Richard Nakka's *Experimental Rocketry* Web Site

Rocket Theory Appendices

Appendix A

Example -- Calculation of the Adiabatic Flame Temperature (AFT) of KN/Sucrose, 65/35 O/F ratio

Consider the combustion of the KN/Sucrose , 65/35 O/F propellant to have the following combustion equation:

$$C_{12}H_{22}O_{11} + 6.288\ KNO_3 \rightarrow 3.796\ CO_2 + 5.205\ CO + 7.794\ H_2O + 3.065\ H_2 + 3.143\ N_2 + 2.998\ K_2CO_3 + 0.274\ KOH$$

The enthalpies of formation for the reactants are obtained from the CRC Handbook of Chemistry and Physics, and for the products, from the JANAF thermochemical tables: (units are kJ/mole)

$C_{12}H_{22}O_{11}$	-2222.1
KNO_3	-494.63
CO_2	-393.52
CO	-110.53
H_2O	-241.83
H_2	0
N_2	0
K_2CO_3	-1150.18

KOH	-424.72

Using the energy balance equation (assuming no changes in K.E. or P.E.):

$$\sum_{R} n_i\left[h_f + \Delta h\right]_i \;=\; \sum_{P} n_e\left[h_f + \Delta h\right]_e$$

Substituting in the values for h_f, n_i and n_e gives :

$1\,(-2222.10 + 0) + 6.288(-494.63 + 0) \;=\; 3.796(-393.52 + \Delta h_{CO2}) + 5.205(-110.53 + \Delta h_{CO}) + 7.794(-241.83 + \Delta h_{H2O}) + 3.065(0 + \Delta h_{H2}) + 3.143\,(0 + \Delta h_{N2}) + 2.998\,(-1150.18 + \Delta h_{K2CO3}) + 0.274\,(-424.72 + \Delta h_{KOH})$

Expanding and gathering terms simplifies the equation to the following form:

$2186.2 \;=\; 3.796\Delta h_{CO2} + 5.205\,\Delta h_{CO} + 7.794\,\Delta h_{H2O} + 3.065\,\Delta h_{H2} + 3.143\,\Delta h_{N2} + 2.998\,\Delta h_{K2CO3} + 0.274\,\Delta h_{KOH}$

Solution of the equation is obtained by simply substituting in values for Δh at a certain temperature. This temperature is equal to the AFT when the the right hand side of the equation is equal to the left hand side (=2186.2).

Take a guess that the AFT lies somewhere between 1700 K and 1800 K (easy for me to guess, as I know the answer! But no matter what the guess, the answer will eventually converge).

From the JANAF tables, the values of Δh are: (units are kJ/mole)

T	CO_2	CO	H_2O	H_2	N_2	K_2CO_3	KOH
1700 K	73.480	45.945	57.758	42.835	45.429	280.275	116.505
1800 K	79.431	49.526	62.693	46.169	48.978	301.195	124.815

For the term on the right side of the equation, substituting in the values at T=1700K :

3.796 (73.480) + 5.205 (45.945) + 7.905 (57.758) + 3.065 (42.835) + 3.143 (45.429) + 2.998 (280.275) + 0.274 (116.505) = **2114.5 kJ/mole**

Substituting in the values at T=1800 K:

3.796 (79.431) + 5.205 (49.526) + 7.794 (62.693) + 3.065 (46.169) + 3.143 (48.978) + 2.998 (301.195) + 0.274 (124.815) = **2280.6 kJ/mole**

Clearly, the actual temperature lies in between 1700 and 1800 K. The actual value may be found by using linear interpolation:

$$T_{AFT} = \frac{2186.2 - 2114.5}{2280.6 - 2114.5}(1800 - 1700) + 1700 = \mathbf{1743}\ \mathrm{K}$$

