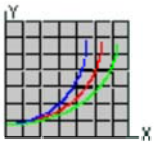
Richard Nakka's *Experimental Rocketry* Web Site



Solid Rocket Motor Theory -- Chamber Pressure

Chamber Pressure

The Chamber Pressure that a rocket motor develops is of crucial importance with regard to the successful operation of a rocket motor. Not only does Chamber Pressure strongly influence propellant burn rate, thermodynamic efficiency and thrust, the Chamber Pressure structurally loads the rocket motor casing and closures to a critical extent. Understanding the nature of Chamber Pressure generation, and accurate prediction of such, is one of the keys to successful rocket motor design.

What causes pressure to develop inside the chamber of a rocket motor? What determines the magnitude of this pressure? Intuitively, the pressure buildup is a result of the combustion of the propellant grain, whereby the gases produced hasten to escape through the nozzle throat. If the throat is sufficiently small, the gases cannot escape quickly enough and the accumulation of gases in the chamber results in pressurization.

In actuality, the intuitive explanation is essentially correct. However, an important factor that determines the *magnitude* of chamber pressure is not at all intuitive -- the concept of *choked flow*. This concept provides for a convenient means to calculate chamber pressure, and is valid for both transient and steady-state modes of motor operation, as discussed below.

By looking at a plot of Chamber Pressure over the operating duration of a rocket motor (Figure 1), one sees that there are three distinct and important phases of operation:

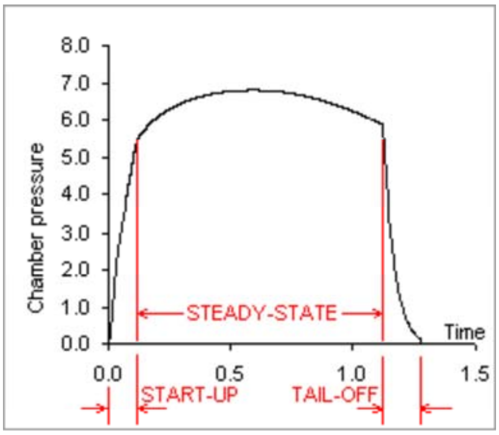


Figure 1 -- Motor chamber pressure

The pressure curve of the rocket motor exhibits *transient* and *steady-state* behaviour. The transient phases are when the pressure varies substantially with time -- during the ignition and *start-up* phase, and following complete (or nearly complete) grain consumption, when the pressure falls down to ambient level during the *tail-off* phase. The variation of chamber pressure during the steady-state burning phase is due mainly to variation of grain geometry (burning surface area) with associated burn rate variation. Other factors may play a role, however, such as nozzle throat erosion and erosive burn rate augmentation.

First of all, the start-up and steady-state pressure phases will be considered. The start-up phase is hypothetically very brief, although in reality, ignition of the complete grain does not occur instantaneously. The actual duration of the start-up phase is strongly dependant upon the effectiveness of the igniter system employed.
The steady-state phase clearly dominates the overall performance of the motor, and as such, constitutes the **design condition**.
In determining the start-up pressure growth, and the steady-state pressure level, it is first noted that the rate of *combustion product generation* is equal to the *rate of consumption of the propellant grain*, given by:

$$\dot{m}_g = A_b \rho_p r \qquad \text{equation 1}$$

where ρ_p is the propellant density, A_b is the grain burning area, and *r* is the propellant burn rate (surface regression rate).
It is important to note that the combustion products may consist of both gaseous and condensed-phase mass. The condensed-phase, which manifests itself as smoke, may be either solid or liquid particles. Only the gaseous products contribute to pressure development. The condensed-phase certainly does, however, contribute to the thrust (overall performance) of the rocket motor, due to its mass and velocity, as shown in equation 1 of the **Thrust Theory** Web page.

The rate at which combustion products are increasingly stored within the combustion chamber is given by:

$$\frac{dM_s}{dt} = \frac{d}{dt}(\rho_o \upsilon_o) = \rho_o \frac{d\upsilon_o}{dt} + \upsilon_o \frac{d\rho_o}{dt} \qquad \text{equation 2}$$

where ρ_o is the instantaneous gas density in the chamber, and υ_o is the instantaneous gas volume (which is equal to the free volume within the chamber).

The change in gas volume with respect to time is equal to the change in volume due to propellant consumption, given by $d\upsilon_o/dt = A_b\, r$. This leads to:

$$\frac{dM_s}{dt} = \rho_o A_b r + \upsilon_o \frac{d\rho_o}{dt} \qquad \text{equation 3}$$

The rate at which combustion products flow through the nozzle throat is limited by the condition of *choked flow*. As described in the **Nozzle Theory** Web Page, the flow achieves sonic (Mach 1) velocity at the narrowest portion of the convergent-divergent nozzle (throat). Flow velocity, at this location, can never exceed the local speed of sound, and is said to be in a *choked* condition. This allows us to determine the rate at which the combustion products flow through the nozzle is given by equation 4: (for derivation, see *Theory Appendix D*)

$$\dot{m}_n = P_o A^* \sqrt{\frac{k}{R\, T_o}} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} \qquad \text{equation 4}$$

Note that R = R'/ M, where *R'* is the *universal gas constant*, and *M* is the *effective molecular weight* of the combustion products. Mass flow rate through the nozzle is seen to be a function of the chamber pressure (which determines the flow density), throat area, and the gas properties (which establish sonic velocity).

