

# RESPOSTAS DOS EXERCÍCIOS PROPOSTOS

## CAPÍTULO 7

### FUNDAMENTOS DE MECÂNICA ANALÍTICA

<b>7.1:</b>	$2m\ddot{x}_1 + c(\dot{x}_1 + \dot{x}_3) + k(2x_1 + x_3 - \ell) = 0;$ $m\ddot{x}_3 + c(\dot{x}_1 + \dot{x}_3) + k(x_1 + x_3 - \ell) - mg = 0$
<b>7.2:</b>	$(m + m_0)\ddot{x} + mL\ddot{\theta}\cos\theta - mL\dot{\theta}^2\sin\theta + c\dot{x} + kx = f(t); L\ddot{\theta} + \ddot{x}\cos\theta + g\sin\theta = 0$
<b>7.3:</b>	$\left[\frac{1}{3}m_h l_h^2 + \frac{3}{2}m_d r_d^2 + m_d l_h(l_h + 2r_d)\right]\ddot{\theta} + k\theta +$ $+\left[\frac{1}{2}m_h l_h + m_d(l_h + r_d)\right]g\sin\theta = \tau(t)$
<b>7.4:</b>	$2m\ddot{x}_1 + c(\dot{x}_1 + \dot{x}_3) + k(2x_1 + x_3 - \ell) = 0;$ $m\ddot{x}_3 + c(\dot{x}_1 + \dot{x}_3) + k(x_1 + x_3 - \ell) - mg = 0$
<b>7.5:</b>	$(m + m_0)\ddot{x} + mL\ddot{\theta}\cos\theta - mL\dot{\theta}^2\sin\theta + c\dot{x} + kx = f(t);$ $L\ddot{\theta} + \ddot{x}\cos\theta + g\sin\theta = 0$
<b>7.6:</b>	$\left[\frac{1}{3}m_h l_h^2 + \frac{3}{2}m_d r_d^2 + m_d l_h(l_h + 2r_d)\right]\ddot{\theta} + k\theta +$ $+\left[\frac{1}{2}m_h l_h + m_d(l_h + r_d)\right]g\sin\theta = \tau(t)$
<b>7.7:</b>	$4m\ddot{x} + 3mL\ddot{\theta}\cos\theta - 3mL\dot{\theta}^2\sin\theta = f(t); \frac{169}{48}L\ddot{\theta} + 3\ddot{x}\cos\theta + 3g\sin\theta = 0$
<b>7.8:</b>	$\left(\frac{1}{2}m_d R^2 + \frac{4}{9}m_h L^2\right)\ddot{\theta} + \frac{1}{3}m_h L^2\ddot{\beta}(\beta - \theta) + \frac{1}{3}m_h L^2\dot{\beta}^2 + kR^2\theta + \frac{2}{3}m_h gL = \tau(t);$ $\frac{1}{3}m_h L^2\ddot{\beta} + \frac{1}{3}m_h L^2\ddot{\theta}(\beta - \theta) - \frac{1}{3}m_h L^2\dot{\theta}^2 + m_h g\frac{L}{2}\beta = 0$
<b>7.9:</b>	$2m\ddot{x}_1 + c(\dot{x}_1 + \dot{x}_3) + k(2x_1 + x_3 - \ell) = 0;$ $m\ddot{x}_3 + c(\dot{x}_1 + \dot{x}_3) + k(x_1 + x_3 - \ell) - mg = 0$

