

NUMERICAL ANALYSIS OF A VERTICAL HELICAL EARTH-AIR HEAT EXCHANGER

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Received: Mar 16, 2023

Reviewed: Apr 10, 2023

Accepted: Apr 18, 2023

ABSTRACT

The Earth-Air Heat Exchanger (EAHE) is an equipment that consists of ducts buried in the ground in which the air is forced to pass through. The heat exchange with the surrounding soil turns the air temperature in the outlet section of the EAHE milder. Due to that, the EAHE is capable of assist air conditioning systems and reduce energy consumption, by taking advantage of the temperature gradient established between the soil surface and its layers. In the current study, the operation of Vertical Helical EAHEs was numerically evaluated with different distances between helicoid curves, for the city of Viamão, located in the southern Brazil. The results stated that the Vertical Helical EAHE with dimensions between the curves equivalent to 100 and 200 mm presented the better thermal performances for cooling mode operation in the hottest seasons of the years, when compared to the Conventional Horizontal EAHE adopted as reference. From this comparison, the obtained average values of the Root Mean Squared Error (RMSE) were, respectively, 0.47 °C and 0.66 °C. At last, it must be highlighted a sevenfold reduction in the soil volume occupied by the installation of the Helical EAHE compared to the Conventional Horizontal EAHE.

Keywords: Earth-Air Heat Exchanger (EAHE), Vertical Helical EAHE, Computational Modeling, Thermal Analysis.

NOMENCLATURE

p pressure, Pa
q turbulent flow of energy, m·K/s
T temperature, K
t time, s
v flow velocity, m/s
x displacement, m

Greek symbols

α thermal diffusivity, m²/s
 ρ density, kg/m³
 τ Reynolds Stress Tensor
 ν kinematic viscosity, m²/s
 δ Kronecker delta

Subscripts

i, j unit vectors
s soil

INTRODUCTION

Energy is fundamental for humanity to live in society, for this reason it is very important to find alternative sources that are not related to conventional fossil fuels (Vaz, 2011). One of the fundamental principles of sustainable development is the use of renewable energy sources, and among them the solar radiation stands out as an inexhaustible source of heat and light. A part of this energy is dissipated in the atmosphere, however more than its half falls to the soil surface. Due to its huge mass and thermal isolation properties, the Earth's surface functions as an inertial and cyclic reservoir for this energy (Brum, 2013). Thus, during the cold periods of the year, the underground does not present itself so cold as the air above it, and in hot periods it is not so warm (Vaz, 2011).

In this context, the Earth-Air Heat Exchangers (EAHE) consist in ducts buried in the ground, in which the ambient air is forced to pass through, taking advantage of the soil thermal inertia, functioning as a tool to reduce the energy consumption for the thermal conditioning of buildings (Rodrigues, 2019). In the present work a Vertical Helical EAHE which requires reduced soil

volume for installation was numerically simulated. Its thermal behavior and occupied soil volume were compared to a Conventional Horizontal EAHE adopted as reference. To do so, the installation of the EAHE was considered to be carried out in the city of Viamão, located in the state of Rio Grande do Sul (RS), southern Brazil, in which the soil presents clayey characteristics (Vaz, 2011).

The software ANSYS Fluent, based on the Finite Volume Method (FVM) was used. First, the computational model was validated and verified for the Conventional Horizontal EAHE. Subsequently, a comparison in terms of thermal performance for the Vertical Helical EAHE was established. Lastly, the case of study considering different dimensions between the helicoid curves was performed, being their results compared to the reference installation.

MATERIAL & METHODS

In Vaz (2011) and Vaz et al. (2011) an experimental and numerical analysis of a Horizontal EAHE was developed for the city of Viamão, during the year of 2007. Gambit and Fluent were the software employed for the computational modeling. The computational model validation achieved a maximum error below 15%, providing a data base of experimental and numerical results for further studies.

