SIMPLIFIED METHOD FOR ESTIMATING THE FLIGHT PERFORMANCE OF A HOBBY ROCKET

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Introduction

As part of the design process for a hobby rocket, it is very useful to be able to **simply** and **quickly** estimate the expected flight performance. **Peak altitude, burnout velocity, burnout altitude, time to apogee, and maximum acceleration** are key flight parameters. As part of the process, studying "what if" scenarios are often useful. i.e. What if mass is reduced? What if rocket were made more aerodynamic (lower drag coefficient, smaller diameter, etc.)? What if a more powerful motor is employed?

To fulfil this need, the following method of *estimating* the flight performance of a rocket was developed.

Three worked examples are provided, as is the theory of the method used.

Method

The first step is to know, estimate, or assume the following basic details of the rocket and motor:

- Motor total impulse
- Motor average thrust
- Propellant mass
- Rocket empty mass
- Rocket maximum diameter
- Rocket drag coefficient

For a given rocket design, the drag coefficient, C_d, can be determined using software such as AEROLAB. Otherwise, for preliminary design purpose, it can be assumed as follows:

1. Smoothly contoured & finely painted rocket with shaped fins, finely finished with a no significant external protrusions, use $C_d = 0.35$
2. Smoothly contoured, painted rocket with shaped fins, with minor external protrusions such as lugs , fin brackets or screw heads, use $C_d = 0.40$
3. Basic rocket with sharp changes in contour and with external protrusions, use $C_d = 0.45$ to 0.50.

The next step is to calculate the"ideal" flight performance, that is, for the condition of **zero aerodynamic drag,** by use of the following equations:

$$z_1 = \frac{1}{2}\left(\frac{F}{m} - g\right) t^2$$ Burnout altitude, zero drag

$$V_1 = \sqrt{\frac{2\,z_1}{m}\,(F - m\,g)}$$ Burnout velocity, zero drag

$$z_2 = \frac{F\ z_1}{m\ g}$$ Peak altitude, zero drag

$$t_2 = t + \sqrt{\frac{2}{g}\left(z_2 - z_1\right)}$$ Time to apogee, zero drag

In reality, aerodynamic drag will reduce the flight performance of the rocket significantly. In order to account for this effect, a **Drag Influence Number** (**N**) is calculated:

$$N = \frac{C_d\ D^2\ V_1^2}{1000\ \ m_d}$$ Drag Influence Number, metric units as shown below

$$N = \frac{C_d\ D^2\ V_1^2}{24353\ \ m_d}$$ Drag Influence Number, English units as shown below

The chart is next used to find the **Drag Reduction Factors** (**fz, fbo, fv, ft**) based on the Drag Influence Number. The ideal (zero drag) values are then multiplied by the appropriate Drag Reduction Factors in order to obtain the flight performance corrected for the effects of drag:

