SHOW THAT THE EXPRESSION FOR ISENTROPIC EXPONENT "m" IN THESIS (Pg C-2) IS SAME AS FORM GIVEN IN ARS JOURNAL (MAY 1962 P 663) IF

① — $m = \dfrac{k'-1}{k'}$ i.e. $\dfrac{T_e}{T_0} = \left(\dfrac{P_e}{P_0}\right)^m$ $\left(\text{FOR A GAS } \dfrac{T_e}{T_0} = \left(\dfrac{P_e}{P_0}\right)^{\frac{k-1}{k}}\right)$

② — $m = \dfrac{R}{\left(\frac{X}{1-X}\right)C_s + C_p}$ (Ref thesis page C-2)

③ — $k' = k\left[\dfrac{1 + \Psi\frac{C_s}{C_p}}{1 + k\,\Psi\frac{C_s}{C_p}}\right]$ (Ref. ARS JOURNAL MAY 62 page 663)

WHERE $\Psi = \dfrac{W_p}{W_g}$

∴ FROM ① and ②

$$\frac{k'-1}{k'} = \frac{R}{\left(\frac{X}{1-X}\right)C_s + C_p}$$

$$= \frac{R}{\left(\frac{1}{\frac{1}{X}-1}\right)C_s + C_p}$$

Since $X = \dfrac{\Psi}{1+\Psi}$ since $X = \dfrac{W_p}{W_g + W_p}$

$$\frac{k'-1}{k'} = \frac{R}{\frac{C_s}{\frac{1+\Psi}{\Psi}-1} + C_p} = \frac{R}{\Psi C_s + C_p}$$

re-arrange

$$(\Psi C_s + C_p)(k'-1) = k'R$$

Since $R = C_p\left(\dfrac{k-1}{k}\right)$ $\left(\text{from } k = \dfrac{C_p}{C_p - R}\right)$

$$(\psi C_s + C_p)(k'-1) = k' C_p\left(\frac{k-1}{k}\right) \checkmark$$

DIVIDE BOTH SIDES BY C_p:

$$\left(\psi \frac{C_s}{C_p} + 1\right)(k'-1) = k'\left(\frac{k-1}{k}\right) \checkmark$$

$$k'\left(1 + \psi\frac{C_s}{C_p}\right) - \left(1 + \psi\frac{C_s}{C_p}\right) = k'\left(\frac{k-1}{k}\right) \checkmark$$

$$k'\left(1 + \psi\frac{C_s}{C_p}\right) - k'\left(\frac{k-1}{k}\right) = 1 + \psi\frac{C_s}{C_p} \checkmark$$

$$k'\left(1 + \psi\frac{C_s}{C_p}\right) - k'\left(1 - \frac{1}{k}\right) = 1 + \psi\frac{C_s}{C_p} \checkmark$$

$$k'\left(\psi\frac{C_s}{C_p} + \frac{1}{k}\right) = 1 + \psi\frac{C_s}{C_p} \checkmark$$

$$\therefore \quad k' = \frac{1 + \psi\frac{C_s}{C_p}}{\frac{1}{k} + \psi\frac{C_s}{C_p}} \checkmark$$

$$\Rightarrow \boxed{k' = k\left(\frac{1 + \psi\frac{C_s}{C_p}}{1 + k\psi\frac{C_s}{C_p}}\right)} \checkmark$$

SAME AS ③

THIS SHOWS THAT THE GAS-PARTICLE MIXTURE BEHAVES LIKE A GAS WITH A MODIFIED ISENTROPIC EXPONENT!