Besides that, in Brum (2013) the Simplified and Reduced computational models were developed to numerically simulate the working principle of a Conventional Horizontal EAHE. Both of these models were validated and verified with the results of Vaz (2011). Basically, the distinction between these models is the considered soil portion height: being 15 m for the Simplified model (considering the distance comprehended between the soil surface and the depth where the average soil temperature is achieved); meanwhile, for the Reduced model it was considered only 1 m above and the same distance below the depth of EAHE installation.

In Rodrigues et al. (2015a) the influence of boundary conditions to model the behavior of an EAHE was evaluated. Thus, a 2 m distance between the soil walls and ducts was defined as the minimum value which did not jeopardize the results by setting the null heat flux condition set for the domain walls. The Design Construtal and exhaustive search methods were used by Rodrigues et al. (2015b) to numerically evaluate criteria for enhancing the thermal potential of EAHEs. In addition to presenting an optimized geometry the study also defines, by a mesh independence test, a convergence of the results with tetrahedral elements sized by three times the diameter of the EAHE duct for the soil, and the diameter divided by three for the flow domain.

Mathur et al. (2017) developed a comparative analysis between a Spiral EAHE and a Conventional Horizontal EAHE, for cooling and heating

conditions. The coefficient of performance (COP) for the horizontal configuration and spiral, respectively, were equal to 5.94 and 6.24 during the summer. Thus, the study presents the Spiral EAHE as an alternative to reduce the physical space required for the EAHE installation.

A numerical analysis was performed by Rodrigues (2019) to evaluate a Y-shaped Horizontal EAHE, through the Construtal Design method and considering the Energy Performance Indicator. It was noted that the installation placed at 1 m depth, in the city of Rio Grande-RS, is capable to reduce the energy consumption in 75 kWh in the hot periods of the year and 120 kWh in the cold ones.

Vaz et al. (2020a) performed a numerical study to evaluate the Thermal Potential (TP), head loss, and the physical occupation of different geometries of EAHE. The Conventional Horizontal, the T-shaped and up to three U-shaped EAHEs operating in series were the geometries established as case of study. The focus was the installation in the city of Rio Grande-RS, which is a coastal city with saturated and sandy soil characteristics. In Vaz et al. (2020b) it was numerically evaluated the thermal performance of a Vertical U-shaped EAHE, operating with up to three devices in series, in comparison with a Horizontal EAHE with a straight duct. The considered cases of study were the cities of Viamão and Rio Grande, both located in the state of RS. The Vertical U-shaped EAHE presented very similar results when compared to the Horizontal EAHE. However, for the city of Viamão, where the soil has clayey characteristics, the vertical configuration did not present itself with the same capability of increasing the air temperature such as the Horizontal configuration adopted as reference.

Applied Methodology

Using the Simplified computational model (Brum, 2013), initially its validation and verification were performed through the numerical simulation of a Conventional Horizontal EAHE, comparing the achieved results with the ones presented by Vaz et al. (2011).

After that, the thermal performance of a Horizontal Spiral EAHE and a Vertical Helical EAHE in relation to a Conventional Horizontal EAHE adopted in the validation and verification procedures was numerically compared. The purpose here was to show that the computational model adequately simulates these configurations.

Lastly, different configurations of the Vertical Helical EAHE were numerically simulated and compared to a Conventional Horizontal EAHE established as reference. All the simulations were performed considering the installations to be placed in the city of Viamão, such as in Vaz et al. (2011) and Brum et al. (2013). Regarding the construction of the computational domains formed by the soil and the duct, the Solid Edge software was used. The spatial

discretization of domains was performed with Meshing tool of the ANSYS software, generating meshes with tetrahedral volumes sized by $d/3$ for the ducts and $3d$ for the soil (where d is the diameter of the duct), according to Rodrigues et al. (2015b). Then, the boundary conditions were set (pre-processing) and the processing procedure carried out by the ANSYS Fluent software. Electronic spreadsheets were used to post-processing and generation of graphs and tables to evaluate the results.

