrm{inv}\alpha'_t$	$2\tan\alpha_t\left(\frac{x_{t1}+x_{t2}}{z_1+z_2}\right)+\mathrm{inv}\alpha_t$	0.0183886	
7	Transverse working pressure angle	α'_t	Find from Involute Function Table	21.3975°	
8	Center distance modification coefficient	y	$\frac{z_1+z_2}{2}\left(\frac{\cos\alpha_t}{\cos\alpha'_t}-1\right)$	0.33333	
9	Center distance	a	$\left(\frac{z_1+z_2}{2}+y\right)m_t$	109.0000	
10	Reference diameter	d	zm_t	36.000	180.000
11	Base diameter	d_b	$d\cos\alpha_t$	33.8289	169.1447
12	Working pitch diameter	d'	$\frac{d_b}{\cos\alpha'_t}$	36.3333	181.6667
13	Addendum	h_{a1} h_{a2}	$(1+y-x_{t2})\,m_t$ $(1+y-x_{t1})\,m_t$	4.000	2.966
14	Tooth depth	h	$\{2.25+y-(x_{t1}+x_{t2})\}\,m_t$	6.716	
15	Tip diameter	d_a	$d+2h_a$	44.000	185.932
16	Root diameter	d_f	d_a-2h	30.568	172.500

Table 4.12 presents the inverse calculation of items 5 to 9 of Table 4.11.

Table 4.12 The calculation of a profile shifted helical gear in the transverse system (2)

No.	Item	Symbol	Formula	Example	
1	Center distance	a		109	
2	Center distance modification coefficient	y	$\frac{a}{m_t}-\frac{z_1+z_2}{2}$	0.33333	
3	Transverse working pressure angle	α'_t	$\cos^{-1}\left(\frac{\cos\alpha_t}{\frac{2y}{z_1+z_2}+1}\right)$	21.39752°	
4	Sum of profile shift coefficient	$x_{t1}+x_{t2}$	$\frac{(z_1+z_2)(\mathrm{inv}\alpha'_t-\mathrm{inv}\alpha_t)}{2\tan\alpha_t}$	0.34462	
5	Transverse profile shift coefficient	x_t		0.34462	0

The transformation from a transverse to a normal system is described by the following equations:

$$\left.\begin{aligned} x_n &= \frac{x_t}{\cos\beta} \\ m_n &= m_t\cos\beta \\ \alpha_n &= \tan^{-1}(\tan\alpha_t\cos\beta) \end{aligned}\right\} \quad (4.17)$$

(3) Sunderland Double Helical Gear

A representative application of transverse system is a double helical gear, or herringbone gear, made with the Sunderland machine.

The transverse pressure angle, α_t, and helix angle, β, are specified as 20° and 22.5°, respectively.

The only differences from the transverse system equations of Table 4.11 are those for addendum and tooth depth.
Table4.13 presents equations for a Sunderland gear.

Table 4.13 The calculation of a double helical gear of SUNDERLAND tooth profile

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Transverse module	m_t		3	
2	Transverse pressure angle	α_t		20°	
3	Reference cylinder helix angle	β		22.5°	
4	Number of teeth	z		12	60
5	Transverse profile shift coefficient	x_t		0.34462	0
6	Involute function α'_t	$\mathrm{inv}\alpha'_t$	$2\tan\alpha_t\left(\frac{x_{t1}+x_{t2}}{z_1+z_2}\right)+\mathrm{inv}\alpha_t$	0.0183886	
7	Transverse working pressure angle	α'_t	Find from Involute Function Table	21.3975°	
8	Center distance modification coefficient	y	$\frac{z_1+z_2}{2}\left(\frac{\cos\alpha_t}{\cos\alpha'_t}-1\right)$	0.33333	
9	Center distance	a	$\left(\frac{z_1+z_2}{2}+y\right)m_t$	109.0000	
10	Reference diameter	d	zm_t	36.000	180.000
11	Base diameter	d_b	$d\cos\alpha_t$	33.8289	169.1447
12	Working pitch diameter	d'	$\frac{d_b}{\cos\alpha'_t}$	36.3333	181.6667
13	Addendum	h_{a1} h_{a2}	$(0.8796+y-x_{t2})\,m_t$ $(0.8796+y-x_{t1})\,m_t$	3.639	2.605
14	Tooth depth	h	$\{1.8849+y-(x_{t1}+x_{t2})\}m_t$	5.621	
15	Tip diameter	d_a	$d+2h_a$	43.278	185.210
16	Root diameter	d_f	d_a-2h	32.036	173.968

(4) Helical Rack

Viewed in the transverse plane, the meshing of a helical rack and gear is the same as a spur gear and rack. Table 4.14 presents the calculation examples for a mated helical rack with normal module and normal pressure angle. Similarily, Table 4.15 presents examples for a helical rack in the transverse system (i.e., perpendicular to gear axis).

Table 4.14 The calculation of a helical rack in the normal system

No.	Item	Symbol	Formula	Example: Gear	Example: Rack
1	Normal module	m_n		2.5	
2	Normal pressure angle	α_n		20°	
3	Reference cylinder helix angle	β		10° 57′ 49″	
4	Number of teeth & helical hand	z		20 (R)	— (L)
5	Normal profile shift coefficient	x_n		0	—
6	Pitch line height	H		—	27.5
7	Transverse pressure angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)$	20.34160°	
8	Mounting distance	a	$\frac{zm_n}{2\cos\beta}+H+x_n m_n$	52.965	
9	Reference diameter	d	$\frac{zm_n}{\cos\beta}$	50.92956	—
10	Base diameter	d_b	$d\cos\alpha_t$	47.75343	
11	Addendum	h_a	$m_n(1+x_n)$	2.500	2.500
12	Tooth depth	h	$2.25m_n$	5.625	
13	Tip diameter	d_a	$d+2h_a$	55.929	—
14	Root diameter	d_f	d_a-2h	44.679	

The formulas of a standard helical rack are similar to those of Table 4.14 with only the normal profile shift coefficient $x_n = 0$. To mesh a helical gear to a helical rack, they must have the same helix angle but with opposite hands.

The displacement of the helical rack, l, for one rotation of the mating gear is the product of the transverse pitch and number of teeth.

$$l=\frac{\pi m_n}{\cos\beta}z \qquad (4.18)$$

According to the equations of Table 4.14, let transverse pitch p_t = 8 mm and displacement l = 160 mm. The transverse pitch and the displacement could be modified into integers, if the helix angle were chosen properly.

