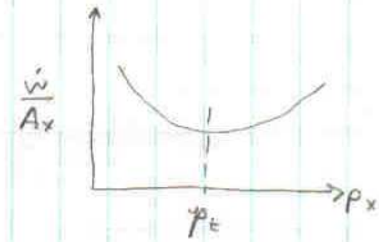
# Derivation of Equation (3-20)

$$\boxed{\frac{P_t}{P_1} = \left(\frac{2}{k+1}\right)^{k/k-1}} \quad (3\text{-}20)$$

The maximum gasflow per unit area occurs at the throat, and the gas pressure corresponding to this, $p_t$, exists and is unique. $P_t$ found by differentiating equation (3-19) and setting derivative $\left(\frac{d\frac{\dot{w}}{A_x}}{dp_x}\right)$ equal to zero.




$$\dot{w} = A_x \frac{p_1}{R}\sqrt{2gJ}\left\{\frac{c_p}{T_1}\left[\left(\frac{p_x}{p_1}\right)^{2/k} - \left(\frac{p_x}{p_1}\right)^{\frac{k+1}{k}}\right]\right\}^{1/2} \quad (3\text{-}19)$$

Clearly, $\frac{d\frac{\dot{w}}{A_x}}{dp_x}$ can be equal to zero only when

$$\frac{d}{dp_x}\left[\left(\frac{p_x}{p_1}\right)^{2/k} - \left(\frac{p_x}{p_1}\right)^{\frac{k+1}{k}}\right] = 0$$

or when $\quad \frac{2}{k}\left(\frac{p_x}{p_1}\right)^{\frac{2-k}{k}} - \frac{k+1}{k}\left(\frac{p_x}{p_1}\right)^{1/k} = 0 \quad$ where $p_x = p_t$

$$\Rightarrow \quad \frac{2}{k}\left(\frac{p_t}{p_1}\right)^{\frac{2-k}{k}} = \frac{k+1}{k}\left(\frac{p_t}{p_1}\right)^{1/k}$$

$$\frac{2}{k+1}\left(\frac{p_t}{p_1}\right)^{\frac{2-k}{k}} = \left(\frac{p_t}{p_1}\right)^{1/k}$$

$$\frac{2}{k+1} = \left(\frac{p_t}{p_1}\right)^{\frac{k-1}{k}}$$

$$\Rightarrow \quad \boxed{\frac{P_t}{P_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}}$$