Mathematical Model

According to Versteeg and Malalasekera (2007), Bejan & Kraus (2003) and Bergman et al. (2011), for the soil the transient temperature field is attained by the solution of the energy conservation equation, given by:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i} \left\{ \alpha_s \frac{\partial T}{\partial x_i} \right\} \quad (1)$$

Regarding the airflow in the duct, which is considered as transient, incompressible, and with turbulent forced convection, the time-averaged conservation equations of mass, momentum and energy are, respectively, described by:

$$\frac{\partial \bar{v}_i}{\partial x_i} = 0 \quad (2)$$

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial (\bar{v}_i \bar{v}_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_j} \delta_{ij} + \frac{\partial}{\partial x_j} \left[\nu \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i} \right) - \tau_{ij} \right] \quad (3)$$

$$\frac{\partial \bar{T}}{\partial t} + \frac{\partial}{\partial x_j} (\bar{v}_j \bar{T}) = \frac{\partial}{\partial x_j} \left(\alpha \frac{\partial \bar{T}}{\partial x_j} - q_j \right) \quad (4)$$

being: T the temperature (K); t the time (s); x a spatial coordinate (m); α_s the soil thermal diffusivity (m^2/s); ν the airflow velocity (m/s); ρ the density (kg/m^3); p the pressure (Pa); δ_{ij} the Kronecker delta; ν the kinematic viscosity (m^2/s); τ_{ij} the Reynolds stress tensor (Pa); α the air thermal diffusivity (m^2/s); q_j the turbulent flow of energy (m-K/s); and $i, j = 1, 2,$ and 3 . Besides that, it is important to mention that the bar above the terms indicates time-averaged quantities, once that the κ - ϵ turbulence model (Launder & Spalding, 1972) was adopted.

Numerical Model

The mathematical model was numerically solved by the ANSYS Fluent software, which is based on the FVM. With regard to the transient solution, the following were adopted: the first order upwind advection scheme for the advective terms; the Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) for the pressure-velocity coupling; and residuals equivalent to 1×10^{-3} for the conservation equations of mass and momentum, and 1×10^{-6} for the energy conservation. All the simulations were processed with 17520 time steps with the size of 1 h (3600 s), which is equivalent to two simulated years. However, only the second-year results are considered, being the first simulated year used to stabilize the soil temperature (Vaz et al., 2011; Brum et al., 2013). Concerning the boundary conditions, the following ones were set: null heat flux in the sides and bottom walls of the soil portion; temperature prescribed on the soil surface and inlet section of duct, representing the annual temperature variation which were fitted experimental data obtained in the year of 2007 (Vaz, 2011); prescribed velocity in the duct inlet, equivalent to 3.3 m/s (Vaz, 2011); and atmospheric pressure in the duct outlet (Brum et al., 2013). Moreover, the initial temperature of the computational domain was set to be the average soil temperature, equivalent to 18.7 °C (291.7 K), as in Vaz et al. (2011).

Computational Domains

The airflow as considered to pass directly through the holes in the soil, which means that the thickness and material of the ducts were disregarded. These considerations avoid problems related to mesh generation, being a simplification adopted by several references, such as Ascione et al (2011), Vaz et al. (2011), Brum et al. (2013), and Rodrigues (2019). The air and the clayey soil were considered to be isotropic and homogeneous materials. Their properties are presented in the Table 1, besides the air dynamic viscosity equivalent to 1.79894×10^{-5} kg/m-s.

Table 1. Thermo-physical properties of the air and soil.