Table 4.15 The calculation of a helical rack in the transverse system

No.	Item	Symbol	Formula	Example	
				Gear	Rack
1	Transverse module	m_t		2.5	
2	Transverse pressure angle	α_t		20°	
3	Reference cylinder helix angle	β		10° 57′ 49″	
4	Number of teeth & helical hand	z		20 (R)	— (L)
5	Transverse profile shift coefficient	x_t		0	—
6	Pitch line height	H		—	27.5
7	Mounting distance	a	$\frac{zm_t}{2} + H + x_t m_t$	52.500	
8	Reference diameter	d	zm_t	50.000	—
9	Base diameter	d_b	$d \cos\alpha_t$	46.98463	
10	Addendum	h_a	$m_t (1 + x_t)$	2.500	2.500
11	Tooth depth	h	$2.25m_t$	5.625	
12	Tip diameter	d_a	$d + 2h_a$	55.000	—
13	Root diameter	d_f	$d_a - 2h$	43.750	

In the meshing of transverse system helical rack and helical gear, the movement, l, for one turn of the helical gear is the transverse pitch multiplied by the number of teeth.

$$l = \pi m_t z \qquad (4.19)$$

4.4 Bevel Gears

Bevel gears, whose pitch surfaces are cones, are used to drive intersecting axes. Bevel gears are classified according to their type of the tooth forms into Straight Bevel Gear, Spiral Bevel Gear, Zerol Bevel Gear, Skew Bevel Gear etc.

The meshing of bevel gears means pitch cone of two gears contact and roll with each other.

Let z_1 and z_2 be pinion and gear tooth numbers; shaft angle Σ; and reference cone angles δ_1 and δ_2; then:

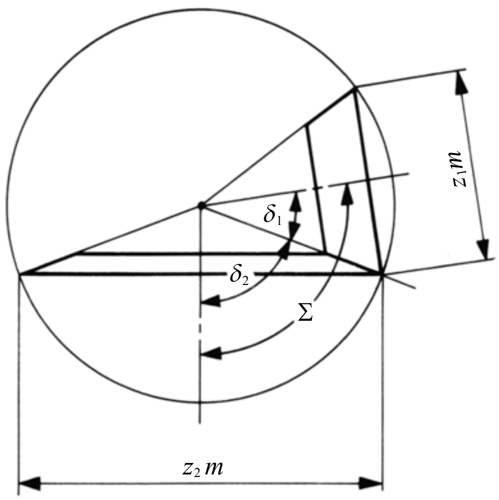


Fig. 4.8 The reference cone angle of bevel gear

$$\left.\begin{aligned} \tan\delta_1 &= \frac{\sin\Sigma}{\dfrac{z_2}{z_1} + \cos\Sigma} \\ \tan\delta_2 &= \frac{\sin\Sigma}{\dfrac{z_1}{z_2} + \cos\Sigma} \end{aligned}\right\} \quad (4.20)$$

Generally, shaft angle $\Sigma = 90°$ is most used. Other angles (Figure 4.8) are sometimes used. Then, it is called "bevel gear in nonright angle drive". The 90° case is called "bevel gear in right angle drive".

When $\Sigma = 90°$, Equation (4.20) becomes:

$$\left.\begin{aligned} \delta_1 &= \tan^{-1}\left(\frac{z_1}{z_2}\right) \\ \delta_2 &= \tan^{-1}\left(\frac{z_2}{z_1}\right) \end{aligned}\right\} \quad (4.21)$$

Miter gears are bevel gears with $\Sigma = 90°$ and $z_1 = z_2$. Their transmission ratio $z_2 / z_1 = 1$.

Figure 4.9 depicts the meshing of bevel gears.

The meshing must be considered in pairs. It is because the reference cone angles δ_1 and δ_2 are restricted by the gear ratio z_1 / z_2. In the facial view, which is normal to the contact line of pitch cones, the meshing of bevel gears appears to be similar to the meshing of spur gears.

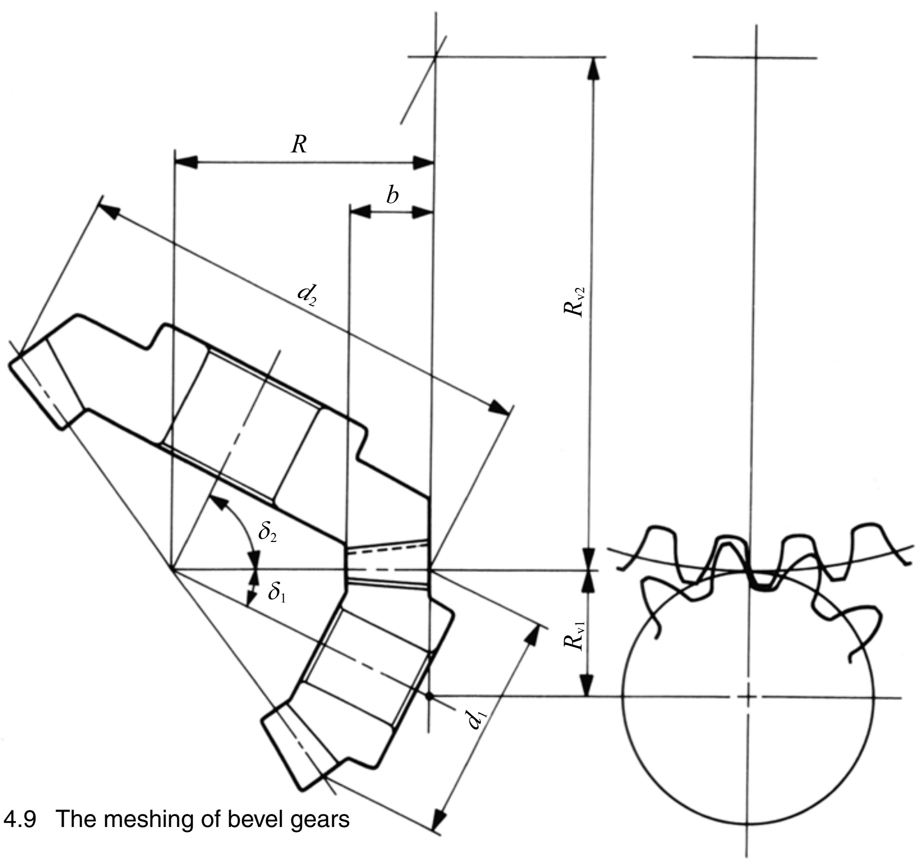


Fig. 4.9 The meshing of bevel gears

(1) Gleason Straight Bevel Gears

A straight bevel gear is a simple form of bevel gear having straight teeth which, if extended inward, would come together at the intersection of the shaft axes. Straight bevel gears can be grouped into the Gleason type and the standard type.