<b>7.10:</b>	$(m + m_0)\ddot{x} + m L \ddot{\theta} \cos \theta - m L \dot{\theta}^2 \operatorname{sen} \theta + c \dot{x} + k x = f(t);$ $L \ddot{\theta} + \ddot{x} \cos \theta + g \operatorname{sen} \theta = 0$
<b>7.11:</b>	$\left[ \frac{1}{3} m_h l_h^2 + \frac{3}{2} m_d r_d^2 + m_d l_h (l_h + 2r_d) \right] \ddot{\theta} + k \theta +$ $+ \left[ \frac{1}{2} m_h l_h + m_d (l_h + r_d) \right] g \operatorname{sen} \theta = \tau(t)$
<b>7.12:</b>	$4 m \ddot{x} + 3 m L \ddot{\theta} \cos \theta - 3 m L \dot{\theta}^2 \operatorname{sen} \theta = f(t); \quad \frac{169}{48} L \ddot{\theta} + 3 \dot{x} \cos \theta + 3 g \operatorname{sen} \theta = 0$
<b>7.13:</b>	$\left( \frac{1}{2} m_d R^2 + \frac{4}{9} m_h L^2 \right) \ddot{\theta} + \frac{1}{3} m_h L^2 \ddot{\beta} (\beta - \theta) + \frac{1}{3} m_h L^2 \dot{\beta}^2 + k R^2 \theta + \frac{2}{3} m_h g L = \tau(t);$ $\frac{1}{3} m_h L^2 \ddot{\beta} + \frac{1}{3} m_h L^2 \ddot{\theta} (\beta - \theta) - \frac{1}{3} m_h L^2 \dot{\theta}^2 + m_h g \frac{L}{2} \beta = 0$
<b>7.14:</b>	$\left( \frac{1}{3} m_1 + m_2 + m \right) L_1^2 \ddot{\theta}_1 + \left( \frac{m_2}{2} + m \right) L_1 L_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) +$ $+ \left( \frac{m_2}{2} + m \right) L_1 L_2 \dot{\theta}_2^2 \operatorname{sen}(\theta_1 - \theta_2) + k_{10} \theta_1 + k_{12} (\theta_1 - \theta_2) + \left( \frac{m_1}{2} + m_2 + m \right) g L_1 \operatorname{sen} \theta_1 = \tau(t);$ $\left( \frac{m_2}{3} + m \right) L_2^2 \ddot{\theta}_2 + \left( \frac{m_2}{2} + m \right) L_1 L_2 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - \left( \frac{m_2}{2} + m \right) L_1 L_2 \dot{\theta}_1^2 \operatorname{sen}(\theta_1 - \theta_2) +$ $+ k_{12} (\theta_2 - \theta_1) + \left( \frac{m_2}{2} + m \right) g L_2 \operatorname{sen} \theta_2 = 0$
<b>7.15:</b>	$m \ddot{x}_1 + k x_1 - m g = \lambda;$ $m \ddot{x}_2 + c (\dot{x}_2 - \dot{x}_3) + k (x_2 - x_3) - m g = \lambda;$ $m \ddot{x}_3 + c (\dot{x}_3 - \dot{x}_2) + k (x_3 - x_2) - m g = 0$
<b>7.16:</b>	$m_0 \ddot{x} + c \dot{x} + k x = f(t) - \lambda_2;$ $L \ddot{\theta} + \ddot{x}_C \cos \theta + \ddot{y}_C \operatorname{sen} \theta + g \operatorname{sen} \theta = 0;$ $\ddot{x}_C + L \ddot{\theta} \cos \theta - L \dot{\theta}^2 \operatorname{sen} \theta = \lambda_2 / m;$ $\ddot{y}_C + L \ddot{\theta} \operatorname{sen} \theta + L \dot{\theta}^2 \cos \theta + m g = \lambda_1 / m$
<b>7.17:</b>	$\frac{1}{2} m_d R^2 \ddot{\theta} + k R^2 \theta = \tau(t) + \frac{2}{3} L (\lambda_1 \operatorname{sen} \theta - \lambda_2 \cos \theta);$ $\frac{1}{3} L \ddot{\beta} + \frac{1}{2} \ddot{x}_B \cos \beta + \frac{1}{2} \ddot{y}_B \operatorname{sen} \beta + \frac{1}{2} g \operatorname{sen} \beta = 0;$ $\ddot{x}_B + \frac{1}{2} L \ddot{\beta} \cos \beta - \frac{1}{2} L \dot{\beta}^2 \operatorname{sen} \beta = \lambda_1 / m_h;$ $\ddot{y}_B + \frac{1}{2} L \ddot{\beta} \operatorname{sen} \beta + \frac{1}{2} L \dot{\beta}^2 \cos \beta + g = \lambda_2 / m_h.$

7.18:

$$\begin{aligned}
 & \frac{1}{3} m_1 L_1^2 \ddot{\theta}_1 + \frac{1}{2} m_1 L_1 \ddot{x}_A \cos \theta_1 + \frac{1}{2} m_1 L_1 \ddot{y}_A \sin \theta_1 + k_{10} \theta_1 + \\
 & + k_{12} (\theta_1 - \theta_2) + m_1 g \frac{L_1}{2} \sin \theta_1 = \tau(t) - L_1 (\lambda_3 \cos \theta_1 + \lambda_4 \sin \theta_1); \\
 & \left( \frac{m_2}{3} + m \right) L_2^2 \ddot{\theta}_2 + \left( \frac{m_2}{2} + m \right) L_2 \ddot{x}_B \cos \theta_2 + \left( \frac{m_2}{2} + m \right) L_2 \ddot{y}_B \sin \theta_2 + \\
 & + k_{12} (\theta_2 - \theta_1) + \left( \frac{m_2}{2} + m \right) g L_2 \sin \theta_2 = 0; \\
 & m_1 \ddot{x}_A + \frac{1}{2} m_1 L_1 \ddot{\theta}_1 \cos \theta_1 - \frac{1}{2} m_1 L_1 \dot{\theta}_1^2 \sin \theta_1 = \lambda_1; \\
 & m_1 \ddot{y}_A + \frac{1}{2} m_1 L_1 \ddot{\theta}_1 \sin \theta_1 + \frac{1}{2} m_1 L_1 \dot{\theta}_1^2 \cos \theta_1 + m_1 g = \lambda_2; \\
 & (m_2 + m) \ddot{x}_B + \left( \frac{m_2}{2} + m \right) L_2 \ddot{\theta}_2 \cos \theta_2 - \left( \frac{m_2}{2} + m \right) L_2 \dot{\theta}_2^2 \sin \theta_2 = \lambda_3; \\
 & (m_2 + m) \ddot{y}_B + \left( \frac{m_2}{2} + m \right) L_2 \ddot{\theta}_2 \sin \theta_2 + \left( \frac{m_2}{2} + m \right) L_2 \dot{\theta}_2^2 \cos \theta_2 + (m_2 + m) g = \lambda_4
 \end{aligned}$$