The principle of mass conservation requires the balance between *mass generation rate* and the sum of the rates at which *mass storage* in the chamber and *outflow through the nozzle*:

$$\dot{m}_g = \frac{dM_s}{dt} + \dot{m}_n \qquad \text{equation 5}$$

Substituting equations 1 & 3 into equation 5 gives:

$$A_b\, \rho_p\, r = \rho_o A_b r + \upsilon_o \frac{d\rho_o}{dt} + \dot{m}_n \qquad \text{equation 6}$$

Propellant burn rate may be expressed in terms of the chamber pressure by the Saint Robert's law (see *Propellant Burn Rate* Web Page):

$$r = a\, P_o{}^n \qquad \text{equation 7}$$

where *a* and *n* are the burn rate coefficient and pressure exponent, respectively. Substituting equations 7 & 4 (mass flowrate through nozzle) into equation 6 leads to the following equation:

$$A_b\, \rho_p\, a\, P_o{}^n = A_b\, \rho_o\, a\, P_o{}^n + \upsilon_o \frac{d\rho_o}{dt} + P_o A^* \sqrt{\frac{k}{R\, T_o}} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} \qquad \text{equation 8}$$

From the *ideal gas law*, the density derivative in the above equation may be expressed as:

$$\frac{d\rho_o}{dt} = \frac{1}{R\, T_o} \frac{dP_o}{dt} \qquad \text{equation 9}$$

As well, considering that chamber temperature, *To*, is essentially independent of chamber pressure, equation 8 may be re-written as:

$$\frac{\upsilon_o}{R\, T_o} \frac{dP_o}{dt} = A_b\, a\, P_o{}^n \left(\rho_p - \rho_o\right) - P_o A^* \sqrt{\frac{k}{R\, T_o}} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} \qquad \text{equation 10}$$

This is a particularly useful equation, as it allows us to determine the rate of change of chamber pressure (dP_o/dt) during the **transient *start-up* phase** of motor operation, where the chamber pressure is rapidly climbing up to the operating steady-state level. Once the steady-state phase is reached, when the outflow of combustion gases is in equilibrium with the production of gases from propellant consumption, $dP_o/dt = 0$, and the left-hand side of equation 10 vanishes. The steady-state chamber pressure may then be expressed as:

$$P_o = \left[\frac{A_b}{A^*} \frac{a \rho_p}{\sqrt{\frac{k}{R T_o} \left(\frac{2}{k+1} \right)^{\frac{k+1}{(k-1)}}}} \right]^{\frac{1}{(1-n)}} \qquad \text{equation 11}$$

Note that the combustion product density term has been dropped, as it is small in comparison to the propellant density.
Equation 11 may be greatly simplified by use of equation 7, letting $K_n = A_b / A^*$ and by noting that the characteristic exhaust velocity (c-star) is given by:

$$c^* = \sqrt{\frac{R T_o}{k \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}}}$$

This leads to the simplified expression for **steady-state chamber pressure**:

$$P_o = K_n \rho_p r c^* \qquad \text{equation 12}$$

where r is the burn rate at the chamber pressure, P_o.

The third and final phase of the pressure curve, the tail-down phase, ideally occurs immediately after the propellant grain has been completely consumed. In actuality, slivers or fragments of propellant grain remain once the bulk of the grain has been consumed. This results in a pressure tail-down that is more gradual than for the ideal case. However, it is impractical to account for this effect, and the tail-down pressure is determined on the assumption that the grain has been fully depleted.
After burnout, when $A_b = 0$, equation 10 becomes

$$\frac{\upsilon_o}{R T_o} \frac{dP_o}{dt} = - \frac{P_o A^*}{c^*} \qquad \text{equation 13}$$

This differential equation may then be solved to express **tail-off chamber pressure** as a function of bleeddown time for choked flow:

$$P_c = P_{bo} \exp\left(\frac{R T_o A^*}{\upsilon_o c^*} t \right) \qquad \text{equation 14}$$

where P_{bo} is the chamber pressure at burn-out and t is the time from burn-out. The pressure is seen to exhibit exponential decay.

In addition to the consequence of sliver burning during tail-off, *nozzle slagging* will tend to make the pressure decay more gradual than predicted by equation 14. Nozzle slagging is the tendency of condensed-phase (in particular liquid matter) to accumulate around the throat, effectively reducing the diameter. Slagging is more significant during tail-off due to the dropping pressure level and lower exhaust velocity.

An example of steady-state chamber pressure calculation, for the Kappa-DX motor, is provided in Theory Appendix E.

Next -- Two-phase Flow

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