In this section, we discuss the Gleason straight bevel gear. The Gleason Company defined the tooth profile as: tooth depth $h = 2.188m$; tip and root clearance $c = 0.188m$; and working depth $h' = 2.000m$.

The characteristics are:

- **Design specified profile shifted gears:**

  In the Gleason system, the pinion is positive shifted and the gear is negative shifted. The reason is to distribute the proper strength between the two gears. Miter gears, thus, do not need any shifted tooth profile.

- **The tip and root clearance is designed to be parallel:**

  The face cone of the blank is turnd parallel to the root cone of the mate in order to eliminate possible fillet interference at the small ends of the teeth.

Table 4.16 shows the minimum number of teeth to prevent undercut in the Gleason system at the shaft angle $\Sigma = 90°$.

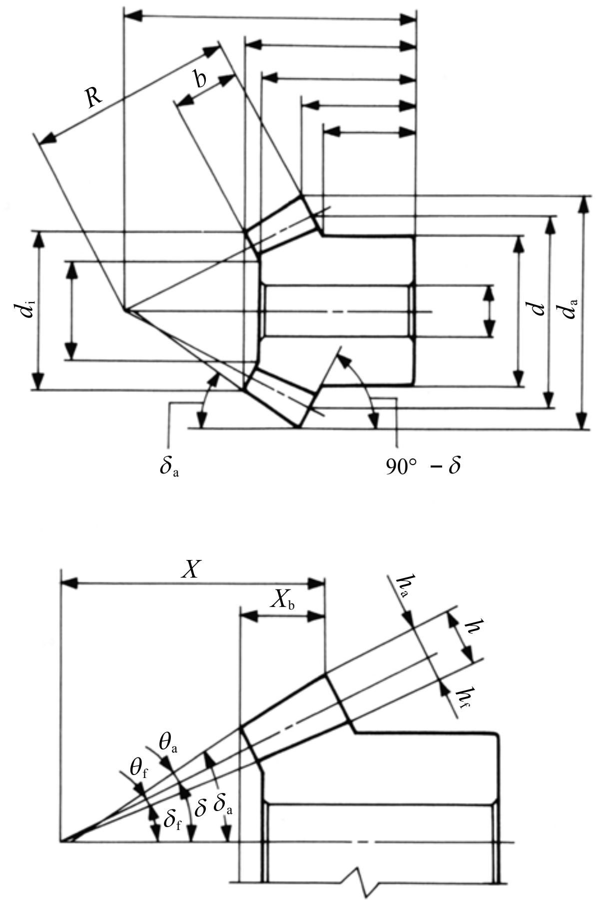


Fig. 4.10 Dimentions and angles of bevel gears

Table 4.16 The minimum numbers of teeth to prevent undercut

Pressure angle	Combination of number of teeth z_1 / z_2					
(14.5°)	29/29 and higher	28/29 and higher	27/31 and higher	26/35 and higher	25/40 and higher	24/57 and higher
20°	16/16 and higher	15/17 and higher	14/20 and higher	13/30 and higher		
(25°)	13/13 and higher					

Table 4.17 presents equations for designing straight bevel gears in the Gleason system. The meanings of the dimensions and angles are shown in Figure 4.10 above. All the equations in Table 4.17 can also be applied to bevel gears with any shaft angle.

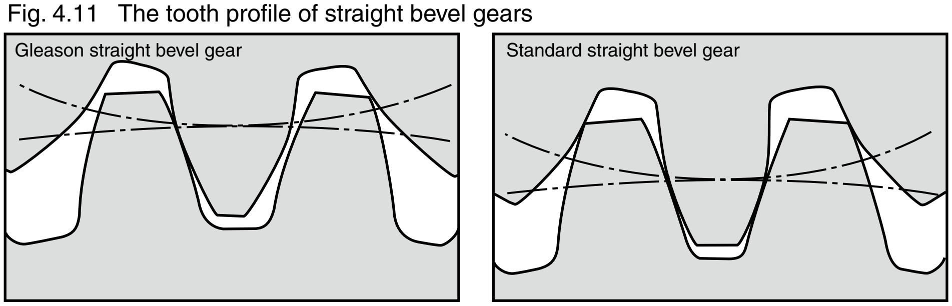
The straight bevel gear with crowning in the Gleason system is called a Coniflex gear. It is manufactured by a special Gleason “Coniflex” machine. It can successfully eliminate poor tooth contact due to improper mounting and assembly.

Tale 4.17 The calcultions of straight bevel gears of the gleason system

No.	Item	Symbol	Formula	Example	
				Pinion (1)	Gear (2)
1	Shaft angle	Σ		90°	
2	Module	m		3	
3	Reference pressure angle	α		20°	
4	Number of teeth	z		20	40
5	Reference diameter	d	zm	60	120
6	Reference cone angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\dfrac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma - \delta_1$	26.56505°	63.43495°
7	Cone distance	R	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
8	Facewidth	b	It should not exceed *R/3* or 10*m*	22	
9	Addendum	h_{a1} h_{a2}	$2.000m - h_{a2}$ $0.540m + \dfrac{0.460m}{\left(\dfrac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	4.035	1.965
10	Dedendum	h_f	$2.188m - h_a$	2.529	4.599
11	Dedendum angle	θ_f	$\tan^{-1}(h_f / R)$	2.15903°	3.92194°
12	Addendum angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.92194°	2.15903°
13	Tip angle	δ_a	$\delta + \theta_a$	30.48699°	65.59398°
14	Root angle	δ_f	$\delta - \theta_f$	24.40602°	59.51301°
15	Tip diameter	d_a	$d + 2h_a\cos\delta$	67.2180	121.7575
16	Pitch apex to crown	X	$R\cos\delta - h_a\sin\delta$	58.1955	28.2425
17	Axial facewidth	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	19.0029	9.0969
18	Inner tip diameter	d_i	$d_a - \dfrac{2b\sin\delta_a}{\cos\theta_a}$	44.8425	81.6609

The first characteristics of a Gleason Straight Bevel Gear is its profile shifted tooth. From Figure 4.11, we can see the tooth profile of Gleason Straight Bevel Gear and the same of Standard Straight Bevel Gear.

Fig. 4.11 The tooth profile of straight bevel gears



(2) Standard Straight Bevel Gears

A bevel gear with no profile shifted tooth is a standard straight bevel gear. The applicable equations are in Table 4.18.

