TRAJETÓRIA/SEM ARRASTO (1) 30 SET 05

BALÍSTICO $\uparrow x$ $\downarrow P$ = peso

$$F = Ma = -P = -Mg \rightarrow a = -g \quad \text{(fase balística)}$$

$$\boxed{M = cte = M_e = \text{massa da estrutura}} \qquad \boxed{D = 0} \text{ arrasto}$$

$$\boxed{P = Mg}$$

PROPULSADO $\uparrow x$ $\uparrow T$ = empuxo $\downarrow P$

$$F = Ma = T - P = T - Mg$$

$$a = \frac{T}{M} - g \qquad \text{(fase propulsada)}$$

$$\boxed{M = \text{variável}}$$

BALÍSTICO $V_b, t_b = \emptyset$ no fim da queima

SUBIDA

$$\frac{dV}{dt} = \boxed{a = -g} \rightarrow \int_{V_b}^{V} dV = -g \int_{t_b}^{t} dt \rightarrow V = V_b - g(t - t_b)$$

No apogeu (H), $\boxed{V_H = 0} \rightarrow 0 = V_b - g(t_H - t_b) \rightarrow \boxed{t_H = t_b + \frac{V_b}{g}}$

Descida: $\int_{V_H}^{V_I} dV = -g \int_{t_H}^{t_I} dt \rightarrow V_I = V_H - g(t_I - t_H) = -g(t_I - t_H)$

Subida:

$$\frac{dh}{dt} = V = V_b - g(t - t_b) \rightarrow \int_{h_b}^{h} dh = \int_{t_b}^{t} [V_b - g(t - t_b)]\, dt$$

$$h = h_b + V_b \int_{t_b}^{t} dt - g \int_{t_b}^{t} t\, dt + g t_b \int_{t_b}^{t} dt = h_b + \left[V_b t - g\frac{t^2}{2} + g t_b t \right]_{t_b}^{t}$$

$$h = h_b + (V_b + g t_b)(t - t_b) - g\frac{(t^2 - t_b^2)}{2}$$

No apogeu (H): $t = t_b + \frac{V_b}{g} \rightarrow H = h_b + (V_b + g t_b)\left(\cancel{t_b} + \frac{V_b}{g} - \cancel{t_b}\right) - \frac{g}{2}\left[\left(t_b + \frac{V_b}{g}\right)^2 - t_b^2\right]$

$$H = h_b + (V_b + g t_b)\frac{V_b}{g} - \frac{g}{2}\left(\cancel{t_b^2} + 2 t_b \frac{V_b}{g} + \frac{V_b^2}{g^2} - \cancel{t_b^2}\right) = h_b + \frac{V_b^2}{g} + \cancel{V_b t_b} - \cancel{V_b t_b} - \frac{V_b^2}{2g}$$

$$\boxed{H = h_b + \frac{V_b^2}{2g}}$$

TRAJETÓRIA / SEM ARRASTO (2) 30 SET 05

BALÍSTICO/DESCIDA

$$\frac{dV}{dt} = \boxed{a = -g} \rightarrow \int_{V_H}^{V} dV = -g\int_{t_H}^{t} dt \longrightarrow V = V_H - g(t - t_H) = -g(t - t_H)$$

No apogeu (H); $V_H = 0 \longrightarrow V = -g(t - t_H)$

$$\frac{dh}{dt} = V = -g(t - t_H) \longrightarrow \int_H^h dh = -\int_{t_H}^{t} g(t - t_H)\,dt \rightarrow h = H + \left[-g\frac{t^2}{2} + g t_H t\right]_{t_H}^{t}$$

$$h = H - g\frac{(t^2 - t_H^2)}{2} + g t_H (t - t_H)$$

No impacto: $h = 0$ e $t = t_I \longrightarrow 0 = H - g\frac{(t_I^2 - t_H^2)}{2} + g t_H (t_I - t_H)$ $\quad \times -1$

$$g\frac{t_I^2}{2} - g\frac{t_H^2}{2} - g t_H t_I + g t_H^2 - H = 0 \quad \times \frac{2}{g}$$

$$t_I^2 - 2t_H t_I + t_H^2 - \frac{2H}{g} = 0 \quad \text{ou} \quad ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 1 \quad b = -2t_H \quad c = t_H^2 - \frac{2H}{g}$$

$$x = \frac{2t_H \pm \sqrt{4t_H^2 - 4\left(t_H^2 - \frac{2H}{g}\right)}}{2} = \frac{2t_H \pm \sqrt{8H/g}}{2} = t_H \pm \sqrt{2H/g}$$

