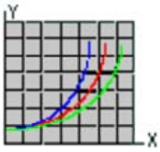
Richard Nakka's *Experimental Rocketry* Web Site



Solid Rocket Motor Theory -- Two-phase Flow

Two-phase Flow

Most solid rocket propellants produce combustion products that are a mixture of gases and *condensed-phase* particles (either liquid or solid) which is evident as visible *smoke* in the exhaust plume. Those propellants containing metals, such as aluminum or magnesium, generate oxides of the metals as condensed-phase combustion products. Metallic-compound oxidizers, such as potassium nitrate (KN) or potassium perchlorate (KP), generate condensed-phase products of particularly high molecular weight, which is rather undesirable. The KN-Sugar propellants produce a dense white cloud of potassium carbonate smoke. In fact, approximately 44% of the exhaust mass is solid matter!

The occurrence of solids or liquids in the exhaust leads to a reduction in performance for a number of reasons:

- This portion of the combustion mass cannot perform any expansion work and therefore does not contribute to acceleration of the exhaust flow.
- The higher effective molecular weight of these products lowers the Characteristic Velocity (c*).
- Due to thermal inertia, the heat of the condensed-phase is partly ejected out of the nozzle before transferring this heat to the surrounding gases, and is therefore not converted to kinetic energy. This is known as **particle thermal lag**.
- Likewise due to the relatively large mass of the particles (compared to the gases), these cannot accelerate as rapidly as the surrounding gases, especially in that portion of the nozzle where flow acceleration is extremely high (throat region). Acceleration of the particles depends upon frictional drag in the gasflow, which necessitates a differential velocity. The net result is that the condensed-phase particles exit the

nozzle at a lower velocity than the gases. This is referred to as **particle velocity lag**.

In terms of the rocket performance parameters, the presence of condensed-phase products is reflected in a reduced Characteristic Velocity, due to the higher effective molecular weight of the gas/particle mixture.
The ideal Thrust Coefficient, C_f, on the other hand, is enhanced with increasing particle fraction, a consequence of a reduced k value. However, the delivered C_f suffers significantly, due to thermal lag and velocity lag. This is probably the largest single efficiency loss experienced by a motor with a significant fraction of particles in the exhaust. Such is especially true with a an underexpanded nozzle (e.g. divergent portion undersized). The apparent importance of having a good divergent portion of the nozzle is clear by examining Figure 3, which shows the variation of ideal Thrust Coefficient for flow through the Kappa rocket motor nozzle. The nozzle is designed with a near-ideal expansion ratio of $A_e/A_t = 11.4$, which gives an ideal Thrust Coefficient of 1.69. However, if the nozzle had been truncated at the throat (red dashed line), the coefficient would only be 0.62. The divergence therefore factors up the ideal thrust by a factor of 2.73 ! Of course, the "delivered" values are probably less pronounced than the ideal values. The delivered Thrust Coefficient for this motor is about 1.5. It is not known what the delivered Thrust Coefficient would be for a truncated nozzle, as this configuration has not been tested.

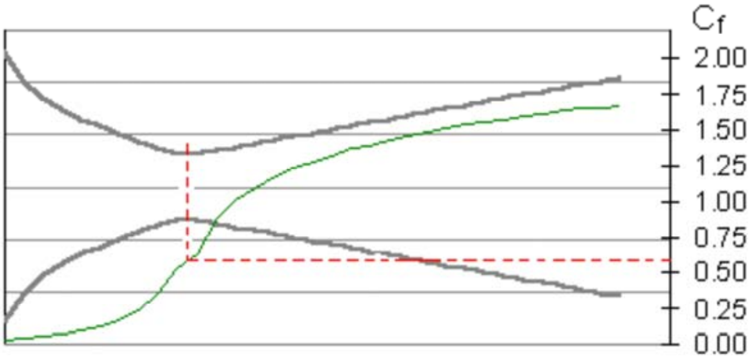


Figure 3 -- Ideal C_f for two-phase flow through Kappa nozzle

Another factor that is important with regard to two-phase flow losses is the *nozzle contour*, especially at the throat region. Figure 4 illustrates the flow acceleration for the Kappa nozzle. The acceleration in the region of the throat (red dashed line) is extremely high, especially just aft, where it is maximum. Most of the particle lag, which is a strong function of acceleration, occurs in this region, thus the importance of designing a nozzle with a well-rounded contour at the throat, without any sharp changes in cross-section.

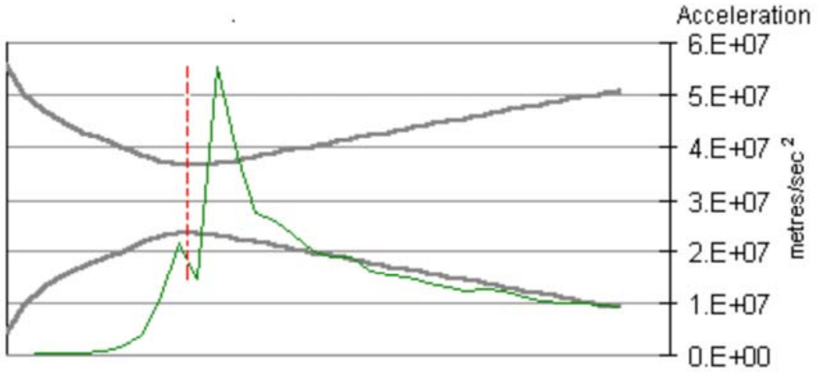


Figure 4 -- Gas/particle acceleration for two-phase flow through Kappa nozzle

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The *size of the rocket motor* as well as *condensed-phase particle size* both play an important role with regard to the influence of two-phase flow effects. This is illustrated in Figure 5, which plots the fraction of Characteristic Velocity loss with respect to:

- Motor size (thrust)
- Particle size

Note that the mass fraction of particles in the exhaust for this study was X = 0.25. For the standard KN-Sugar propellants, X = 0.44.