Table 4.18 Calculation of a standard straight bevel gears

No.	Item	Symbol	Formula	Example Pinion (1)	Example Gear (2)
1	Shaft angle	Σ		90°	
2	Module	m		3	
3	Reference pressure angle	α		20°	
4	Number of teeth	z		20	40
5	Reference diameter	d	zm	60	120
6	Reference cone angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\dfrac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505°	63.43495°
7	Cone distance	R	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
8	Facewidth	b	It should not exceed $R/3$ or $10m$	22	
9	Addendum	h_a	$1.00m$	3.00	
10	Dedendum	h_f	$1.25m$	3.75	
11	Deddendum angle	θ_f	$\tan^{-1}(h_f/R)$	3.19960°	
12	Addendum angle	θ_a	$\tan^{-1}(h_a/R)$	2.56064°	
13	Tip angle	δ_a	$\delta+\theta_a$	29.12569°	65.99559°
14	Root angle	δ_f	$\delta-\theta_f$	23.36545°	60.23535°
15	Tip ciameter	d_a	$d+2h_a\cos\delta$	65.3666	122.6833
16	Pitch apex to crown	X	$R\cos\delta-h_a\sin\delta$	58.6584	27.3167
17	Axial facewidth	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	19.2374	8.9587
18	Inner tip diameter	d_i	$d_a-\dfrac{2b\sin\delta_a}{\cos\theta_a}$	43.9292	82.4485

These equations can also be applied to bevel gear sets with other than 90° shaft angle.

(3) Gleason Spiral Bevel Gears

A spiral bevel gear is one with a spiral tooth flank as in Figure 4.12. The spiral is generally consistent with the curve of a cutter with the diameter d_c. The spiral angle β is the angle between a generatrix element of the pitch cone and the tooth flank. The spiral angle just at the tooth flank center is called mean spiral angle β_m. In practice, spiral angle means mean spiral angle.

All equations in Table 4.21 are dedicated for the manufacturing method of Spread Blade or of Single Side from Gleason. If a gear is not cut per the Gleason system, the equations will be different from these.

The tooth profile of a Gleason spiral bevel gear shown here has the tooth depth $h = 1.888m$; tip and root clearance $c = 0.188m$; and working depth $h' = 1.700m$. These Gleason spiral bevel gears belong to a stub gear system. This is applicable to gears with modules $m > 2.1$.

Table 4.19 shows the minimum number of teeth to avoid undercut in the Gleason system with shaft angle $\Sigma = 90°$ and pressure angle $\alpha_n = 20°$.

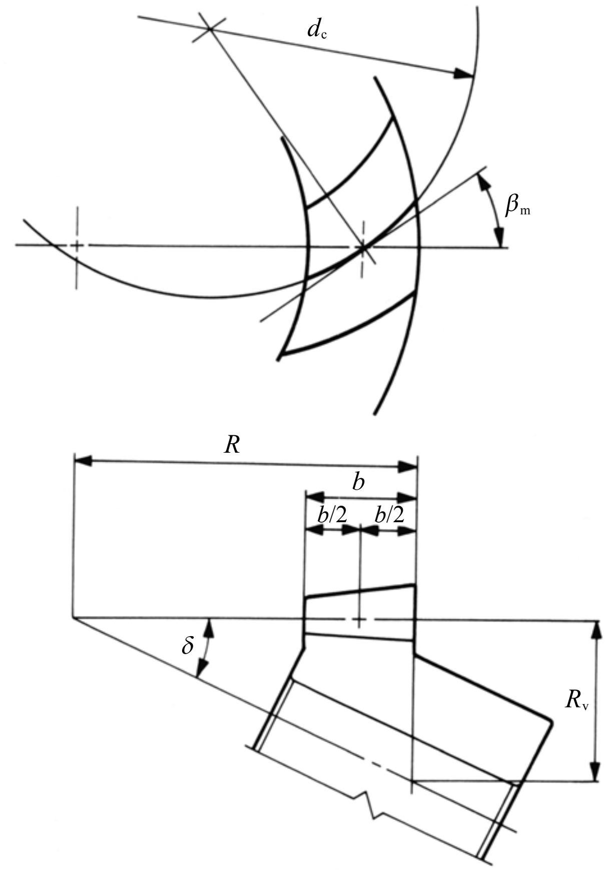


Fig.4.12 spiral bevel gear (Left-hand)

Table 4.19 The minimum numbers of teeth to prevent undercut

$\beta = 35°$

Pressure angle	Combination of numbers of teeth z_1 / z_2					
20°	17/17 and higher	16/18 and higher	15/19 and higher	14/20 and higher	13/22 and higher	12/26 and higher

If the number of teeth is less than 12, Table 4.20 is used to determine the gear sizes.

Table 4.20 Dimentions for pinions with number of teeth less than 12

Number of teeth in pinion	z_1	6	7	8	9	10	11
Number of teeth in gear	z_2	34 and higher	33 and higher	32 and higher	31 and higher	30 and higher	29 and higher
Working depth	h'	1.500	1.560	1.610	1.650	1.680	1.695
Tooth depth	h	1.666	1.733	1.788	1.832	1.865	1.882
Gear addendum	h_{a2}	0.215	0.270	0.325	0.380	0.435	0.490
Pinion addendum	h_{a1}	1.285	1.290	1.285	1.270	1.245	1.205
Tooth thickness of gear s_2	30	0.911	0.957	0.975	0.997	1.023	1.053
	40	0.803	0.818	0.837	0.860	0.888	0.948
	50	—	0.757	0.777	0.828	0.884	0.946
	60	—	—	0.777	0.828	0.883	0.945
Normal pressure angle	α_n	20°					
Spiral angle	β	35°~ 40°					
Shaft angle	Σ	90°					

NOTE: All values in the table are based on $m = 1$.