$$\boxed{t_I = t_H + \sqrt{\frac{2H}{g}}}$$ ou ~~$t_I = t_b + \frac{V_b}{g} + \sqrt{\frac{2H}{g}}$~~ $$\boxed{t_I = t_b + \frac{\left(V_b + \sqrt{2gh_b + V_b^2}\right)}{g}}$$

$$V_I = V_H - g(t_I - t_H) = 0 - g\left(t_H + \sqrt{\frac{2H}{g}}\right) + g t_H = -\cancel{g t_H} - g\sqrt{\frac{2H}{g}} + \cancel{g t_H}$$

$$\boxed{V_I = -\sqrt{2gH}} \boxed{= -\sqrt{2gh_b + V_b^2}} \qquad |V_{max}| = V_I$$

$$\boxed{h_I = 0}$$

$$\boxed{t_I = t_b + \frac{(V_b - V_I)}{g}}$$

TRAJETÓRIA / SEM ARRASTO (3) ~~30/~~ 1º OUT 05

PROPULSADO: $a = \frac{T}{M} - g$

$$\frac{dv}{dt} = \boxed{a = \frac{T}{M} - g} \longrightarrow \int_0^V dV = \int_0^t \left(\frac{T}{M} - g\right) dt$$

Com $M = M_e + M_p\left(1 - \frac{t}{t_b}\right)$ [variação linear da massa de propelante]

ou $\boxed{M = M_0 - M_p \frac{t}{t_b}}$ onde $\boxed{M_0 = M_e + M_p}$

$$\int_0^V dV = \int_0^t \left[\frac{T}{\left(M_0 - M_p \frac{t}{t_b}\right)} - g\right] dt$$

Com $Z = M = M_0 - M_p \frac{t}{t_b} \rightarrow dZ = -\frac{M_p dt}{t_b}$

e $dt = -\frac{t_b}{M_p} dZ$

em $t = 0 \rightarrow Z_0 = M_0$ em $t = t \longrightarrow Z = M$

$$\int_0^V dV = \int_{Z_0}^{Z} \left(\frac{T}{Z} - g\right)\left(-\frac{t_b}{M_p} dZ\right) = -\frac{t_b}{M_p}\int_{Z_0}^{Z}\left(\frac{T}{Z} - g\right) dZ = -\frac{t_b}{M_p}\left[T\int_{Z_0}^{Z}\frac{dZ}{Z} - g\int_{Z_0}^{Z} dZ\right]$$

$$V = -\frac{t_b}{M_p}\left[T \ln Z - gZ\right]_{Z_0}^{Z} = -\frac{t_b}{M_p}\left[T(\ln Z - \ln Z_0) - g(Z - Z_0)\right] = -\frac{t_b}{M_p}\left[T \ln\left(\frac{Z}{Z_0}\right) - g(Z - Z_0)\right]$$

$$V = -\frac{t_b}{M_p}\left[T \ln\left(\frac{M_0 - M_p\frac{t}{t_b}}{M_0}\right) - g\left(\cancel{M_0} - M_p\frac{t}{t_b} - \cancel{M_0}\right)\right] = -\frac{t_b}{M_p}\left[T \ln\left(1 - \frac{M_p}{M_0}\frac{t}{t_b}\right) + g\frac{M_p t}{t_b}\right]$$

$$V = -\left[\frac{t_b}{M_p} T \ln\left(1 - \frac{M_p}{M_0}\frac{t}{t_b}\right) + gt\right]$$

Para $t = t_b$:

$$\boxed{V_b = -\left[\frac{t_b}{M_p} T \ln\left(1 - \frac{M_p}{M_0}\right) + g t_b\right]} = \boxed{-\left[\frac{I_t}{M_p}\ln(1 - R) + g t_b\right]}$$

$R = \frac{M_p}{M_0}$

$I_t = t_b T$

$$M(t_b) = M_0 - M_p\frac{t_b}{t_b} = M_0 - M_p = M_e \longrightarrow \boxed{a_b = \frac{T}{M_e} - g}$$

$\boxed{t_b = \text{DADO}}$ $\boxed{\text{DADOS: } T, M_p, M_e, g,} \rightarrow M_0, I_t, I_{sp}$