Material	Density (kg/m^3)	Thermal Conductivity ($\text{W}/\text{m}\cdot\text{K}$)	Specific Heat ($\text{J}/\text{kg}\cdot\text{K}$)
Soil	1800	2.1	1780
Air	1.16	0.0242	1010

The computational domains were developed with a soil height (H) of 15 m, ensuring that on the lower surface of the soil temperature is constant and equal to the average soil temperature (Vaz et al., 2011; Brum et al., 2013). In addition, as presented by Rodrigues et al. (2015a), a 2 m distance was kept between the duct and the domain walls. For the procedures of validation and verification, the adopted computational domain is presented by Figure 1, being

designed with a total length $L_s = 25.77$ m, such as in Brum et al. (2013), and with a width equivalent to $W = 4.11$ m. The duct was considered to be placed at a depth equivalent to 1.6 m, and its diameter equivalent to 110 mm (Vaz et al., 2011).

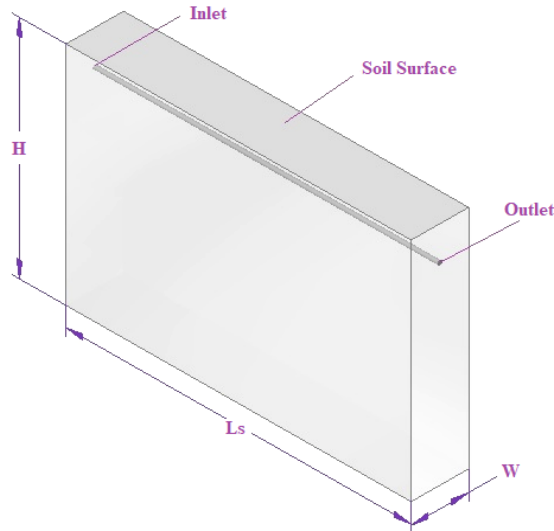


Figure 1. Conventional Horizontal EAHE.

Regarding the analysis of the Horizontal Spiral EAHE, the duct was kept with the 110 mm (see Fig. 2a), being adopted a 1 m distance between the curves of the spiral, according to Mathur et al. (2017). The external diameter of the spiral was equal to 5.675 m, and consequently the diameter of the soil portion was 9.675 m. The installation depth was also 1.6 m. In turn, the Vertical Helical EAHE (see Fig. 2b) was constructed with a 110 mm diameter, distance between the helical curves equal to 500 mm, total diameter of 1.45 m, and total depth of 2.8 m. The soil portion achieved a diameter equal to 5.45 m. It should be noted that in the Fig. 2 cases the total length of the ducts was approximately 25.77 m, allowing a coherent comparison with the Conventional Horizontal EAHE used in the validation and verification procedures.

At last, the case study was carried out for a Vertical Helical EAHE, installed in a borehole with a diameter of 400 mm and 3 m depth, which are the possible dimensions to be attained by manual soil drilling tools. Thus, considering the possible dimensions for the holes in the ground, 50 mm duct was established for the EAHE, considering different dimensions between the helical curves: 100 mm, 200 mm, 300 mm and 400 mm. Due to that, the simulated Vertical Helical EAHE had different total lengths according to the distance between the curves, which were, respectively, equal to: 41.19 m; 22.54 m; 16.22 m; and 12.40 m. The variation of dimensions did not change in the installation depth, which was held constant at 3 m. Figure 3(a) presents the computational domain, in which the portion of soil

has $H = 15$ m and $W = 4.4$ m. Meanwhile, the Figs. 3(b)-3(e) show, in detail, the geometric configurations of the four evaluated dimensions. Their results were compared to the Conventional Horizontal EAHE (see Fig. 1), with diameter of 50 mm and placed at 3 m depth, according to Brum et al. (2013).

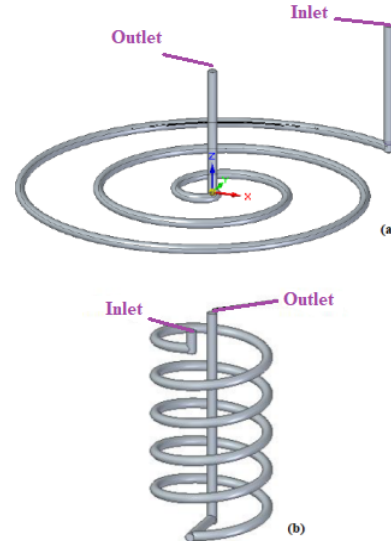


Figure 2. Ducts details of the EAHE:
(a) Horizontal Spiral and (b) Vertical Helical.