$z_{peak} = f_z\ \ z_2$ Peak altitude, corrected for drag

$z_{burnout} = f_{zbo}\ \ z_1$ Burnout altitude, corrected for drag

$V_{max} = f_v\ \ V_1$ Maximum velocity, corrected for drag

$t_{peak} = f_t\ \ t_2$ Time to apogee, corrected for drag

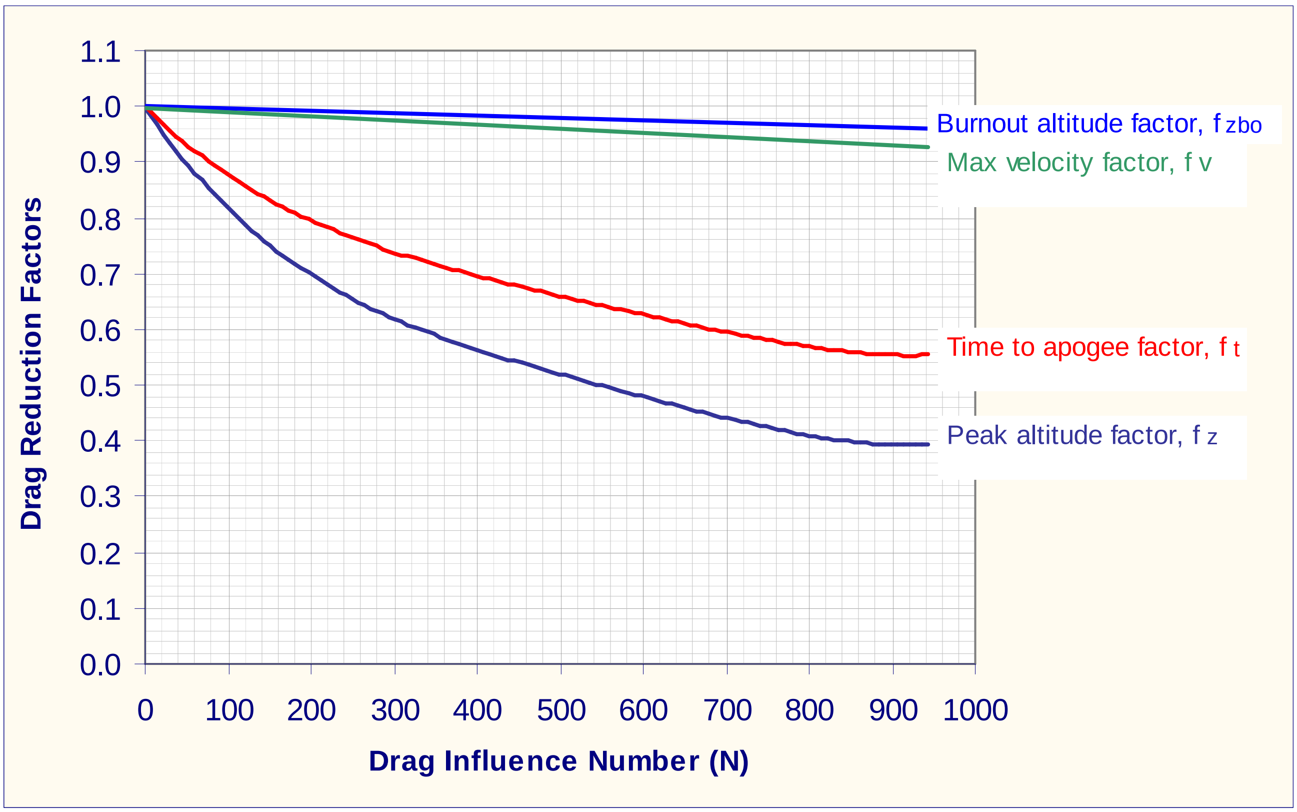
In addition to the above, the rocket **acceleration** can be calculated. Aerodynamic drag does not significantly affect the acceleration of the rocket and as such drag may be neglected.

$$a = \frac{F}{m\ g}$$ Rocket average acceleration

$$a_{max} = \frac{F_{max}}{m_d\ g}$$ Rocket maximum acceleration (conservative)

Definition of terms, units are metric (English):

F = motor average thrust, N (lbf)
m = rocket average mass = $m_d + ½ m_p$ where
m_d = rocket dead mass (i.e. rocket less propellant mass), kg (slugs)
m_p = propellant mass, kg (slugs)
t = motor thrust time, sec. where t = I/F (since Total Impulse, I = F t)
g = acceleration due to gravity= 9.81 metres/sec.= 32.2 ft/sec.
C_d = rocket aerodynamic drag coefficient, dimensionless
D = rocket maximum body diameter, cm (inch.)



Drag reduction factor chart

Example 1:

Consider a rocket of 8 cm. diameter, having a dead mass of 1.5 kg., and a propellant mass of 0.15 kg. The estimated drag coefficient is 0.4.

The H class motor that will power the rocket produces an average thrust of 120 N., and has a total impulse of 180 N-sec.

Estimate the maximum altitude for the rocket, as well as maximum velocity, burnout velocity, and time to apogee.

First, determine the motor thrust time and the average rocket mass:

t = I/F = 180/120 = 1.5 sec.
$m = m_d + \frac{1}{2} m_p$ = 1.5 + ½ (0.15) = 1.575 kg.