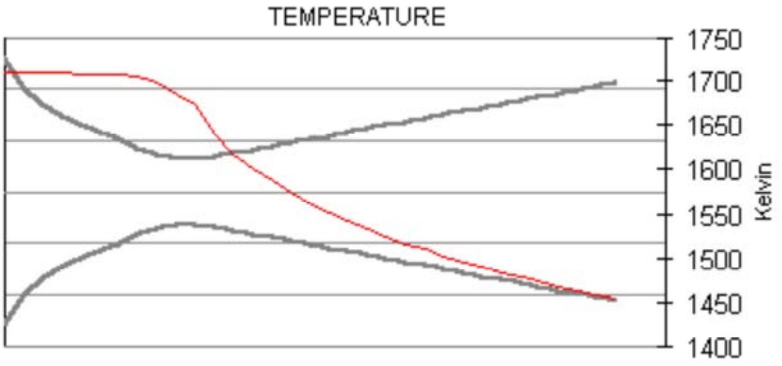
This is in close agreement with the combustion temperature predicted by GUIPEP (1720 K.), that being about 1% lower. The small deviation is a result of the simplified combustion equation assumed in this example. In reality, some trace products such as NH_3 and monatomic K form, consuming energy in the process.

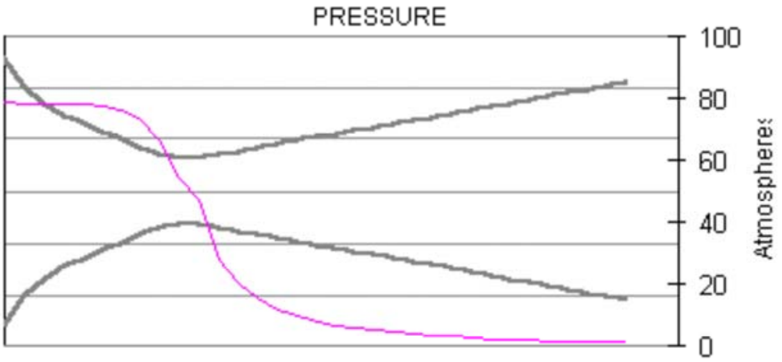
Appendix B

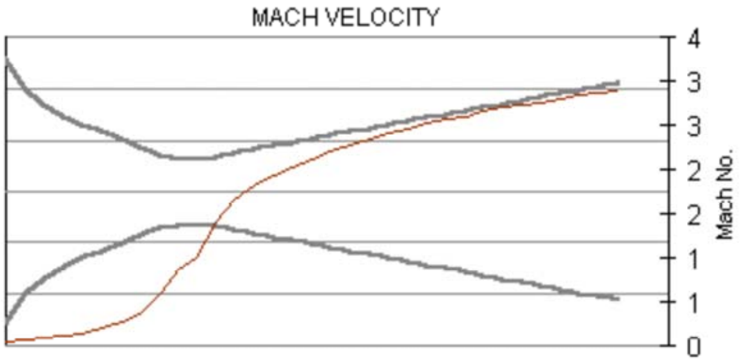
Reserved for future use.

