13 Feb 93

Eqn (5-32) p. 147 SUTTON

ENERGY REQ'D TO BRING UNIT MASS INTO SATELLITE ORBIT

$$E = \underbrace{\tfrac{1}{2} V_s^2}_{①} + \underbrace{\int_{R_0}^{R} g \, dR}_{②}$$

TERM ① – see eqn. 5-30 $= \frac{1}{2} R_0^2 \frac{g_0}{R_0+h}$ ✓

TERM ②

$$\int_{R_0}^{R} g \, dR = \int_{R_0}^{R} g_0 \frac{R_0^2}{R^2} \, dR \quad \text{(see eqn - 5-7)}$$

$$= g_0 R_0^2 \int_{R_0}^{R} \frac{1}{R^2} \, dR = g_0 R_0^2 \left[-\frac{1}{R} \right]_{R_0}^{R}$$

$$= g_0 R_0^2 \left(-\frac{1}{R} + \frac{1}{R_0} \right)$$

since $R = R_0 + h$

$$= g_0 R_0^2 \left(-\frac{1}{R_0+h} + \frac{1}{R_0} \right)$$

$$= g_0 R_0 - \frac{g_0 R_0^2}{R_0+h}$$

combine terms ① and ②

$$E = \frac{1}{2} R_0^2 \frac{g_0}{R_0+h} + g_0 R_0 - \frac{g_0 R_0^2}{R_0+h}$$

$$= -\frac{1}{2} R_0^2 \frac{g_0}{R_0+h} + g_0 R_0$$

$$= \frac{-\frac{1}{2} R_0^2 g_0 + g_0 R_0 (R_0+h)}{R_0+h}$$

$$= \frac{-\frac{1}{2} R_0^2 g_0 + \frac{1}{2} g_0 R_0 (2R_0+2h)}{R_0+h}$$

②

$$= \frac{1}{2} R_0 g_0 \frac{(-R_0 + 2R_0 + 2h)}{R_0+h}$$

$$\boxed{E = \frac{1}{2} R_0 g_0 \frac{R_0 + 2h}{R_0 + h}}$$

EQN. (5-32)