Table 4.21 shows the calculations of spiral bevel gears of the Gleason system

Table 4.21 The calculations of spiral bevel gears of the Gleason system

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Shaft angle	Σ		90°	
2	Module	m		3	
3	Normal pressure angle	α_n		20°	
4	Mean spiral angle	β_m		35°	
5	Number of teeth and spiral hand	z		20 (L)	40 (R)
6	Transverse pressure angle	α_t	$\tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\beta}\right)$	23.95680	
7	Reference diameter	d	zm	60	120
8	Reference cone angle	δ_1 δ_2	$\tan^{-1}\left(\dfrac{\sin\Sigma}{\dfrac{z_2}{z_1}+\cos\Sigma}\right)$ $\Sigma-\delta_1$	26.56505°	63.43495°
9	Cone distance	R	$\dfrac{d_2}{2\sin\delta_2}$	67.08204	
10	Facewidth	b	It should be less than 0.3R or 10m	20	
11	Addendum	h_{a1} h_{a2}	$1.700m-h_{a2}$ $0.460m+\dfrac{0.390m}{\left(\dfrac{z_2\cos\delta_1}{z_1\cos\delta_2}\right)}$	3.4275	1.6725
12	Dedendum	h_f	$1.888m-h_a$	2.2365	3.9915
13	Dedendum angle	θ_f	$\tan^{-1}(h_f/R)$	1.90952°	3.40519°
14	Addendum angle	θ_{a1} θ_{a2}	θ_{f2} θ_{f1}	3.40519°	1.90952°
15	Tip angle	δ_a	$\delta+\theta_a$	29.97024°	65.34447°
16	Root angle	δ_f	$\delta-\theta_f$	24.65553°	60.02976°
17	Tip diameter	d_a	$d+2h_a\cos\delta$	66.1313	121.4959
18	Pitch apex to crown	X	$R\cos\delta-h_a\sin\delta$	58.4672	28.5041
19	Axial facewidth	X_b	$\dfrac{b\cos\delta_a}{\cos\theta_a}$	17.3563	8.3479
20	Inner tip diameter	d_i	$d_a-\dfrac{2b\sin\delta_a}{\cos\theta_a}$	46.1140	85.1224

All equations in Table 4.21 are also applicable to Gleason bevel gears with any shaft angle. A spiral bevel gear set requires matching of hands; left-hand and right-hand as a pair.

(4) Gleason Zerol Bevel Gears

When the spiral angle $\beta_m = 0$, the bevel gear is called a Zerol bevel gear. The calculation equations of Table 4.17 for Gleason straight bevel gears are applicable. They also should take care again of the rule of hands; left and right of a pair must be matched.

Figure 4.13 is a left-hand Zerol bevel gear.

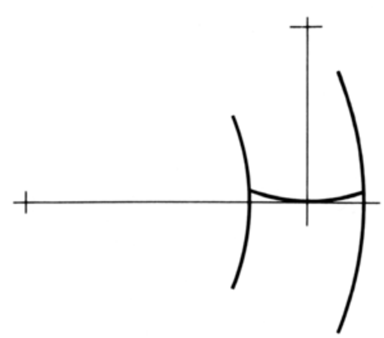


Fig. 4.13 Left-hand zerol bevel gear

4.5 Screw Gears

Screw gearing includes various types of gears used to drive nonparallel and nonintersecting shafts where the teeth of one or both members of the pair are of screw form. Figure 4.14 shows the meshing of screw gears.
Two screw gears can only mesh together under the conditions that normal modules (m_{n1}) and (m_{n2}) and normal pressure angles (a_{n1}, a_{n2}) are the same.

Let a pair of screw gears have the shaft angle Σ and helix angles β_1 and β_2:

If they have the same hands, then:

$$\Sigma = \beta_1 + \beta_2$$

If they have the opposite hands, then:

$$\Sigma = \beta_1 - \beta_2 \text{ or } \Sigma = \beta_2 - \beta_1 \qquad (4.22)$$

If the screw gears were profile shifted, the meshing would become a little more complex. Let β'_1, β'_2 represent the working pitch cylinder;

If they have the same hands, then:

$$\Sigma = \beta'_1 + \beta'_2$$

If they have the opposite hands, then:

$$\Sigma = \beta'_1 - \beta'_2 \text{ or } \Sigma = \beta'_2 - \beta'_1 \qquad (4.23)$$

Table 4.22 presents equations for a profile shifted screw gear pair. When the normal profile shift coefficients $x_{n1} = x_{n2} = 0$, the equations and calculations are the same as for standard gears.

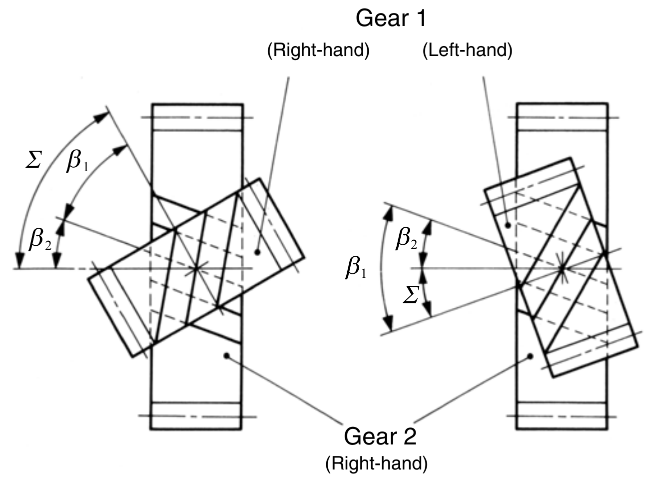


Fig.4.14 Screw gears of nonparallel and nonintersecting axes

Table 4.22 The equations for a screw gear pair on nonparallel and Nonintersecting axes in the normal system

No.	Item	Symbol	Formula	Example Pinion	Example Gear
1	Normal module	m_n		3	
2	Normal pressure angle	α_n		20°	
3	Reference cylinder helix angle	β		20°	30°
4	Number of teeth & helical hand	z		15 (R)	24 (R)
5	Number of teeth of an Equivalent spur gear	z_v	$\frac{z}{\cos^3\beta}$	18.0773	36.9504
6	Transverse pressure angle	α_t	$\tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)$	21.1728°	22.7959°
7	Normal profile shift coefficient	x_n		0.4	0.2
8	Involute function α'_n	$\mathrm{inv}\alpha'_n$	$2\tan\alpha_n\left(\frac{x_{n1}+x_{n2}}{z_{v1}+z_{v2}}\right)+\mathrm{inv}\alpha_n$	0.0228415	
9	Normal working pressure angle	α'_n	Find from involute function table	22.9338°	
10	Transverse working pressure angle	α'_t	$\tan^{-1}\left(\frac{\tan\alpha'_n}{\cos\beta}\right)$	24.2404°	26.0386°
11	Center distance modification coefficient	y	$\frac{1}{2}(z_{v1}+z_{v2})\left(\frac{\cos\alpha_n}{\cos\alpha'_n}-1\right)$	0.55977	
12	Center distance	a	$\left(\frac{z_1}{2\cos\beta_1}+\frac{z_2}{2\cos\beta_2}+y\right)m_n$	67.1925	
13	Reference diameter	d	$\frac{zm_n}{\cos\beta}$	47.8880	83.1384
14	Base diameter	d_b	$d\cos\alpha_t$	44.6553	76.6445
15	Working pitch diameter	d'_1 d'_2	$2a\frac{d_1}{d_1+d_2}$ $2a\frac{d_2}{d_1+d_2}$	49.1155	85.2695
16	Working helix angle	β'	$\tan^{-1}\left(\frac{d'}{d}\tan\beta\right)$	20.4706°	30.6319°
17	Shaft angle	Σ	$\beta'_1+\beta'_2$ or $\beta'_1-\beta'_2$	51.1025°	
18	Addendum	h_{a1} h_{a2}	$(1+y-x_{n2})m_n$ $(1+y-x_{n1})m_n$	4.0793	3.4793
19	Tooth depth	h	$\{2.25+y-(x_{n1}+x_{n2})\}m_n$	6.6293	
20	Tip diameter	d_a	$d+2h_a$	56.0466	90.0970
21	Root diameter	d_f	d_a-2h	42.7880	76.8384