Knowing these basic parameters, the ideal burnout altitude is calculated:

$$z_1 = \frac{1}{2}\left(\frac{F}{m} - g\right)t^2$$ Burnout altitude, zero drag

$$z_1 = \frac{1}{2}\left(\frac{120}{1.575} - 9.81\right)(1.5)^2 = 74.7 \text{ m}$$

$$V_1 = \sqrt{\frac{2\,z_1}{m}\,(F - m\,g)}$$ Burnout velocity, zero drag

$$V_1 = \sqrt{\frac{2\,(74.7)}{1.575}\left[120 - 1.575\,(9.81)\right]} = 99.6 \text{ m/sec.}$$

$$z_2 = \frac{F\ z_1}{m\ g}$$ Peak altitude, zero drag

$$z_2 = \frac{120\ (74.7)}{1.575\ (9.81)} = 580 \text{ m}$$

$$t_2 = t + \sqrt{\frac{2}{g}\left(z_2 - z_1\right)}$$ Time to apogee, zero drag

$$t_2 = 1.5 + \sqrt{\frac{2}{9.81}\left(580 - 74.7\right)} = 11.7 \text{ sec.}$$

The Drag Influence Number in next calculated:

$$N = \frac{C_d \ D^2 \ V_1^2}{1000 \ \ m_d}$$ Drag Influence Number, metric units

$$N = \frac{0.4 \ \ (8)^2 \ (99.6)^2}{1000 \ \ (1.5)} = 169$$

From the graph, the drag reduction factors are:

$f_z = 0.73$
$f_{zbo} = 0.99$
$f_v = 0.98$
$f_t = 0.82$

Giving the final estimates for peak altitude, burnout altitude, maximum velocity and time to apogee:

Zpeak **=** 0.73 (580) = **423 m.**	**Peak altitude**
Zburnout **=** 0.99 (74.7) = **74 m.**	**Burnout altitude**
Vmax **=** 0.98 (99.6) = **98 m/sec.**	**Maximum (burnout) velocity**
tpeak **=** 0.82 (11.7) = **9.6 sec.**	**Time to apogee**

Example 2:

Consider a rocket of 4 inch. diameter, having a dead weight of 11.5 pounds, and a propellant weight of 3.2 pounds. The estimated drag coefficient is 0.45

The K class motor that will power this rocket produces an average thrust of 190 lb., and has a total impulse of 418 lbf-sec.

Estimate the maximum altitude for the rocket, as well as maximum velocity, burnout velocity, and time to apogee.

First, determine the motor burn time and the average rocket mass:

t = I/F = 418/190 = 2.2 sec.
Since mass units must be slugs, divide weight by "g":
m_d = 11.5/32.2 = 0.357 slugs
m_p = 3.2/32.2 = 0.099 slugs

$m = m_d + ½\, m_p$ = 0.357 + ½ (0.099) = 0.406 slugs

Knowing these basic parameters, the ideal burnout altitude is calculated:

$$z_1 = \frac{1}{2}\left(\frac{F}{m} - g\right) t^2$$ Burnout altitude, zero drag

$$z_1 = \frac{1}{2}\left(\frac{190}{0.406} - 32.2\right)(2.2)^2 = 1055 \text{ ft.}$$

$$V_1 = \sqrt{\frac{2\, z_1}{m}\,(F - m\,g)}$$ Burnout velocity, zero drag

$$V_1 = \sqrt{\frac{2\,(1055)}{0.406}\,[190 - 0.406\,(32.2)]} = 959 \text{ ft/sec.}$$

$$z_2 = \frac{F\; z_1}{m\; g}$$ Peak altitude, zero drag

$$z_2 = \frac{190\;\;(1055)}{0.406\;\;(32.2)} = 15333 \text{ ft}$$

$$t_2 = t + \sqrt{\frac{2}{g}\,(z_2 - z_1)}$$ Time to apogee, zero drag

$$t_2 = 2.2 + \sqrt{\frac{2}{32.2}\left(15333 - 1055\right)} = 32.0 \text{ sec.}$$