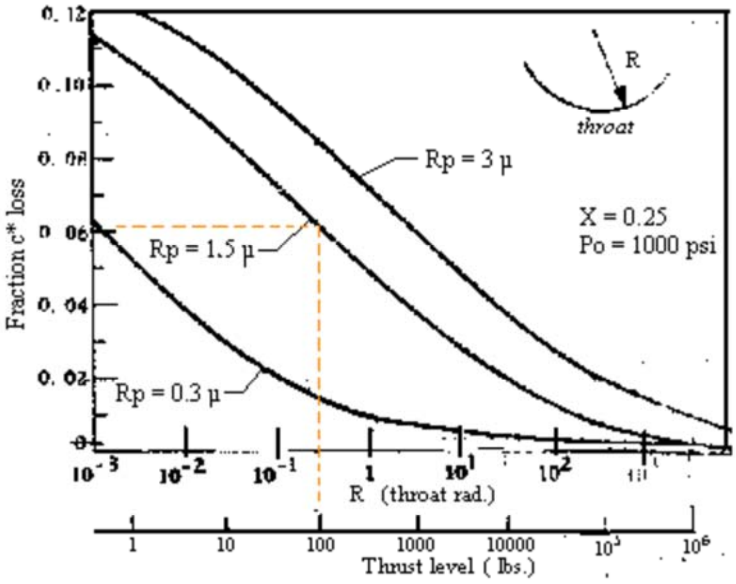

Figure 5 -- Influence of motor size and particle size on c*
Ref. Dynamics of Two-Phase Flow in Rocket Nozzles, ARS Journal, Dec.1962

For example, for a 100 lb. thrust motor, the motor suffers a 6% loss in Characteristic Velocity if the average particle size is 1.5 micron, as shown by the red dashed line.

It is clear from this plot that for amateur experimental motors, which are typically of 1000 lb. thrust or less, that two-phase flow losses can be significant, but can probably be disregarded for large "professional" motors.

How is two-phase flow taken into account with regard to motor performance calculations, such as those presented in the preceding Theory Pages? I asked myself that very question when I began researching the KN-Sucrose propellant from a theoretical performance aspect, back in 1983 when I began work on my B.Sc. thesis *Solid Propellant Rocket Motor Design and Testing*. All the equations in Sutton and other textbooks seemed to ignore the existence of particles in the exhaust, but I knew that I could not ignore this, not when the propellant exhaust contains 44% solid matter! After much consternation, I eventually managed to find a couple of books, and in particular, ARS Journal articles, that touched on this topic. I ended up re-deriving all the pertinent performance equations from basic principles, then made the necessary modifications to account for the presence of condensed-phase. A key assumption required was that the particles flow at the same velocity as the gas (i.e. no velocity lag), so the modified equations represent an upper limit on performance. The details are too involved to present here, so I will only present the final outcome, which fortunately, is quite simple. As it turns out, the gas-particle mixture behaves like a gas with a modified isentropic exponent, *k*. All the fundamental equations remain the same and are fully applicable to two-phase flow, with the only modifications being:

1. **Molecular Weight**, *M*, must take into account the presence of the condensed-phase by calculating the *effective Molecular Weight*, which is obtained by dividing the system mass by the number of moles of *gas* in the system.
   For example, if the system mass is 100 grams and the number of gas moles is 2.3819, then:

   $$M = \frac{100}{2.3819} = 41.98 \ \text{g/mole}$$

2. The modified **isentropic exponent** takes two forms, one for conditions where flow velocity (or actually, acceleration) is low, and the other for conditions of flow with high acceleration. Where flow acceleration is low, such as in the combustion chamber,

   $$\mathbf{k} = \frac{Cp_{mix}}{Cp_{mix} - R'} \qquad \text{equation 1}$$

   where Cp_{mix} is the *effective specific heat of the gas & solid mixture* and *R*' is the universal gas constant. The method of calculating Cp_{mix} is provided in the Technical Notepad Web Page.
   This is the form of *k* to be used, therefore, when calculating **chamber pressure** and **Characteristic Velocity**.

Where flow velocity and acceleration are high, that is, in the nozzle:

$$k = k'\left[\frac{1 + \psi \dfrac{Cs}{Cp_{gas}}}{1 + k'\psi \dfrac{Cs}{Cp_{gas}}}\right] \qquad \text{equation 2}$$

where *k'* is the isentropic exponent for the *gas only mixture* , $\psi = X/(1-X)$, where X is the mass fraction of particles in the exhaust. *Cs* is the specific heat for the solid (or liquid) mixture in the exhaust, and Cp_{gas} is the specific heat for the gas only mixture.

The derivation of this form of modified isentropic exponent assumes a frozen flow condition where no thermal or velocity particle lag is assumed to exist, and is based on the momentum and energy equations for steady isentropic flow. Additional details on the calculation of this modified isentropic exponent may be found in the Technical Notepad Web Page and in the ARS Journal article "*Recent Advances in Gas-Particle Nozzle Flows*", R.F. Hoglund, May 1962.
This is the form of *k* to be used, therefore, when calculating **Exhaust velocity, Thrust, Thrust Coefficient** and the other nozzle flow parameters.

For those interested in more of the theoretical treatment of two-phase flow, I'd suggest perusing *Solid Propellant Rocket Motor Design and Testing* which is available for downloading in PDF format.

Next -- Corrections for Actual Motors

Last updated August 19, 2001

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