Standard screw gears have relations as follows:

$$\left.\begin{aligned} d'_1 &= d_1 \qquad d'_2 = d_2 \\ \beta'_1 &= \beta_1 \qquad \beta'_2 = \beta_2 \end{aligned}\right\} \qquad (4.24)$$

4.6 Cylindrical Worm Gear Pair

Cylindrical worms may be considered cylindrical type gears with screw threads. Generally, the mesh has a 90° shaft angle. The number of threads in the worm is equivalent to the number of teeth in a gear of a screw type gear mesh. Thus, a one-thread worm is equivalent to a one-tooth gear; and two-threads equivalent to two-teeth, etc. Referring to Figure 4.15, for a reference cylinder lead angle γ, measured on the pitch cylinder, each rotation of the worm makes the thread advance one lead p_z.

There are four worm tooth profiles in JIS B 1723, as defined on the right.

Table 4.23 Axial module of cylindrical worm gear pair

1	1.25	1.60	2.00	2.50	3.15	4.00	5.00
6.30	8.00	10.00	12.50	16.00	20.00	25.00	

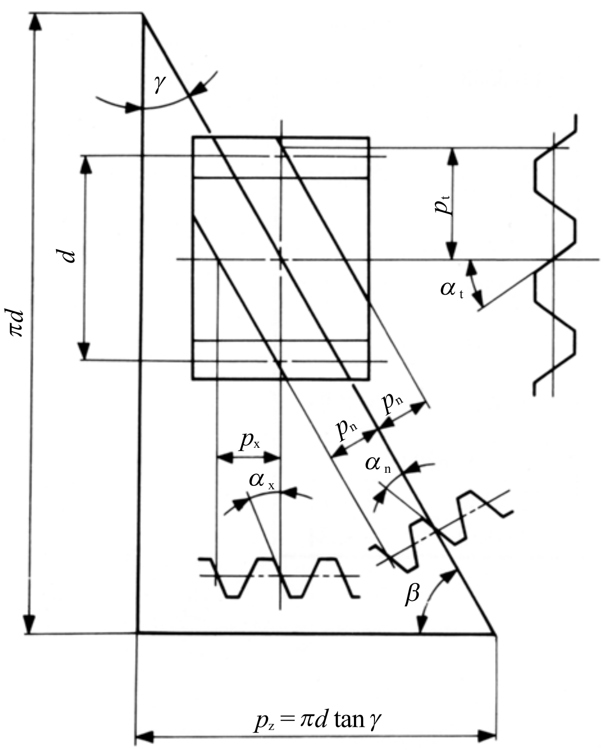


Fig. 4.15 Cylindrical worm (Right-hand)

Type I Worm: The tooth profile is trapezoidal on the axial plane.
Type II Worm: The tooth profile is trapezoid on the plane normal to the space.
Type III Worm: The tooth profile which is obtained by inclining the axis of the milling or grinding, of which cutter shape is trapezoidal on the cutter axis, by the lead angle to the worm axis.
Type IV Worm: The tooth profile is of involute curve on the plane of rotation.

Type III worm is the most popular. In this type, the normal pressure angle α_n has the tendency to become smaller than that of the cutter, α_0.
Per JIS, Type III worm uses a axial module m_x and cutter pressure angle $\alpha_0 = 20°$ as the module and pressure angle. A special worm hob is required to cut a Type III worm wheel.
Standard values of axial module, m_x , are presented in Table 4.23.

Because the worm mesh couples nonparallel and nonintersecting axes, the axial plane of worm does not correspond with the axial plane of worm wheel. The axial plane of worm corresponds with the trsnsverse plane of worm wheel. The transverse plane of worm corresponds with the axial plane of worm wheel. The common plane of the worm and worm wheel is the normal plane. Using the normal module, m_n, is most popular. Then, an ordinary hob can be used to cut the worm wheel.

Table 4.24 presents the relationships among worm and worm wheel axial plane, transverse plane, normal plane, module, pressure angle, pitch and lead.

Table 4.24 The relations of cross sections of worm gear pair

Worm		
Axial plane	Normal plane	Transverse plane
$m_x = \dfrac{m_n}{\cos\gamma}$	m_n	$m_t = \dfrac{m_n}{\sin\gamma}$
$\alpha_x = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\cos\gamma}\right)$	α_n	$\alpha_t = \tan^{-1}\left(\dfrac{\tan\alpha_n}{\sin\gamma}\right)$
$p_x = \pi m_x$	$p_n = \pi m_n$	$p_t = \pi m_t$
$p_z = \pi m_x z$	$p_z = \dfrac{\pi m_n z}{\cos\gamma}$	$p_z = \pi m_t z \tan\gamma$
Transverse plane	Norml plane	Axial plane
Worm wheel		

NOTE: The transverse plane is the plane perpendicular to the axis.

Reference to Figure 4.15 can help the understanding of the relationships in Table 4.24. They are similar to the relations in Formulas (4.16) and (4.17) that the helix angle β be substituted by $(90° - \gamma)$. We can consider that a worm with lead angle γ is almost the same as a helical gear with helix angle $(90° - \gamma)$.

(1) Axial Module Worm Gear Pair

Table 4.25 presents the equations, for dimensions shown in Figure 4.16, for worm gears with axial module, m_x, and normal pressure angle $\alpha_n = 20°$.