The Drag Influence Number in next calculated:

$$N = \frac{C_d \ D^2 \ V_1^2}{24353 \ m_d}$$ Drag Influence Number, English units

$$N = \frac{0.45 \ (4)^2 \ (959)^2}{24353 \ (0.357)} = 761$$

From the graph, the drag reduction factors are:

$f_z = 0.40$
$f_{zbo} = 0.96$
$f_v = 0.94$
$f_t = 0.56$

Giving the final estimates for peak altitude, burnout altitude, maximum velocity and time to apogee:

$\mathbf{Z}_{peak}$ **=** 0.40 (15333) = **6133 ft.**	**Peak altitude**
$\mathbf{Z}_{burnout}$ **=** 0.96 (1055) = **1013 ft.**	**Burnout altitude**
$\mathbf{V}_{max}$ **=** 0.94 (959) = **901 ft/sec.**	**Maximum (burnout) velocity**
$\mathbf{t}_{peak}$ **=** 0.56 (32.0) = **17. 9 sec.**	**Time to apogee**

Example 3:

Consider a model rocket of 2.54 cm. diameter, having a basic mass of 65 grams and powered by a C6-5 motor. The assumed drag coefficient is 0.40

From the manufacturer specification sheet, the C6-5 motor that will power this rocket produces a total impulse of 10 N-sec and has a burn time of 1.6 seconds. The loaded mass of the motor is 25.8 grams of which 12.5 grams is propellant.

Estimate the maximum altitude for the rocket, as well as maximum velocity, burnout velocity, and time to apogee.

Since the motor total impulse and burn time are provided, the average thrust may be calculated:

F = I/t = 10/1.6 = 6.25 N.

Before calculating the average mass of the rocket, the motor empty mass must be determined and added to the rocket basic mass:

m_d = 65 + (25.8 – 12.5) = 78.3 grams, or 0.0783 kg.
m = m_d + ½ m_p = 78.3 + ½ (12.5) = 84.6 grams, or 0.0846 kg.

Knowing these basic parameters, the ideal burnout altitude is calculated:

$$z_1 = \frac{1}{2}\left(\frac{F}{m} - g\right)t^2$$ Burnout altitude, zero drag

$$z_1 = \frac{1}{2}\left(\frac{6.25}{0.0846} - 9.81\right)(1.6)^2 = 82.0 \text{ m}$$

$$V_1 = \sqrt{\frac{2\,z_1}{m}\left(F - m\,g\right)}$$ Burnout velocity, zero drag

$$V_1 = \sqrt{\frac{2\,(82.0)}{0.0846}\left[6.25 - 0.0846\,(9.81)\right]} = 102.5 \text{ m/sec.}$$

$$z_2 = \frac{F\,z_1}{m\,g}$$ Peak altitude, zero drag

$$z_2 = \frac{6.25\;(82.0)}{0.0846\;(9.81)} = 617.5 \text{ m}$$

$$t_2 = t + \sqrt{\frac{2}{g}(z_2 - z_1)}$$ Time to apogee, zero drag

$$t_2 = 1.6 + \sqrt{\frac{2}{9.81}(617.5 - 82.0)} = 12.0 \text{ sec.}$$

The Drag Influence Number in next calculated

$$N = \frac{C_d \ D^2 \ V_1^2}{1000 \ m_d}$$ Drag Influence Number, metric units

$$N = \frac{0.4 \ (2.54)^2 \ (102.5)^2}{1000 \ (0.0783)} = 346$$

From the graph, the drag reduction factors are:

f_z = 0.58
f_{zbo} = 0.98
f_v = 0.97
f_t = 0.71

Giving the final estimates for peak altitude, burnout altitude, maximum velocity and time to apogee:

Zpeak **=** 0.58 (617.5) = **358 m.**	**Peak altitude**
Zburnout **=** 0.98 (82.0) = **80 m.**	**Burnout altitude**
Vmax **=** 0.97 (102.5) = **99 m/sec.**	**Maximum (burnout) velocity**
tpeak **=** 0.71 (12.0) = **8.5 sec.**	**Time to apogee**

Note that the delay element for this motor provides a nominal delay of 5 seconds. As such, the time to peak (parachute deployment) would be 1.6 + 5 = 6.6 seconds, and the rocket would not achieve the calculated apogee.