TRAJETÓRIA / SEM ARRASTO (4) 2 OUT 05

$$\frac{dh}{dt} = V = -\frac{t_b}{M_p} T \ln\left(1 - \frac{M_p}{M_0}\frac{t}{t_b}\right) - gt$$

$$\int_0^h dh = \int_0^t \left[-\frac{t_b}{M_p} T \ln\left(1 - \frac{M_p}{M_0}\frac{t}{t_b}\right) - gt\right] dt$$

$$Z = 1 - \frac{M_p}{M_0}\frac{t}{t_b} \rightarrow dZ = -\frac{M_p}{M_0}\frac{dt}{t_b} \rightarrow dt = -\frac{M_0 t_b}{M_p} dZ$$

$$h = -\frac{t_b}{M_p} T \int_0^Z \ln Z \left(-\frac{M_0 t_b}{M_p} dZ\right) - g\int_0^t t\,dt = +\left(\frac{t_b}{M_p}\right)^2 M_0 T \int_0^Z \ln Z\, dZ - g\frac{t^2}{2}$$

$$h = \left(\frac{t_b}{M_p}\right)^2 M_0 T \left[Z\ln Z - Z\right]_0^Z - g\frac{t^2}{2} = \left(\frac{t_b}{M_p}\right)^2 M_0 T \left(\underbrace{Z\ln Z - Z}_{Z(\ln Z - 1)} - \underbrace{Z_0 \overset{1\times 0}{\ln Z_0}}_{0} + \overset{1}{Z_0}\right) - g\frac{t^2}{2}$$

$$t = 0 \rightarrow Z_0 = 1 \rightarrow [\,] = 1\,\overset{0}{\widehat{\ln 1}} - 1 = -1$$

$$h = M_0\left(\frac{t_b}{M_p}\right)^2 T \left\{\left(1 - \frac{M_p}{M_0}\frac{t}{t_b}\right)\left[\ln\left(1 - \frac{M_p}{M_0}\frac{t}{t_b}\right) - 1\right] + 1\right\} - g\frac{t^2}{2}$$

Para $t = t_b$:

$$\boxed{h_b = M_0 T \left(\frac{t_b}{M_p}\right)^2 \left\{\left(1 - \frac{M_p}{M_0}\right)\left[\ln\left(1 - \frac{M_p}{M_0}\right) - 1\right] + 1\right\} - g\frac{t_b^2}{2}}$$

$$\boxed{h_b = \frac{I_t t_b}{M_p}\left[\left(\frac{1}{R} - 1\right)\ln(1-R) + 1\right] - g\frac{t_b^2}{2}}$$

$$R = \frac{M_p}{M_0}$$

$$I_t = t_b T$$

$$\boxed{h_b = \frac{I_t}{\dot M}\left[1 + \left(\frac{1}{R} - 1\right)\ln(1-R)\right] - g\frac{t_b^2}{2}}$$

$$\dot M = \frac{M_p}{t_b}$$

$$\boxed{h_b = \frac{I_t}{\dot M}\left[1 + \frac{(1-R)}{R}\ln(1-R)\right] - g\frac{t_b^2}{2}}$$

$$R = \frac{M_p}{M_0}$$

TRAJETÓRIA / SEM ARRASTO (5) 9 OUT 05

EXEMPLO: SONDINHA II

DADOS

$M_p = 0.010$ kg, $M_e = 0.020$ kg $\Big\}$ $M_0 = M_p + M_e = 0.030$ kg $\rightarrow R = \frac{M_p}{M_0} = 0.333$

$T = 3$ N, $t_b = 0.7$ s $\Big\}$ $I_t = t_b T = 2.1$ Ns

$g = 9.8$ m/s^2

$\dot{M}_p = \frac{M_p}{t_b} \simeq 0.0143$ kg/s

$I_{sp} = \frac{I_t}{M_p g} \simeq 21.4$ s

FASE PROPULSADA

$t_b = 0.7$ s

$h_b = 25,5$ m

$V_b = 78,3$ m/s $\simeq 282$ km/h

$a_b = 140$ m/s$^2 \simeq 14,3g$

FASE BALÍSTICA ASCENDENTE

$t_H = 8,69$ s

$H = 338$ m

$V_H = 0$

$a_H = -9.8$ m/s$^2 = -1g$

FASE BALÍSTICA DESCENDENTE

$t_I = 17.0$ s

$h_I = 0$ m

$V_I = -81.4$ m/s $\simeq 293$ km/h

$a_I = -9.8$ m/s$^2 = -1g$

EXPERIMENTAL

RESULTADO: $t_I = 10$ s

Desconsiderando a fase propulsada,
($h_b = t_b = 0$)

~~t_I~~ $\boxed{V_b = \frac{t_I}{2} g}$ $V_b = 49$ m/s $= V_I$
(perda 37%)

$\boxed{H = \frac{t_I^2}{8} g}$ $H = 122$ m

$\boxed{t_H = \frac{t_I}{2}}$ $t_H = 5$ s

CONCLUSÃO: Desprezar o arrasto leva a grandes erros nos resultados (H, t_I, V_b etc)