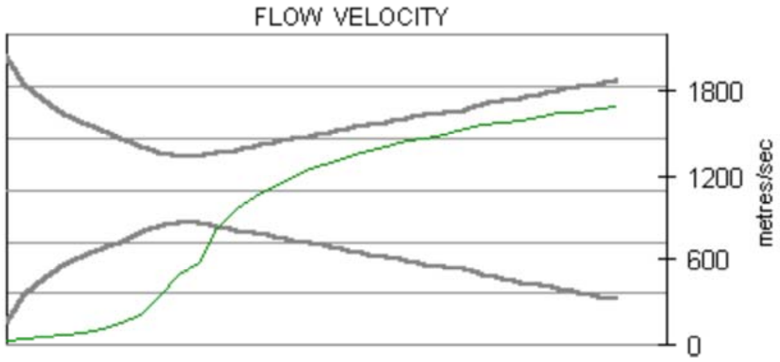
Appendix C

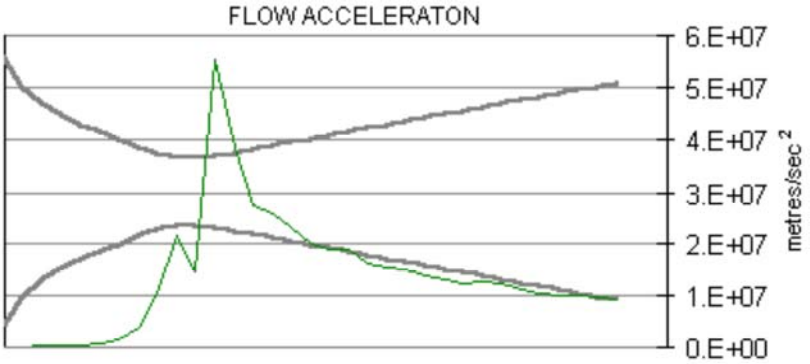
The following are plots of the nozzle flow properties for the Kappa-DX rocket motor:

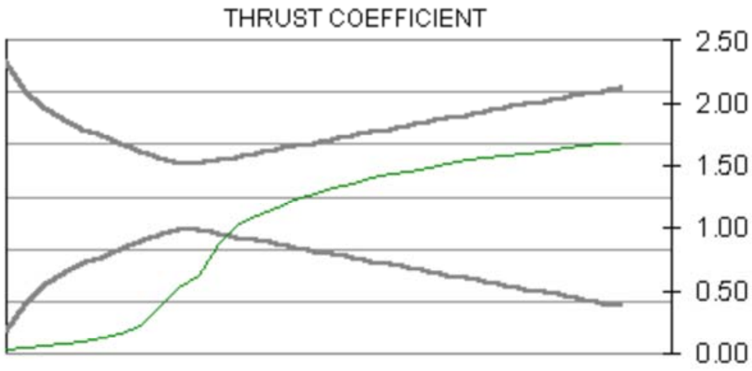












Time for flow to travel through nozzle = 430 microseconds.

Appendix D

The derivation of the expression for **mass flow rate through the nozzle** is presented here.
From *Equation 9* of the Nozzle Theory Web Page, the continuity equation for mass flow rate through the nozzle is given by:

$$\dot{m} = \rho^* \, v^* \, A^*$$

where * designates *critical* (throat) conditions. From *Equation 7* of the referenced web page, the critical flow density may be written as:

$$\rho^* = \frac{\rho_o}{\left(1 + \frac{k-1}{2}\right)^{\frac{1}{k-1}}} = \frac{\rho_o}{\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}}$$

and from *Equations 3 & 4*, the critical (sonic) velocity may be given by:

$$v^* = \sqrt{\frac{2k}{k+1} R T_o}$$

From the ideal gas law, the chamber density may be expressed as:

$$\rho_o = \frac{P_o}{R T_o}$$

Substitutionof this equation and those for critical density and velocity into the mass flow rate expression gives:

$$\dot{m} = \frac{P_o}{R T_o} \frac{\sqrt{\frac{2k}{k+1} R T_o}}{\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}} A^*$$

which may be rearranged to the form of the expression shown as Equation 4 of the Chamber Pressure Theory Web Page:

$$\dot{m}_n = P_o A^* \sqrt{\frac{k}{R T_o}} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$

Appendix E

Example: Calculate the maximum steady-state chamber pressure for the design of the Kappa-DX rocket motor.

Units of measure:

The most prudent (botch-proof) system of units is *mks* (metre : kilogram : second), however, for this example, appropriate English units will used, as well.

Equation 12 of the Chamber Pressure Theory Web Page:

$$P_o = K_n \rho_p \, r \, c^*$$

Burn/throat area	Kn, max. = 378
Propellant density	ρ_p = 1.806 g/cm^3 = 1806 kg/m^3 = 0.00203 slug/in^3
Burn rate	r = 12.65 mm/s = 0.01265 m/s = 0.50 in/s
Propellant c-star	c* = 912 m/s = 2992 ft/s

Therefore,
Po = 378 (1806) .01265 (926) = 7.9 x 10^6 N/m^2 (7.9 MPa)
or
Po = 378 (0.00203) 0.50 (2992) = 1148 psi

Last updated August 19, 2001

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