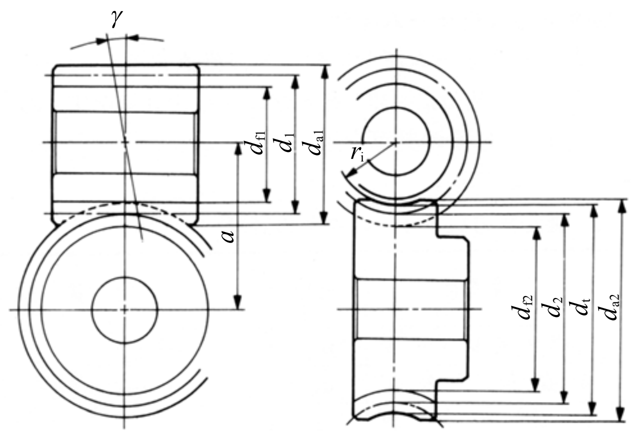


Fig. 4.16 Dimentions of cylindrical worm gear pair

Table 4.25 The calculations of axial module system worm gear pair

No.	Item	Symbol	Formula	Example: Worm	Example: Wheel
1	Axial module	m_x		3	
2	Normal pressure angle	α_n		(20°)	
3	No. of threads, no. of teeth	z		Double(R) *	30 (R)
4	Reference diameter	d_1 d_2	(Qm_x) NOTE 1 $z_2 m_x$	44.000	90.000
5	Reference cylinder lead angle	γ	$\tan^{-1}\left(\frac{m_x z_1}{d_1}\right)$	7.76517°	
6	Profile shift coefficient	x_{t2}		–	0
7	Center distance	a	$\frac{d_1 + d_2}{2} + x_{t2} m_x$	67.000	
8	Addendum	h_{a1} h_{a2}	$1.00\ m_x$ $(1.00 + x_{t2})\ m_x$	3.000	3.000
9	Tooth depth	h	$2.25\ m_x$	6.750	
10	Tip diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 + 2h_{a2} + m_x$ NOTE 2	50.000	99.000
11	Throat diameter	d_t	$d_2 + 2h_{a2}$	–	96.000
12	Throat surface radius	r_i	$\frac{d_1}{2} - h_{a1}$	–	19.000
13	Root diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_t - 2h$	36.500	82.500

* Double−threaded right-hand worm .

NOTE 1: Diameter factor, Q, means reference diameter of worm, d_1, over axial module, m_x.

$$Q = \frac{d_1}{m_x}$$

NOTE 2: There are several calculation methods of worm wheel tip diameter d_{a2} besides those in Table 4.25.

NOTE 3: The facewidth of worm, b_1, would be sufficient if:
$b_1 = \pi m_x\ (4.5 + 0.02 z_2)$

NOTE 4: Effective facewidth of worm wheel $b' = 2m_x\sqrt{Q + 1}$. So the actual facewidth of $b_2 \geqq b' + 1.5\ m_x$ would be enough.

(2) Normal Module System Worm Gear Pair

The equations for normal module system worm gears are based on a normal module, m_n, and normal pressure angle, $\alpha_n = 20°$. See Table 4.26.

Table 4.26 The calculations of normal module system worm gear pair

No.	Item	Symbol	Formula	Example: Worm	Example: Worm Wheel
1	Normal module	m_n		3	
2	Normal pressure angle	α_n		(20°)	
3	No. of threads, No. of teeth	z		Double(R) *	30 (R)
4	Reference diameter of worm	d_1		44.000	—
5	Reference cylinder lead angle	γ	$\sin^{-1}\left(\frac{m_n z_1}{d_1}\right)$	7.83748°	
6	Reference diameter of worm wheel	d_2	$\frac{z_2 m_n}{\cos\gamma}$	—	90.8486
7	Normal profile shift coefficient	x_{n2}		—	− 0.1414
8	Center distance	a	$\frac{d_1 + d_2}{2} + x_{n2} m_n$	67.000	
9	Addendum	h_{a1} h_{a2}	$1.00\ m_n$ $(1.00 + x_{n2})\ m_n$	3.000	2.5758
10	Tooth depth	h	$2.25\ m_n$	6.75	
11	Tip diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 + 2h_{a2} + m_n$	50.000	99.000
12	Throat diameter	d_t	$d_2 + 2h_{a2}$	—	96.000
13	Throat surface radius	r_i	$\frac{d_1}{2} - h_{a1}$	—	19.000
14	Root diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_t - 2h$	36.500	82.500

* Double-threaded right-hand worm .

NOTE: All notes are the same as those of Table 4.25.

(3) Crowning of the Tooth

Crowning is critically important to worm gears. Not only can it eliminate abnormal tooth contact due to incorrect assembly, but it also provides for the forming of an oil film, which enhances the lubrication effect of the mesh. This can favorably impact endurance and transmission efficiency of the worm mesh. There are four methods of crowning worm gear pair:

(a) Cut Worm Wheel with a Hob Cutter of Greater Reference Diameter than the Worm.

A crownless worm wheel results when it is made by using a hob that has an identical pitch diameter as that of the worm. This crownless worm wheel is very difficult to assemble correctly. Proper tooth contact and a complete oil film are usually not possible.
However, it is relatively easy to obtain a crowned worm wheel by cutting it with a hob whose reference diameter is slightly larger than that of the worm.

This is shown in Figure 4.17. This creates teeth contact in the center region with space for oil film formation.

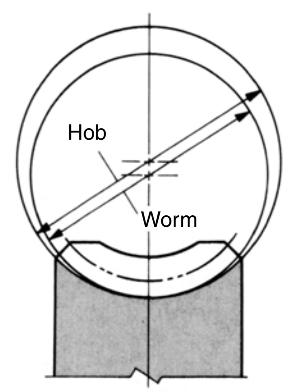


Fig.4.17 The method of using a greater diameter hob

(b) Recut With Hob Center Position Adjustment.

The first step is to cut the worm wheel at standard center distance. This results in no crowning. Then the worm wheel is finished with the same hob by recutting with the hob axis shifted parallel to the worm wheel axis by $\pm \Delta h$. This results in a crowning effect, shown in Figure 4.18.

(c) Hob Axis Inclining $\Delta\theta$ From Standard Position.

In standard cutting, the hob axis is oriented at the proper angle to the worm wheel axis. After that, the hob axis is shifted slightly left and then right, $\Delta\theta$, in a plane parallel to the worm wheel axis, to cut a crown effect on the worm wheel tooth.

This is shown in Figure 4.19. Only method (a) is popular. Methods (b) and (c) are seldom used.

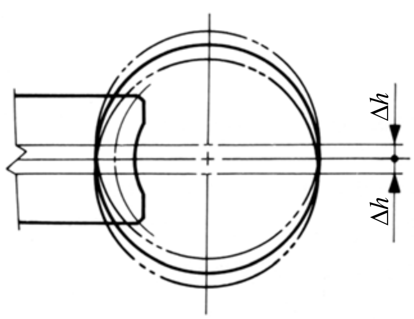


Fig.4.18 Offsetting up or down

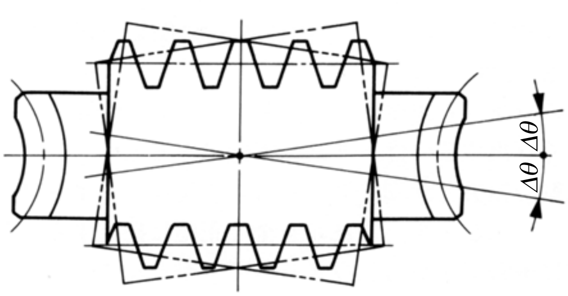


Fig. 4.19 Inclining right or left

(d) Use a Worm with a Larger Pressure Angle than the Worm Wheel.