Theory of method

The method is based largely on the important principle of **Conservation of Energy**. Energy can be defined as the capacity for doing work. It may exist in a variety of forms and may be transformed from one type of energy to another. However, these energy transformations are constrained by this fundamental principle. One way to state this principle is to say that the total energy of an isolated system remains constant

Work = Force × distance, or W = F d
Kinetic Energy = ½ mass × velocity squared, or $KE = ½ mV^2$
Potential Energy = mass × gravitational acceleration × height, or $PE = m g z$

Neglecting the effect of air resistance (aerodynamic drag), the work performed by the rocket motor is equal to the rocket's potential energy at apogee:

W = *PE*, or
$F (z_1 - z_o) = m g (z_2 - z_o)$

where zo = ground (datum) level, taken to be zero
z_1 = altitude at motor burnout
z_2 = peak height (apogee)
F = motor average thrust
m = rocket average mass. *See note [1]*
Since zo = 0, simplifying gives
$F z_1 = m g z_2$

Rearranging gives the expression for Peak Altitude for the condition of zero drag:

$$z_2 = \frac{F\ z_1}{m\ g}$$ Peak Altitude, zero drag

The ideal (zero drag) altitude at motor burnout may be determined from the basic equations of motion

$$F = m (a + g)$$

which may be rearranged to give the expression for acceleration of the rocket

$$a = \frac{F}{m} - g$$

Also, from the equations of motion

$$z_1 = \frac{1}{2} a t^2 + v_o t + z_o$$

With V_o and Z_o taken as being zero, the expression simplifies to

$$z_1 = \frac{1}{2} a\, t^2$$

Substituting in the expression for acceleration gives the expression for Burnout Altitude for the condition of zero drag:

$$z_1 = \frac{1}{2}\left(\frac{F}{m} - g\right) t^2$$ Burnout Altitude, zero drag

where t is the motor burn time.

The burnout velocity, which corresponds to the maximum velocity of the rocket, may also be determined for the condition of zero drag, since:

Work performed by motor = Rocket's Kinetic Energy at burnout + Rocket's Potential Energy at burnout

or, W = *KE* + *PE*, where work is given by Force × Distance

$$F\, z_1 = \frac{1}{2} m\, V_1^2 + m\, g\, z_1$$

Rearranging the equation in terms of burnout velocity, V_1:

$$V_1 = \sqrt{\frac{2\, z_1}{m} (F - m\, g)}$$ Maximum (burnout) Velocity

To derive the expression for Time to reach Apogee (Peak), the equation given earlier for altitude as a function of acceleration and time can be modified to determine the delta altitude from burnout to peak:

$$z_2 - z_1 = \frac{1}{2} g\, (t_2 - t_1)^2$$ where **g** is substituted for **a**

Rearranging to solve for time to peak

$$t = t_b + \sqrt{\frac{2}{g} (z_2 - z_1)}$$ Time to Apogee

where t_b = motor burn time

To account for losses in flight performance due to aerodynamic drag (work is performed to overcome drag force), *drag reduction factors* are applied to these ideal (zero drag) values. Drag reduction factors are obtained from the figure, which plots these factors against a *Drag Influence Number* (*N*). The Drag Influence Number is a convenient parameter that relates the degree to which drag can be expected to adversely affect a given rocket's flight performance.

The drag force acting on a subsonic rocket is given by

$$F_d = \frac{1}{2} \rho C_d A V^2$$

where
F_d = drag force
ρ = air mass density
C_d = drag coefficient of rocket vehicle
A = reference cross-section area upon which Cd is based, usually nose cone base area
V = rocket velocity

The drag force tends to decelerate the rocket, or in other words, tends to accelerate the rocket downward.