It should be noted that to establish the comparison between the studied cases, it was considered the Root Mean Squared Error (RMSE), that is, the square root of the average error. In addition, the TP, which is defined by the temperature difference between the inlet and the outlet of the EAHE (Rodrigues et al., 2015b), was also measured and compared.

RESULTS AND DISCUSSION

Initially, the validation and verification procedures of the numerical model are presented. Then, the results of the Horizontal Spiral and Vertical Helical EAHE. Finally, the results for the case study of the Vertical Helical EAHE are presented.

Validation and Verification of the Computational Model

Figure 4 illustrates the temperature variation in the outlet of the simulated Conventional Horizontal EAHE (see Fig. 1) in comparison with the experimental and numerical results of Vaz et al. (2011), achieving RMSE values of 1.82 °C and 0.71 °C, respectively, validating and verifying the computational model. In addition, in Fig. 4 one can note that the thermal performance of the EAHE is in agreement with the reference results.

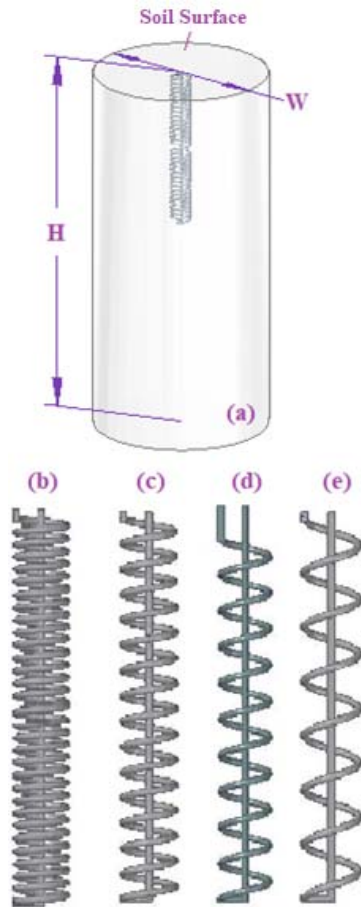


Figure 3. Vertical Helical EAHE: (a) computational domain; and detail of the different dimensions between the helical curves, equal to: (b) 100 mm, (c) 200 mm, (d) 300 mm, and (e) 400 mm.

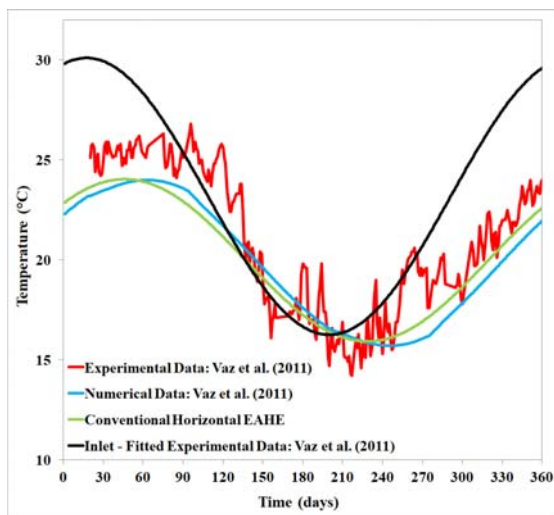


Figure 4. Validation and verification of the computational model.