This is a very complex method, both theoretically and practically. Usually, the crowning is done to the worm wheel, but in this method the modification is on the worm. That is, to change the pressure angle and pitch of the worm without changing base pitch, in accordance with the relationships shown in Equations 4.25:

$$p_x \cos\alpha_x = p_x' \cos\alpha_x' \tag{4.25}$$

In order to raise the pressure angle from before change, α_x', to after change, α_x, it is necessary to increase the axial pitch, p_x', to a new value, p_x, per Equation (4.25). The amount of crowning is represented as the space between the worm and worm wheel at the meshing point A in Figure 4.21. This amount may be approximated by the following equation:

$$\text{Amount of crowning} \fallingdotseq k\,\frac{p_x - p_x'}{p_x'}\,\frac{d_1}{2} \tag{4.26}$$

where d_1 : Reference diameter of worm
k : Factor from Table 4.27 and Figure 4.20
p_x : Axial pitch after change
p_x' : Axial pitch before change

Table 4.27 The value of factor k

α_x	14.5°	17.5°	20°	22.5°
k	0.55	0.46	0.41	0.375

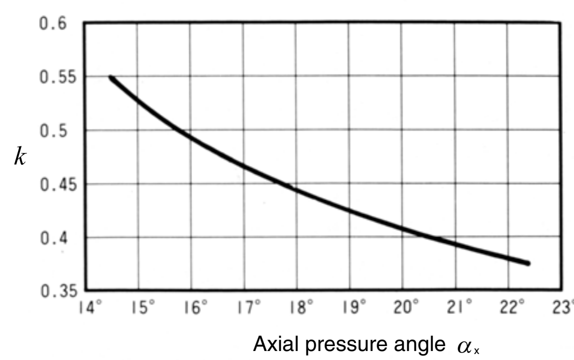


Fig. 4.20 The value of factor (k)

Table 4.28 shows an example of calculating worm crowning.

Table 4.28 The calculation of worm crowning

No.	Item	Symbol	Formula	Example
1	Axial module	m_x'	NOTE: These are the data before crowning.	3
2	Normal pressure angle	α_n'		20°
3	Number of threads of worm	z_1		2
4	Reference diameter of worm	d_1		44.000
5	Reference cylinder lead angle	γ'	$\tan^{-1}\left(\frac{m_x' z_1}{d_1}\right)$	7.765166°
6	Axial pressure angle	α_x'	$\tan^{-1}\left(\frac{\tan\alpha_n'}{\cos\gamma'}\right)$	20.170236°
7	Axial pitch	p_x'	$\pi m_x'$	9.424778
8	Lead	p_z'	$\pi m_x z_1$	18.849556
9	Amount of crowning	C_R	*	0.04
10	Factor	k	From Table 4.27	0.41
After crowning				
11	Axial pitch	p_x	$p_x'\left(\frac{2C_R}{kd_1}+1\right)$	9.466573
12	Axial pressure angle	α_x	$\cos^{-1}\left(\frac{p_x'}{p_x}\cos\alpha_x'\right)$	20.847973°
13	Axial module	m_x	$\frac{p_x}{\pi}$	3.013304
14	Reference cylinder lead angle	γ	$\tan^{-1}\left(\frac{m_x z_1}{d_1}\right)$	7.799179°
15	Normal pressure angle	α_n	$\tan^{-1}(\tan\alpha_x \cos\gamma)$	20.671494°
16	Lead	p_z	$\pi m_x z_1$	18.933146

* It should be determined by considering the size of tooth contact .

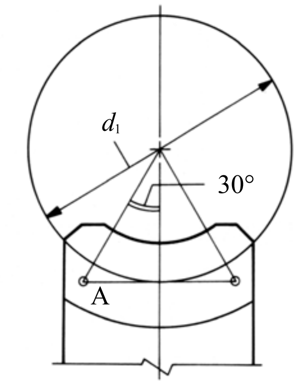


Fig.4.21 Position A is the point of determining crowning amount

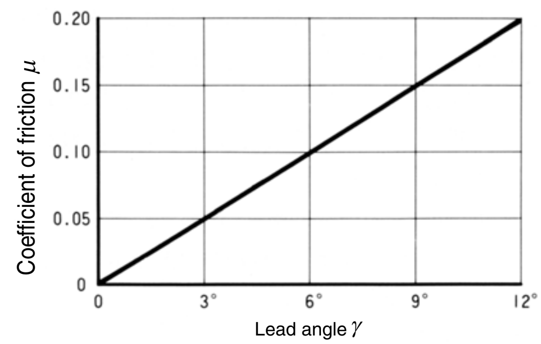


Fig. 4.22 The critical limit of self-locking of lead angle γ and

(4) Self-Locking Of Worm Gear Pair

Self-locking is a unique characteristic of worm meshes that can be put to advantage. It is the feature that a worm cannot be driven by the worm wheel. It is very useful in the design of some equipment, such as lifting, in that the drive can stop at any position without concern that it can slip in reverse. However, in some situations it can be detrimental if the system requires reverse sensitivity, such as a servomechanism.

Self-locking does not occur in all worm meshes, since it requires special conditions as outlined here. In this analysis, only the driving force acting upon the tooth surfaces is considered without any regard to losses due to bearing friction, lubricant agitation, etc. The governing conditions are as follows:

Let F_{t1} = tangential driving force of worm

Then,

$$F_{t1} = F_n(\cos\alpha_n \sin\gamma - \mu\cos\gamma) \qquad (4.27)$$

If $F_{t1} > 0$ then there is no self-locking effect at all. Therefore, $F_{t1} \leq 0$ is the critical limit of self-locking.

Let α_n in Equation (4.27) be 20°, then the condition:

$F_{t1} \leq 0$ will become:

$$(\cos 20° \sin\gamma - \mu\cos\gamma) \leq 0$$

Figure 4.22 shows the critical limit of self-locking for lead angle γ and coefficient of friction μ. Practically, it is very hard to assess the exact value of coefficient of friction μ. Further, the bearing loss, lubricant agitation loss, etc. can add many side effects. Therefore, it is not easy to establish precise self-locking conditions.
However, it is true that the smaller the lead angle γ, the more likely the self-locking condition will occur.