$a = F_d/m$, or

$$a = \frac{\rho C_d A V^2}{2 m}$$

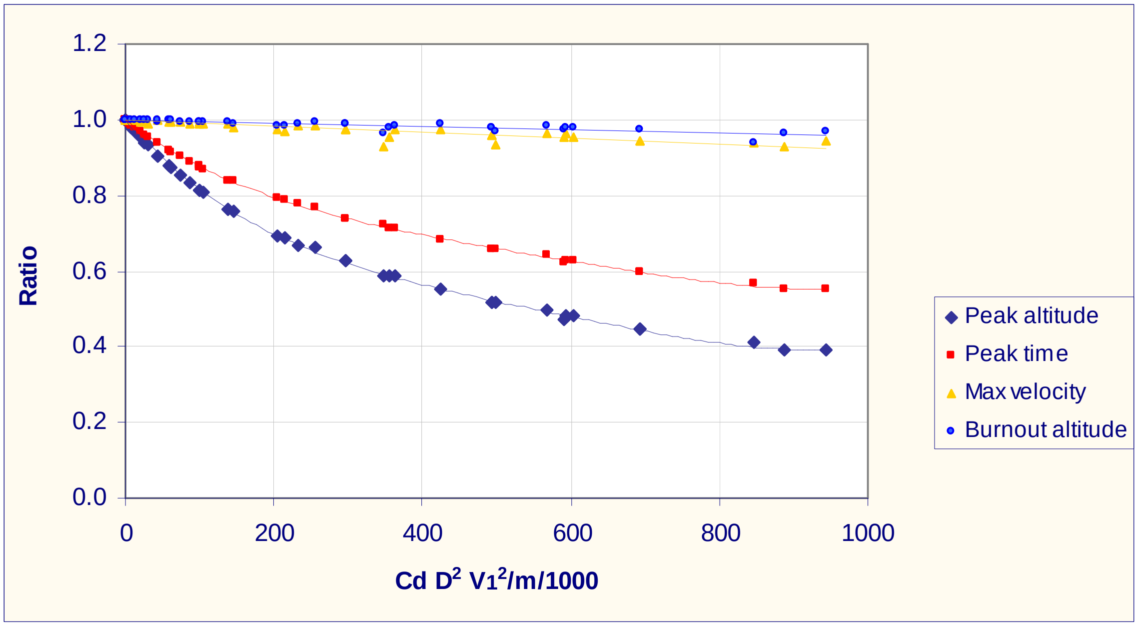
Making the simplifying assumption that air density is a constant, and noting that the base area is proportional to base diameter squared gives

$$a \propto \frac{C_d D^2 V^2}{m}$$

For convenience, the value of the right hand side of the proportionality sign is divided by 1000, when using conventional metric units (m-k-s system). The appropriate velocity is the maximum (burnout) velocity, V_1, and the appropriate mass is the “dead mass” (since most of the flight occurs following burnout).
The result is referred to as the *Drag Influence Number* , N

$$N = \frac{C_d \; D^2 \; V_1^2}{1000 \; m_d}$$

For a given value of N, it may be expected that the flight parameters of interest (peak altitude, time to apogee, maximum velocity, and burnout velocity) will be reduced to a certain degree, compared to the ideal (zero drag) values, and by a consistent factor. This was confirmed by running a large number of simulations (using SOAR software) with various rocket configurations over a range of $0 < N < 900$. The resulting curves were best-fit using suitable mathematical functions. The results showed little scatter for peak altitude and peak time ratios. Some scatter was seen for maximum velocity and burnout altitude ratios. This was likely a result of the similifying assumption regard rocket mass variation during the powered phase of flight. The results of this exercise are graphed in the following figure.



Range of rocket configurations used in simulations

Motor class range: A to K (1.3 to 2000 N-s.)
Rocket dead mass: 0.04 to 8 kg. (0.1 to 18 lb.)
Rocket diameter: 2.5 to 15 cm (1 to 6 inch)

Outside this range, accuracy of results is uncertain.

Notes:
[1] This is a simplifying assumption, since in reality the mass of the rocket is not constant. It has been found that using the average mass of the rocket provides a value of z_2 very close to that obtained when variable mass (due to propellant consumption) is taken into account.