Thermal Performance Evaluation of the Spiral Horizontal and Vertical Helical EAHE

With the reference based in the Conventional Horizontal EAHE (see Fig. 1) used in the validation and verification procedures, the Horizontal Spiral EAHE (see Fig. 2a) and the Vertical Helical EAHE (see Fig. 2b) presented, respectively, RMSE values of 0.68 °C and 1.21 °C. Thus, it is possible to infer that the computational model is capable to adequately simulate the thermal performance of the EAHE with spiral and helical geometries (see Fig. 5), showing a similar trend with those of the reference installation.

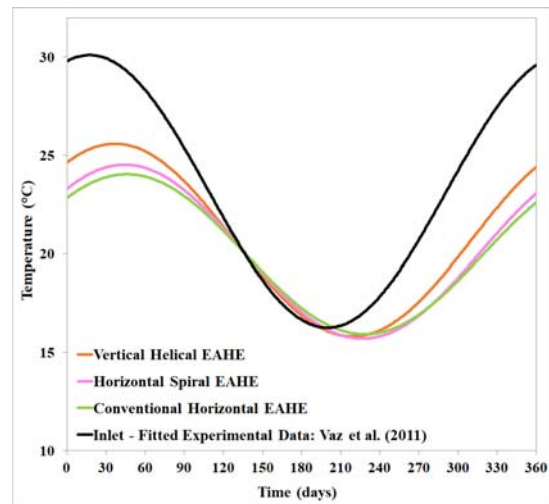


Figure 5. Thermal Performance evaluation of the Horizontal Spiral and Vertical Helical EAHE.

Case Study

The results of the outlet air temperature variation of the Vertical Helical EAHE with different dimensions between the curves (see Fig. 3) were compared to the ones presented by the Conventional Horizontal EAHE adopted as reference, as it is possible to see in Fig. 6. As expected, one can observe that the four helical EAHE proposed presented an analogous thermal behavior, being in agreement with the reference installation (especially in hot periods).

In addition, Table 2 presents the monthly calculated RMSE values of the studied Vertical Helical EAHE with different distances between the curves, in comparison with the reference installation.

From the results shown in Table 2, one can note that during the period comprehended between the months of April and September the higher values of RMSE are attained. The explanation to this fact is that during the colder months the Vertical Helical EAHE reached results of outlet temperature very close to the values in its entrance, as can be seen in Fig. 6.

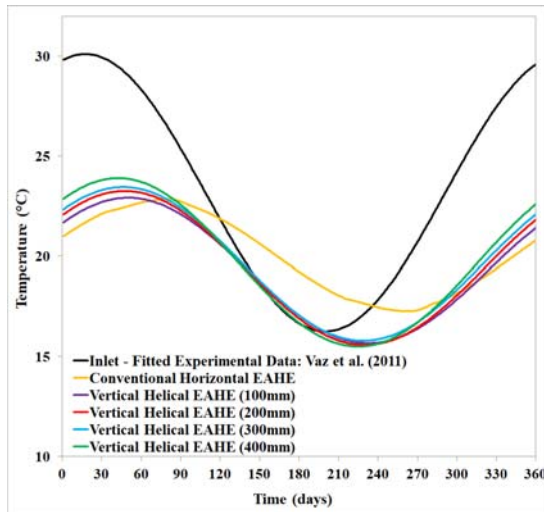


Figure 6. Temperature variation of the Vertical Helical EAHE and Horizontal EAHE.

Table 2. Monthly RMSE values (in °C) obtained by the comparison of the Vertical Helical EAHEs with the Conventional Horizontal EAHE.

Distance/ Month	Jan.	Feb.	Mar.	Apr.	May	June
100 mm	0.67	0.47	0.35	0.96	1.15	2.07
200 mm	1.06	0.81	0.30	0.87	1.13	2.19
300 mm	1.30	1.01	0.35	0.74	1.15	2.05
400 mm	1.82	1.45	0.57	0.73	1.06	2.36
Distance/ Month	July	Aug.	Sept.	Oct.	Nov.	Dec.
100 mm	1.98	1.17	1.28	0.59	0.19	0.52
200 mm	2.00	1.21	1.28	0.48	0.43	0.87
300 mm	1.99	1.08	1.04	0.27	0.69	1.14
400 mm	1.93	1.22	1.15	0.30	1.04	1.60

This behavior is presented through the TP values shown in Fig. 7. Null or very close to zero values for heating TP of the Vertical Helical EAHE were found, at the same time the Conventional Horizontal EAHE reached values between 1 °C and 3 °C. However, for cooling, the TP of the helical configuration were 7 °C up to 8 °C, and for the horizontal one almost 9 °C.

It should be noted that the Vertical Helical EAHE with 100 mm between its curves presents a total length around 70% larger than the installation with 200 mm. Although, there's a maximum TP difference of 0.4 °C for cooling operation. Thus, it is

possible to recommend the Vertical Helical EAHE with 200 mm, without a significant TP reduction.

With regard to the occupied soil volume, the Vertical Helical EAHE, regardless of the dimension between its curves, requires a total volume equal to 228.08 m³ while the Conventional Horizontal EAHE demands 1565.53 m³. Thus, a reduction of 6.86 times in the occupied soil volume for installation was reached for the Vertical Helical EAHE, in comparison with the reference installation.

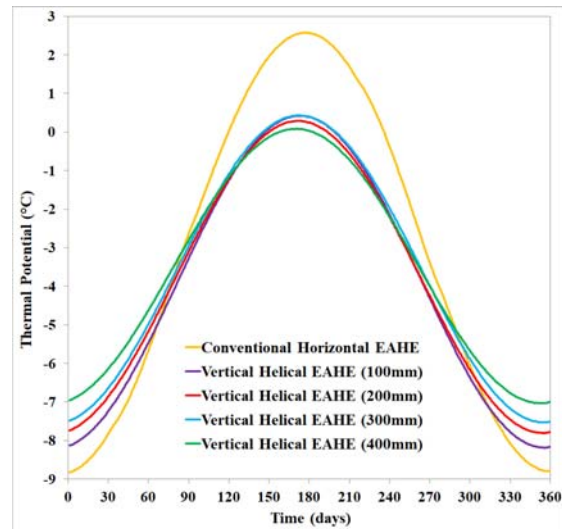


Figure 7. TP variation of the Vertical Helical EAHE and the Conventional Horizontal EAHE.

CONCLUSIONS

In the current work, it was developed an analysis based on a validated and verified computational model, being the purpose to evaluate the thermal performance of Vertical Helical EAHE considering different dimensions between the helical curves, in the city of Viamão, in the year of 2007. To do so, it was adopted as reference a Conventional Horizontal EAHE. It was concluded that the Vertical Helical EAHE with 100 mm distance between the curves presented an average RMSE value, for cooling operation, equal to 0.47 °C when compared to the reference installation. Although, it is recommended the installation of the EAHE with a 200 mm distance, which achieved the RMSE value of 0.66 °C but with 70% less total length of ducts. In terms of TP, a maximum reduction of 0.4 °C was observed by comparing the Vertical Helical EAHE with 100 mm and 200 mm.

Also, the inability of the Vertical Helical EAHE of operating for air heating in cold periods of the year when simulated for the conditions of Viamão, is in agreement with the results presented by the Vertical U-Shaped EAHE of Vaz et al. (2020b).

Finally, it was noted that the Vertical Helical EAHEs installation takes almost seven times less soil

volume than the reference one, being recommended specially for urban areas where the physical space is very limited.

For future studies, it is intended to evaluate the viability of Vertical Helical EAHE in coastal regions where the soil has saturated conditions.

ACKNOWLEDGEMENTS

The authors would like to thank the Fundação de Amparo à Pesquisa do Rio Grande do Sul (FAPERGS), process: 21/2551-0002231-0, and the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), processes: 307791/2019-0, 308396/2021-9, and 309648/2021-1, for the financial support for the development of this research.

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